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Exercises for Modern Experimental Physics III (Experimental Particle and Astroparticle Physics)

Summer term 2024

Exercise sheet Nr. 5 To be worked on until 18.07.2024

The principle of an *electrostatic filter with magnetic adiabatic collimation* (MAC-E filter) has been used successfully for many years to determine the mass of the electron neutrino (ν_e) [1, 2]. The **KA**rlsruher **TRI**tium **N**eutrino experiment [3] aims to achieve a sensitivity of better than 0.3 eV to the mass of the electron neutrino – an improvement of an order of magnitude over the sensitivity of previous "neutrino scales".[1] A MAC-E filter with 9.8 m diameter and just under 24 m length is used for this purpose. Figure 1 shows the principle of an MAC-E filter at the KATRIN experiment schematically. It is an instrument for high-precision measurement of the kinetic energy of charged particles (spectrometer). The photo in Fig. 2 shows this main spectrometer being transported to the experimental site at KIT Campus North in November 2006.

Exercise 1: MAC-E Filter (10 points)

a) Describe and discuss the operating principle of the MAC-E filter. Characterise the trajectory of an electron on its path through the spectrometer. Discuss the splitting of the kinetic energy into the various components

$$\begin{aligned} E_\mathrm{kin} &= E_\parallel + E_\perp \\ &= E_\mathrm{kin}\cdot\cos^2\theta + E_\mathrm{kin}\cdot\sin^2\theta. \end{aligned}$$

Where $\theta = \angle(\vec{B}, \vec{p})$ is the angle between the magnetic field vector $\vec{B}$ and the instantaneous momentum vector $\vec{p}$ of the electron.

[1] Recently, the KATRIN experiment has improved its upper limit on the neutrino mass to 0.45 eV (see Ref. [4]).

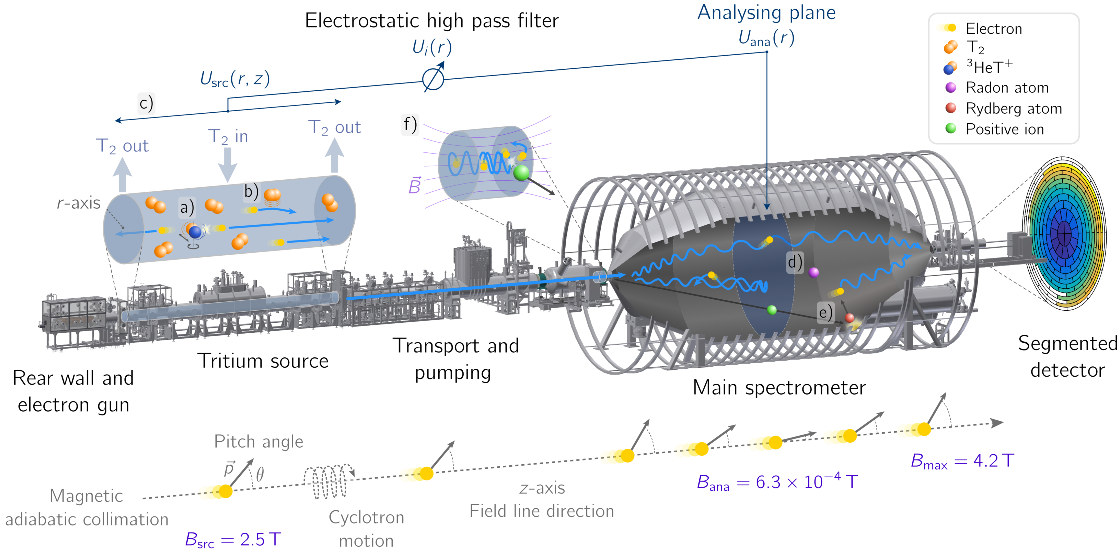

Figure 1: Illustration of the KATRIN main spectrometer within the beam line as an example for an electrostatic filter with magnetic adiabatic collimation: *top:* Overview of the electrode and magnet system, *below:* Evolution of the momentum vector $\vec{p}$ of an electron as it passes through the spectrometer under the influence of magnetic collimation.

Figure 2: Current application example of a MAC-E filter: the main spectrometer of the KATRIN experiment.

To understand the "adiabatic collimation" of the momentum, use the fact that the magnetic moment of the electron is

$$\mu = |\vec{\mu}| = \frac{e}{2m_e}|\vec{\ell}| = \frac{E_\perp}{B} \tag{1}$$

(in non-relativistic approximation, $\vec{\ell}$ denotes the orbital angular momentum) is a conservation law.

b) How are the electric field $\vec{E}$ and the magnetic field $\vec{B}$ aligned in the central cylindrical part of the spectrometer?

c) The arrows in the lower part of Fig. 1 only show the momentum of the electron correctly if the influence of the electrostatic field is neglected and only the magnetic collimation is considered. How should the representation of the arrows be adapted if the effect of the electrostatic field is added?

d) Sketch each of the following

- the course of the electric potential $\phi(z)$ and the magnetic field strength $B(z)$ along the length axis z in the spectrometer,
- the trajectory $r(z)$ of an electron with starting angle $\theta = 45°$ through the entire spectrometer, and
- the course of the longitudinal kinetic energy $E_{\|}(z)$ of the electron from the entrance to the exit of the spectrometer.

Assume that the electron has just enough kinetic energy to just overcome the potential barrier qU_0 in the centre of the spectrometer. Set up a condition for an electron to pass from the source to the detector (so-called "transmission condition"). Use the start energy $E_{\text{kin, start}}$ and the start angle θ_{start}.

e) Please reason why the MAC-E filter can also be described as a high-pass filter for electrons.

f) Can you think of a way to convert this *high-pass filter* into a *band-pass filter*?

g) Determine an expression for the relative energy resolution $\Delta E/E$. Use Eq. (1) from subtask (a) and the original design values for the magnetic fields: $B_{\text{src}} = 3.6\,\text{T}$, $B_{\text{max}} = 6\,\text{T}$, $B_{\text{ana}} = 3 \times 10^{-4}\,\text{T}$, and $B_{\text{detector}} = 5.6\,\text{T}$.

- What condition on the magnetic field results from this in order to achieve a resolution of $\Delta E \lesssim 1\,\text{eV}$ for particles with an energy of $E = 18.6\,\text{keV}$ (corresponding to the maximum energy of electrons from tritium β decay), as is required for KATRIN?
- What effects does this have on the required dimensions of the spectrometer? Use the fact that, in addition to the magnetic moment μ, the magnetic flux

$$\Phi_{\text{mag}} = \int \vec{B} \cdot \mathrm{d}\vec{A}$$

is a conservation quantity and note that the following value for $\Phi_{\text{mag, source}}$ was chosen for the tritium source (for technical reasons):

$$\Phi_{\text{mag, source}} = 191\,\text{T}\,\text{cm}^2. \tag{2}$$

- What technical difficulties arise from these requirements?

Exercise 2: Active and Sterile Neutrinos (10 points)

This exercise will give a first hands-on introduction to Beyond Standard Model searches at KATRIN. The tasks are provided in form of a jupyter notebook, which you can find on the git repository. For this exercise we will use the `Datenanalyse` container on the jupytermachine.

- Influence of the neutrino mass
- Differential tritium spectrum
- Sterile neutrino mass search and statistical sensitivity

References

[1] V. M. Lobashev and P. E. Spivak, "A method for measuring the electron antineutrino rest mass", *Nucl. Instrum. Meth. A* **240** (1985) 305–310. `doi:10.1016/0168-9002(85)90640-0`.

[2] A. Picard et al., "A solenoid retarding spectrometer with high resolution and transmission for keV electrons", *Nucl. Instrum. Meth. B* **63** (1992), no. 3, 345–358. `doi:10.1016/0168-583X(92)95119-C`.

[3] KATRIN Collaboration, "Direct neutrino-mass measurement with sub-electronvolt sensitivity", *Nature Phys.* **18** (2022), no. 2, 160–166, `arXiv:2105.08533`. `doi:10.1038/s41567-021-01463-1`.

[4] M. Aker et al., "Direct neutrino-mass measurement based on 259 days of KATRIN data", `arXiv:2406.13516`.