33)

Blatt 10

a)

mit $\iiint |\psi_{2,0}|^2 d\tau = \iiint |\psi_{2,1,0}|^2 d\tau = 1$

und $\iiint \psi_{2,0}\, \psi_{2,1,0}\, d\tau = 0$

$$\Rightarrow \iiint |\psi_-|^2 d\tau = \cos^2\beta + \sin^2\beta = 1$$

$$\iiint |\psi_+|^2 d\tau = \cos^2\beta + \sin^2\beta = 1$$

$$\iiint \psi_+ \psi_- \, d\tau = \cos\beta \sin\beta - \sin\beta\cos\beta = 0$$

b)

$$\langle p_{el,z} \rangle_+ = -e \iiint \psi_+ \, z \, \psi_+ \, d\tau$$

$$= -e \Big\{ \sin^2\beta \underbrace{\iiint z\, \psi_{2,0}^2 \, d\tau}_{0} + \cos^2\beta \underbrace{\iiint z\, \psi_{2,1,0}^2 \, d\tau}_{0}$$

$$+ 2 \sin\beta \cos\beta \underbrace{\iiint \psi_{2,0}\, z\, \psi_{2,1,0} \, d\tau}_{\text{Nebenrechnung:}}$$

$$\iiint \psi_{2,0}\, z\, \psi_{2,1,0}\, d\tau = \langle z \rangle_+$$

$$= \iiint 2\left(\frac{1}{2a_0}\right)^{\frac{3}{2}} \left(1 - \frac{r}{2a_0}\right) e^{-\frac{r}{2a_0}} \frac{1}{\sqrt{4\pi}}\, r\cos\theta$$

$$\cdot \sqrt{\frac{3}{4\pi}} \cos\theta \, \frac{2}{\sqrt{3}} \left(\frac{1}{2a_0}\right)^{\frac{3}{2}} \frac{r}{2a_0}\, e^{-\frac{r}{2a_0}}\, r^2 dr \, \sin\theta \, d\theta \, d\varphi$$

$$= \underbrace{\iint \frac{1}{\sqrt{4\pi}} \cos\theta \sqrt{\frac{3}{4\pi}} \cos\theta \, \sin\theta \, d\theta \, d\varphi}_{A} \cdot \int 2\left(\frac{1}{2a_0}\right)^{\frac{3}{2}} \left(1 - \frac{r}{2a_0}\right) e^{-\frac{r}{2a_0}}$$

$$\cdot r \cdot \frac{2}{\sqrt{3}} \left(\frac{1}{2a_0}\right)^{\frac{3}{2}} \frac{r}{2a_0}\, e^{-\frac{r}{2a_0}}\, r^2 dr$$

$$A = \frac{\sqrt{3}}{4\pi}\, 2\pi \int_0^{180^\circ} \cos^2\theta \, \sin\theta \, d\theta = \frac{\sqrt{3}}{2} \int_{-1}^{+1} u^2 du = \frac{\sqrt{3}}{2} \left.\frac{u^3}{3}\right|_{-1}^{+1}$$

$$= \frac{1}{\sqrt{3}}$$

$$\langle z \rangle_+ = \frac{4}{3}\left(\frac{1}{2a_0}\right)^3 \int e^{-\frac{r}{a_0}} \left(1-\frac{r}{2a_0}\right)\frac{r^4}{2a_0}\, dr$$

$$= \frac{1}{6a_0^3}\int_0^\infty \left(\frac{r^4}{2a_0} - \frac{r^5}{(2a_0)^2}\right) e^{-\frac{r}{a_0}}\, dr$$

$$= \frac{a_0}{12}\int_0^\infty \frac{r^4}{a_0^4} e^{-\frac{r}{a_0}} \frac{dr}{a_0} - \frac{a_0}{24}\int_0^\infty \frac{r^5}{a_0^5} e^{-\frac{r}{a_0}} \frac{dr}{a_0}$$

Hinweis $\Rightarrow$ $= \frac{a_0}{12}\, 4! - \frac{a_0}{24}\, 5! = -3a_0$

$$\langle p_{el,z} \rangle_+ = 2e \sin\beta \cos\beta\, 3a_0$$

$$\langle p_{el,z} \rangle_- = -\langle p_{el,z} \rangle_+ = -2e \sin\beta \cos\beta\, 3a_0$$

c) Maximal für $2\sin\beta\cos\beta = \sin 2\beta \overset{!}{=} 1 \Rightarrow \beta = 45°$

$W_{el} = -p_{el,z} E_z$ mit $\langle p_{el,z} \rangle_\pm = \pm 3ea_0$

35) Klausuraufgabe:

Moseleysches Gesetz: $E_n = - hc R_\infty \frac{(z-\sigma)^2}{n^2}$

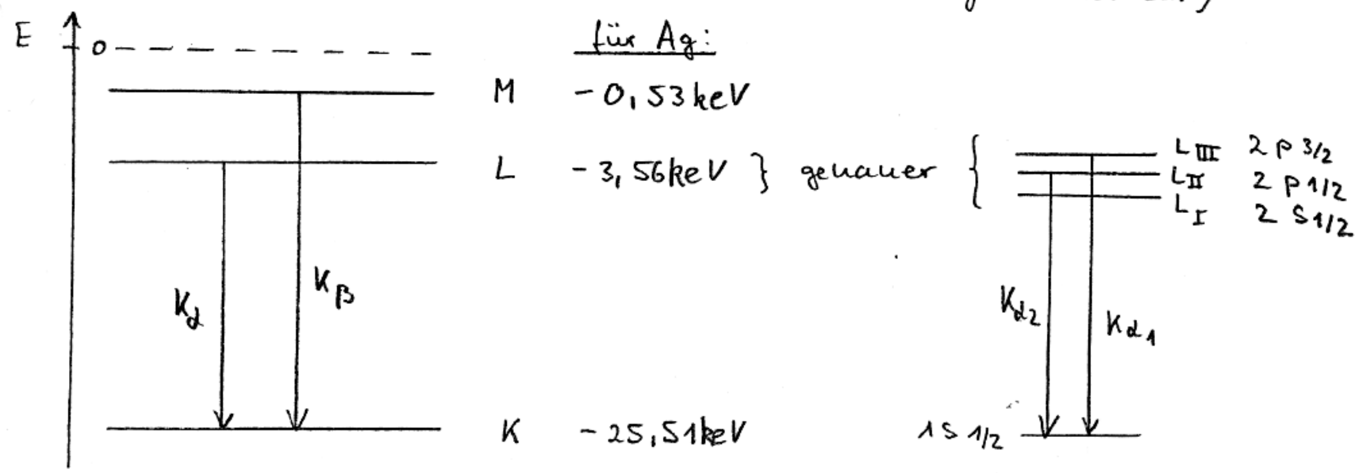
$z = 26$

$\sigma_K = 1$

$\Delta E_{n=2 \to n=1} = \frac{3}{4} hc R_\infty (z-1)^2 = 6375 \text{ eV}$

34)

Skizze in Aufgabe: – K_α, K_β: charakteristische Linien (für jew. Element)



λ_{min}: $W_{kin} = \frac{1}{2} m v^2 = W_{pot} = eU$

$W_{kin} \stackrel{!}{=} h\nu$

$\Leftrightarrow \quad eU = h\frac{c}{\lambda} \quad \Leftrightarrow \quad \lambda = \lambda_{min} = \frac{hc}{eU}$

$\underline{\underline{\lambda_{min} = 35{,}5\,pm}}$ mit $eU = 35\,keV$

λ_{min} unabhängig vom Element der Anode (bis auf Unterschied in Austrittsarbeit)

$\frac{hc}{\lambda_{K\alpha}} = |\Delta W_{LK}| \rightsquigarrow \lambda_{K\alpha} = \frac{hc}{(25{,}51 - 3{,}56)\,keV} \Rightarrow \underline{\underline{\lambda_{K\alpha} = 56{,}6\,pm}}$

$\frac{hc}{\lambda_{K\beta}} = |\Delta W_{MK}| \rightsquigarrow \lambda_{K\beta} = \frac{hc}{(25{,}51 - 0{,}53)\,keV} \Rightarrow \underline{\underline{\lambda_{K\beta} = 49{,}7\,pm}}$