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Exercises for Modern Experimental Physics III (Experimental Particle and Astroparticle Physics)

Summer term 2024

Exercise sheet Nr. 1 To be worked on until 02.05.2024

Exercise 1: Orders of magnitude and center-of-mass energy (5 points)

a) In everyday life you have been used to work with the so-called SI units. In particle physics it is customary to describe the processes that occur in other units. For example, energies are expressed in electron volts (eV) instead of the SI unit joule (J). Convert the specified energies into eV and enter the wavelengths for photons of this energy in meters (m). Which microscopic systems can be resolved with photons of the corresponding wavelengths?

- 10^{-16} J
- $4.5 \cdot 10^{-15}$ J
- 10^{-10} J

b) Calculate the center-of-mass energy of particle reactions at following particle accelerators:

Accelerator	Colliding particles	Energies
LEP 2	e^+e^-	$E_{e^+} = E_{e^-} = 103\,\mathrm{GeV}$
SuperKEKB	e^+e^- (crossing angle 83 mrad)	$E_{e^+} = 4\,\mathrm{GeV}, E_{e^-} = 7\,\mathrm{GeV}$
HERA	$e^\pm p$	$E_e = 27.5\,\mathrm{GeV}, E_p = 920\,\mathrm{GeV}$
	ep	$E_e = 27.5\,\mathrm{GeV}$, p at rest
Tevatron Run II	$p\bar{p}$	$E_p = E_{\bar{p}} = 980\,\mathrm{GeV}$
LHC Run 3	pp	$E_p = 6.8\,\mathrm{TeV}$
	${}^{208}_{82}\mathrm{Pb}\ {}^{208}_{82}\mathrm{Pb}$ (lead nuclei)	$E = 5.12\,\mathrm{TeV}$ per nuclei

What approximation can be made for these energies?

Consider the collision of an accelerated lead nucleus with another stationary lead nucleus. What is the energy needed for the lead nucleus to achieve the same center-of-mass energy as in PbPb collisions at the LHC?

Exercise 2: Mandelstam variables (5 points)

Consider a reaction between two particles with initial four-momenta p_1 and p_2. The outgoing particles have four-momenta p_3 and p_4. The reaction should be considered once in the center-of-mass frame and once in the rest frame of particle 2, which should coincide with the laboratory frame ("fixed-target experiment"). Assuming small masses of the particles ($m \ll E$), show that the following relationships apply (in natural units):

a) $s = (2E_1^*)^2 = 2m_2E_1$

b) $t = \frac{s}{2}(\cos\theta^* - 1) = 2m_2(E_3 - E_1)$

c) $u = \frac{s}{2}(\cos\theta^* + 1) = 2m_2E_3$

d) $s + t + u = \sum_{i=1}^{4} m_i^2$

Variables with * refer to the center-of-mass frame and variables without refer to the laboratory frame. Let θ be the scattering angle of particle 1, i.e., the angle between p_1 and p_3.

Exercise 3: Rapidity and pseudorapidity (5 points)

For a given mass, the kinematics of a particle are completely determined by three independent quantities. In particle physics, the variables p_{T}, ϕ and y are generally used for this purpose. Here, p_{T} corresponds to the transverse momentum, ϕ to the azimuthal angle, and y to the rapidity of the particle. The quantities p_{T} and ϕ are determined in the plane perpendicular to the scattering axis. The scattering axis corresponds the z-axis of the coordinate system used. The rapidity is defined as

$$y = \frac{1}{2}\ln\left(\frac{E + p_z}{E - p_z}\right) , \quad (1)$$

with the energy E and the momentum along the z-axis p_z. If the energy is very large compared to the mass of the particle ($E \gg m$) one can approximate the rapidity y with the pseudorapidity

$$\eta = -\ln\left(\tan(\theta/2)\right) , \quad (2)$$

where θ is the scattering angle of the particle with respect to the beam axis.

a) Show that y is form-invariant under Lorentz boosts along the z-axis. What does form invariance mean in this context?

b) Confirm by calculation that y for $E \gg m$ approaches η. Calculate y and η for the following particles, as they often occur in collisions at the LHC:

Particle	Mass	Energy	Scattering angle
Charged pion ($\pi^\pm$)	140 MeV	10 GeV	5°
W boson ($W^\pm$)	80.4 GeV	300 GeV	10°
Top quark (t)	172.5 GeV	300 GeV	10°

<u>Exercise 4</u>: Standard model processes (5 points)

a) Consider the following particle production processes:

- $e^+e^- \to \mu^+\mu^-$
- $e^+e^- \to u\bar{t}$
- $e^+e^- \to \nu_\tau\bar{\nu}_\tau$
- $pp \to K^+\Sigma^+$
- $pn \to \Lambda^0\Sigma^+$

Which of these interactions is possible according to the rules of the standard model? If the process is allowed draw the corresponding Feynman diagram, if not explain why this process is forbidden.

b) Similarly, explain if the following particle decays are possible or not, and if they are allowed draw the Feynman diagram.

- $\mu^- \to e^-\gamma$
- $\tau^- \to e^-\bar{\nu}_e\nu_\tau$
- $\Sigma^+ \to K^+\Lambda^0$
- $\Xi^- \to n\pi^-$
- $p \to e^+\pi^0$

Hint: use the website of the Particle Data Group to look up the particles and their properties if you are not familiar with them anymore.