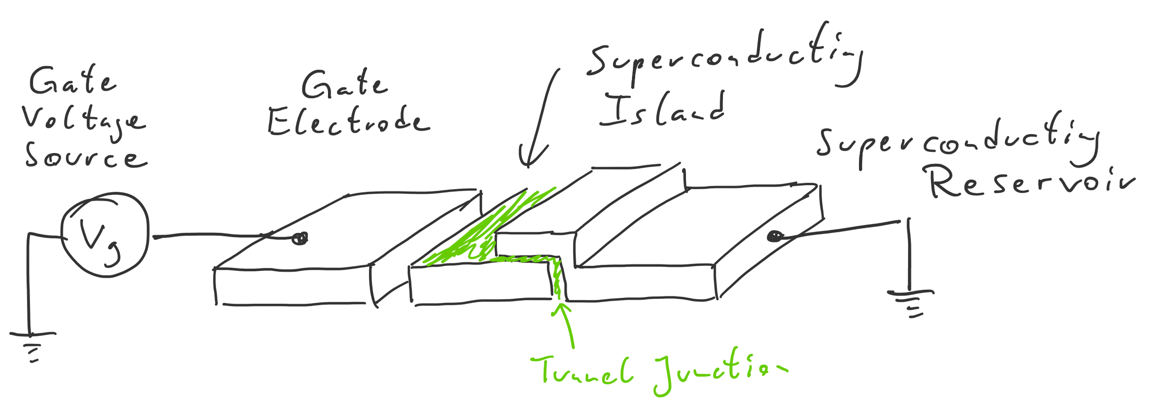
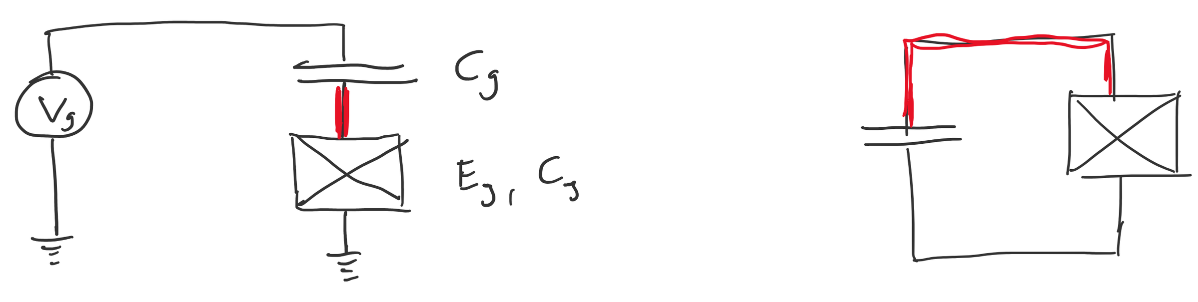
Vorlesung 16.8

16.8 Cooper Pair Box (CPB) (late 90's)

- charge qubit: charge degree of freedom is used for coupling and interaction
- CPB consists of a superconducting island into which Cooper pairs may tunnel via a Josephson junction





Hamiltonian

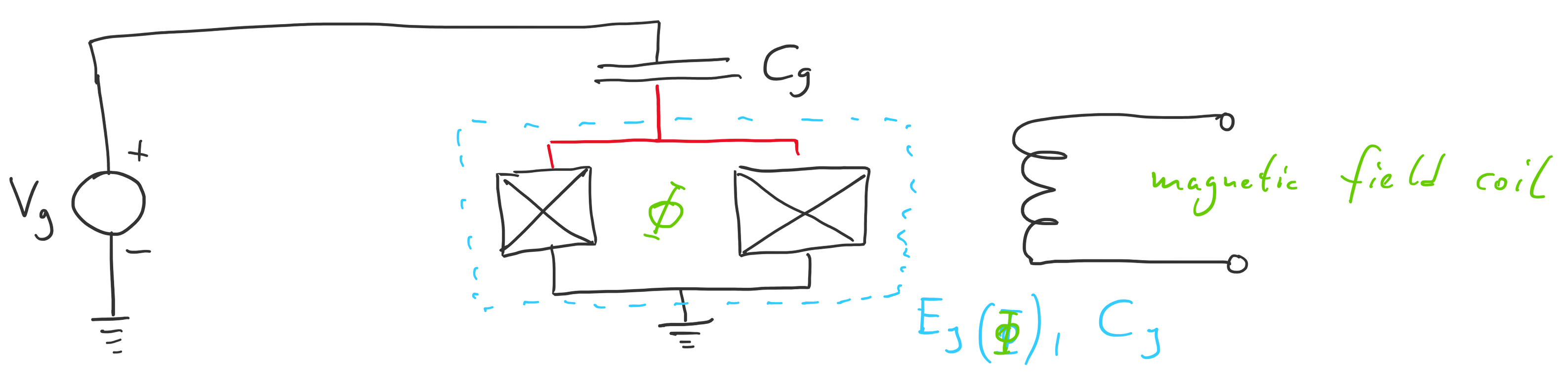
$$\mathcal{H} = \underbrace{\frac{(Q_J - C_g V_g)^2}{2\,C_\Sigma}}_{\text{capacitive / charging energy}} - \underbrace{E_J \cos\delta}_{\text{Josephson inductive energy}} \qquad C_\Sigma = C_g + C_J$$

δ: phase across the junction
Q_J: charge of the island

- with charging energy scale $E_c = \frac{e^2}{2\,C_\Sigma}$
  number of Cooper pairs $\boxed{n = Q_J/2e}$ $\quad n_g = \frac{C_g V_g}{2e}$

$$\mathcal{H} = 4\,E_c\,(n - n_g)^2 - E_J \cos\delta$$

Split Junction – split Cooper Pair Box



- two parallel junctions replace the single junction
- this pair merely creates an effective single junction
- Josephson energy can be tuned in situ by applying a magnetic field

Quantization of Hamiltonian at the sweet spot

- quantize CPB circuit using the commutation relation
  $$[n, \delta] = i$$
- for dominating charging energy of the island $E_c \gg E_J$
  $\Rightarrow$ choice of basis states $\{|n\rangle\}$ number of Cooper pairs

a) charge basis

Eigenvalue probleme:

$$\left[4\,E_c\,(\hat n - n_g)^2 - E_J \cos\left(2\pi\frac{\phi}{\phi_0}\right)\right]\Psi_m = E_m \Psi_m$$

i) $\hat n|n\rangle = n|n\rangle \qquad n \in \mathbb{Z}$

ii) $\hat n = \sum_{n=-\infty}^{\infty} n\,|n\rangle\langle n| \qquad \sum_n |n\rangle\langle n| = \mathbb{1}$

negativ in respect to the offset

(iii) $\cos\left(2\pi\frac{\phi}{\phi_0}\right) = \frac{1}{2}\left(e^{i2\pi\phi/\phi_0} + e^{-i2\pi\phi/\phi_0}\right)$

$[\hat n, \hat\varphi] = i \qquad [\hat n, e^{i\varphi}] = e^{i\varphi}$

$e^{-i2\pi\phi/\phi_0}|n\rangle = |n \pm 1\rangle$

$$\rightsquigarrow \quad H = \sum_n 4\tilde E_c\,(n - n_g)^2\,|n\rangle\langle n| - \sum_n \frac{E_J}{2}\Big[\underbrace{|n+1\rangle\langle n|}_{e^{i2\pi\frac{\phi}{\phi_0}}} + \underbrace{|n\rangle\langle n+1|}_{e^{-i2\pi\frac{\phi}{\phi_0}}}\Big]$$

$$\begin{array}{c} \langle -N| \\ \langle -N+1| \\ \vdots \\ \\ \langle N+1| \end{array} \begin{pmatrix} 4\tilde E_c(-N-n_g)^2 & -\frac{\tilde E_J}{2} & 0 & \cdots \\ -\frac{\tilde E_J}{2} & 4E_c(-N+1-n_g)^2 & -\frac{E_J}{2} & \cdots \\ 0 & -\frac{E_J}{2} & \ddots & -\tilde E_J/2 \\ \vdots & \ddots & -\tilde E_J/2 & 4\tilde E_c(+N-n_g)^2 \end{pmatrix}$$

b) phase basis

$$\hat\varphi|\varphi\rangle = \varphi|\varphi\rangle$$

$\hat n = -i\frac{\partial}{\partial\varphi}$

$H = 4\tilde E_c\left(-\frac{\partial}{\partial\varphi} - n_g\right)^2 - E_J\cos(\hat\varphi)$

boundary condition: $\Psi_k(\varphi) = \Psi_k(\varphi + 2\pi)$

transformation:

$$\tilde\Psi_k = e^{-in_g\hat\varphi}\,\Psi_k(\varphi)$$

$$\Rightarrow H\tilde\Psi_k = -4\tilde E_c \frac{\partial^2}{\partial\varphi^2}\tilde\Psi_k(\varphi) - E_J\cos(\varphi)\,\tilde\Psi_k(\varphi)$$

Bloch – theorem: The energy eigenstates of a particle in a periodic potential are Bloch waves

$$\varphi_{mk}(r) = \underbrace{e^{ikr}}_{\text{plane wave}}\,u(r)$$

periodic fct with same periodicity as potential

$$\varphi_{mk}(\varphi) = e^{ikr}\,u(\varphi)$$

$u(\varphi) = u(\varphi + 2\pi) = \cos(\varphi)$

Ansatz: $\tilde\Psi_k(\varphi) = e^{ik\varphi}\,u(\varphi)$

$$= e^{-in_g\varphi}\,u(\varphi)$$

$\hookrightarrow u(\varphi) = \Psi(\varphi)$

Fourier Ansatz:

$$\tilde\Psi_{m,n_g}(\varphi) = e^{-in_g\varphi}\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} C_{m,l}\,e^{il\varphi} \qquad l \in \mathbb{Z}$$

$$= \frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} C_{m,l}\,e^{i(l-n_g)\varphi}$$

l has an integer value (spectrum) to satisfy periodicity (2π)

$$-4\tilde E_c\frac{\partial^2}{\partial\varphi^2}\left(e^{-in_g\varphi}\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} C_{m,l}\,e^{+il\varphi}\right) - \frac{E_J}{2}\left(e^{+i\varphi} + e^{-i\varphi}\right)\left(\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} e^{i(l-n_g)\varphi}\right) = E_{m,n_g}\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} e^{i(l-n_g)\varphi}\,c_{m,l}$$

$$\sum_{l=-\infty}^{\infty} 4\tilde E_c\,(l-n_g)^2\,c_{m,l} - \frac{E_J}{2}\,c_{m,l}\left(e^{i(l+1-n_g)\varphi} + e^{i(l-1-n_g)\varphi}\right) = E_{m,n_g}\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} e^{i(l-n_g)\varphi}\,c_{m,l}$$

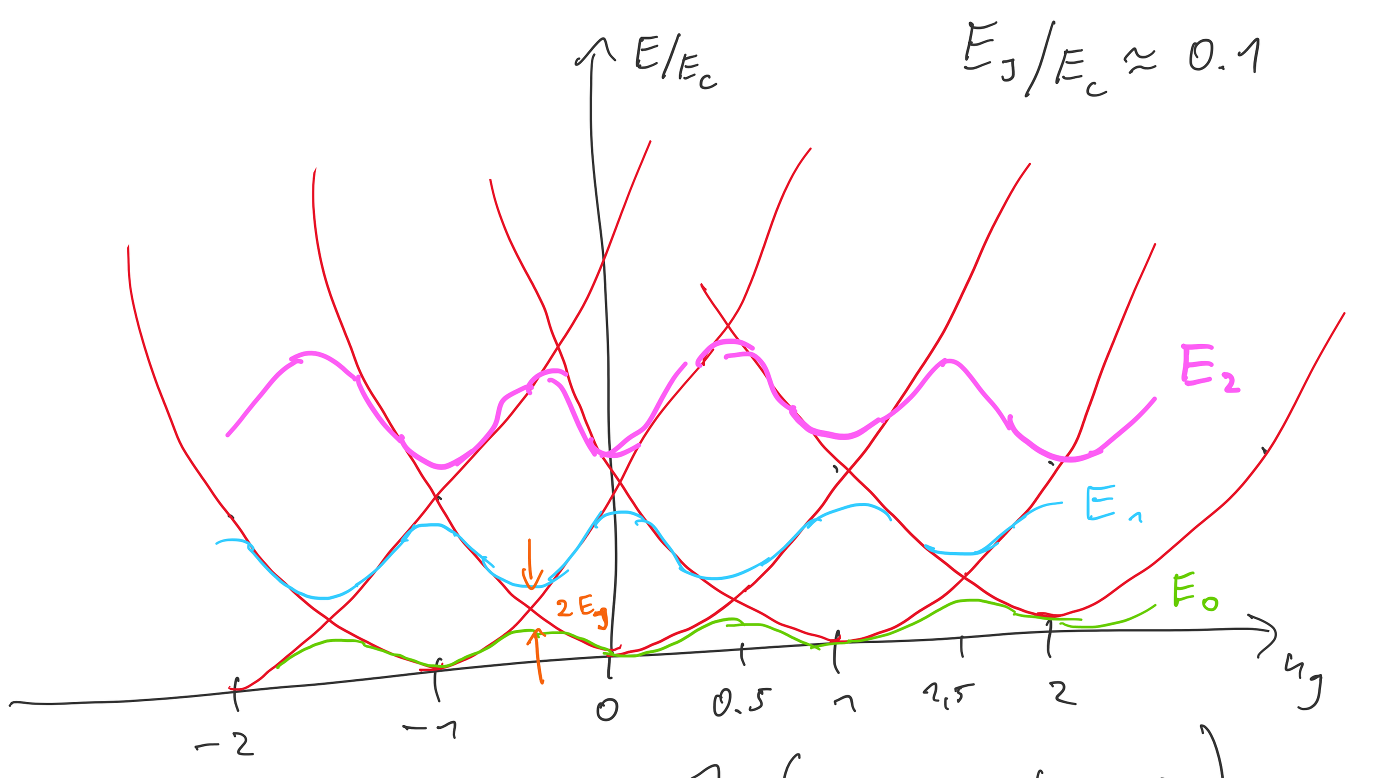
$\frac{1}{\sqrt{2\pi}}e^{i(l-n_g)l}$

Matrix representation:

$$4\tilde E_c\,(l-n_g)^2\,c_{m,l} - \frac{E_J}{2}\left(c_{m,l+1} + c_{m,l-1}\right) - E_{m,n_g}\,c_{m,l} = 0$$

$$\tilde H - E\,\mathbb{1} = 0 \qquad \text{basis: Fourier coefficients} \quad \tilde c_m = \begin{pmatrix} c_{m,\infty} \\ \vdots \\ c_{m,-\infty} \end{pmatrix}$$

Band structure $\quad E_m(n_g)$



$$\left|{}^{1}_{0}\right\rangle = \frac{1}{\sqrt2}\left(|n=0\rangle \mp |n=1\rangle\right)$$