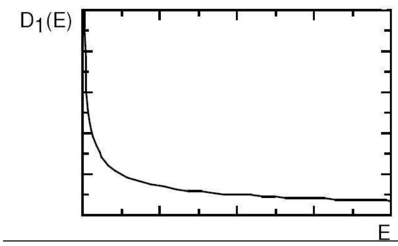
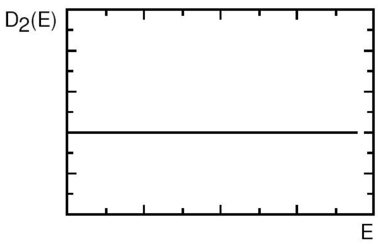
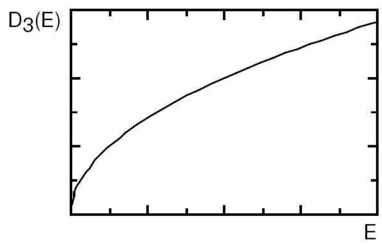
Ergänzung zum Übungsblatt 7

Aufgabe 3 a) Zustandsdichte des Elektronengases

Dimension d		
1	2	3
Volumen im k-Raum		
$\frac{2\pi}{L}$	$\left(\frac{2\pi}{L}\right)^2$	$\left(\frac{2\pi}{L}\right)^3$
Zahl der Elektronen im Intervall $(k, k+dk) \rightarrow D_d^*(k)dk$		
2 wegen Spin $2\cdot\frac{2dk}{2\pi/L}=\frac{2L}{\pi}dk$ noch eine 2 wegen $\lvert k\rvert$	Kreisumfang $2\pi k$ $2\cdot\frac{2\pi k dk}{(2\pi/L)^2}=\frac{L^2k}{\pi}dk$	Oberfläche $4\pi k^2$ $2\frac{4\pi k^2}{(2\pi/L)^3}dk=\frac{L^3k^2}{\pi^2}dk$
Zustandsdichte in Abhängigkeit der Energie $D_d^*(E)=D_d^*(k)\frac{dk}{dE}$		
$\frac{2L}{\pi}\cdot\frac{m}{\hbar^2k}=\frac{2L}{\pi}\frac{m}{\hbar^2\left(\frac{2mE}{\hbar^2}\right)^{1/2}}$ $D_1^*(E)=\frac{L(2m)^{1/2}}{\pi\hbar}E^{-1/2}$	$\frac{L^2k}{\pi}\cdot\frac{m}{\hbar^2k}=\frac{L^2m}{\pi\hbar^2}$ $D_2^*(E)=\frac{L^2(2m)^{2/2}}{2\pi\hbar^2}E^0$	$\frac{L^3k^2}{\pi^2}\cdot\frac{m}{\hbar^2k}=\frac{L^3m}{\pi^2\hbar^2}\left(\frac{2mE}{\hbar^2}\right)^{1/2}$ $D_3^*(E)=\frac{L^3(2m)^{3/2}}{2\pi^2\hbar^3}E^{1/2}$
Zustandsdichte pro Volumeneinheit des Ortsraumes $D_d(E)=D_d^*(E)/L^d$		
$D_1(E)=\frac{(2m)^{1/2}}{\pi\hbar}\frac{1}{\sqrt{E}}$ 	$D_2(E)=\frac{m}{\pi\hbar^2}$ 	$D_3(E)=\frac{(2m)^{3/2}}{2\pi^2\hbar^3}\sqrt{E}$ 
Fermiwellenvektor $k_F(T=0)$		
$2\cdot\frac{2k_F}{2\pi/L}=\frac{2L}{\pi}k_F\overset{!}{=}N$ $k_F=\frac{N}{2L}\pi=\frac{\pi}{2}n$	$2\cdot\frac{\pi k_F^2}{(2\pi/L)^2}=\frac{L^2}{2\pi}k_F^2\overset{!}{=}N$ $k_F=\left(\frac{N}{A}2\pi\right)^{1/2}=(2\pi n)^{1/2}$	$2\cdot\frac{\frac{4}{3}\pi k_F^3}{(2\pi/L)^3}=\frac{L^3}{3\pi^2}k_F^3\overset{!}{=}N$ $k_F=(3\pi^2n)^{1/3}$
Fermienergie $E_F=\frac{\hbar^2k_F^2}{2m}(T=0)$		
$E_F=\frac{\hbar^2}{2m}\frac{\pi^2}{4}n^2$ $\Rightarrow E_F=\frac{\hbar^2\pi^2}{8m}n^2$	$E_F=\frac{\hbar^2}{2m}2\pi n$ $\Rightarrow E_F=\frac{\hbar^2\pi}{m}n$	$E_F=\frac{\hbar^2}{2m}(3\pi^2n)^{2/3}$
Zustandsdichte bei E_F und n bei $T=0$		
$D_1(E_F)=\frac{(2m)^{1/2}}{\pi\hbar}\cdot\frac{(8m)^{1/2}}{\hbar\pi n}$ $\Rightarrow D_1(E_F)=\frac{1}{2}\frac{n}{E_F}$	$D_2(E_F)=\frac{2}{2}\frac{n}{E_F}$	$D_3(E_F)=\frac{(2m)^{3/2}}{2\pi^2\hbar^3}\left(\frac{\hbar^2}{2m}\right)^{1/2}(3\pi^2n)^{1/3}$ $\Rightarrow D_3(E_F)=\frac{3}{2}\frac{n}{E_F}$