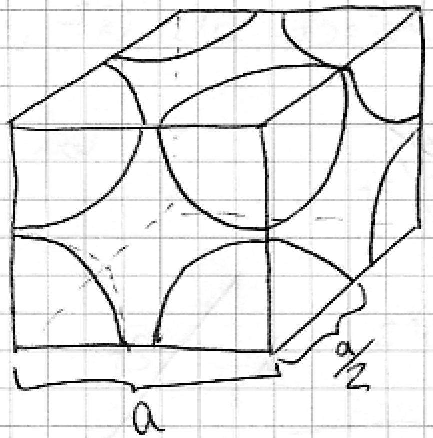
A1

a) <u>SC:</u>



$8\cdot\frac{1}{8}$-Kugel mit Radius $\frac{a}{2}$

$\rightarrow V_{Kugel} = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$

$pV = \frac{V_{Kugel}}{a^3} = \frac{4}{3}\pi\cdot\frac{1}{8}$

$= \frac{\pi}{6} \simeq 52{,}4\%$

<u>bcc:</u> $8\cdot\frac{1}{8}$-Kugel + 1 Kugel

Raumdiag. $\sqrt{3}a = r + r + 2r = 4r$

$\rightarrow r = \frac{\sqrt{3}a}{4}$

$\rightarrow V_{Kugel} = 2\cdot\frac{4}{3}\pi\left(\frac{\sqrt{3}a}{4}\right)^3$

$pV = \frac{2\cancel{8}\pi}{\cancel{3}}\cdot\frac{\cancel{3}\sqrt{3}}{\cancel{4}\cdot 16} = \frac{\sqrt{3}}{8}\pi \simeq 0{,}680$

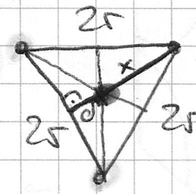
<u>fcc:</u> $8\cdot\frac{1}{8}$-Kugel + $6\cdot\frac{1}{2}$-Kugel

Flächendiag. $\sqrt{2}a = 4r \rightarrow r = \frac{1}{2\sqrt{2}}a$

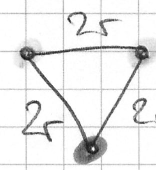
$\rightarrow pV = 4\cdot\frac{4}{3}\pi r^3/a^3 = \frac{2\cancel{16}\pi}{3}\cdot\frac{1}{\cancel{8}\,2^{3/2}} = \frac{\pi}{3\sqrt{2}} \simeq 74{,}0\%$

b) hcp besteht aus zwei gedacht gestapelten Tetraedern $\hat{=}$ Pyramiden mit gleichseitigem Dreieck als GF und gleichen Seiten

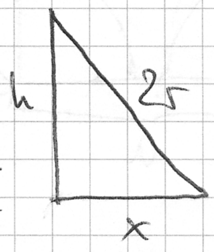
von oben:



von der Seite:



Höhe h eines Tetraeders:



1) $h^2 + x^2 = (2r)^2 \rightarrow h = \sqrt{4r^2 - x^2}$

2) $(x + \delta)^2 + r^2 = 4r^2$

3) $x^2 = \delta^2 + r^2 \Rightarrow \delta = \sqrt{x^2 - r^2}$

in 2): $\left(x + \sqrt{x^2 - r^2}\right)^2 = 3r^2 \Rightarrow x = \frac{2r}{\sqrt{3}}$

in 1): $h = 2\sqrt{\frac{2}{3}}\, r = \frac{c}{2} \Rightarrow c = 4\sqrt{\frac{2}{3}}\, r$

$a = 2r \Rightarrow \frac{c}{a} = \sqrt{\frac{8}{3}}$

$V_{\frac{1}{2}EZ} = 2h \cdot A_\Delta = 2h(x+\delta)\, r = 2\sqrt{3}\, h r^2 = 4\sqrt{2}\, r^3$

$V_{Kugel} = \frac{4}{3}\pi r^3$ ($\frac{1}{2}$ EZ enthält genau 1 Kugel)

$\rightarrow pV = \frac{V_{Kugel}}{V_{\frac{1}{2}EZ}} = \frac{1}{3\sqrt{2}}\pi \approx 0{,}740$

c) hcp: $\rho = \frac{m_{ges}}{V_{ges}} = \frac{\cancel{N} \cdot m_{Kugel}}{\cancel{N} \cdot V_{\frac{1}{2}EZ}} = \frac{m_{Kugel}}{V_{Kugel}} \cdot pV = \frac{\frac{M}{N_A}}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3} pV$

$= \frac{3M \cdot \cancel{8}\,2}{\cancel{4} N_A \pi a^3} pV = \frac{6MpV}{\pi N_A a^3} \rightarrow \rho_{Mg} = 2331 \frac{kg}{m^3} \cdot pV$

$= 1725 \frac{kg}{m^3}$

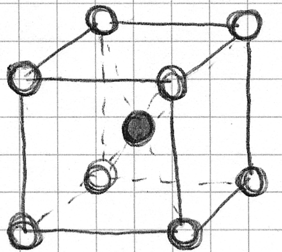
bcc: ~~$\rho = \frac{m_{Kugel}}{\ldots} = 6MpV \ldots$ Ba $2341 \frac{kg}{\ldots}$~~

Ba: <u>bcc:</u> $\rho = \dfrac{2 m_{Kugel}}{V_{EZ}} = \dfrac{2M}{N_A a^3} = \overset{3605}{\cancel{376}} \dfrac{kg}{m^3}$

Pb: <u>fcc:</u> $\rho = \dfrac{4 m_{Kugel}}{V_{EZ}} = \dfrac{4M}{N_A a^3} = 11{,}35 \dfrac{kg}{m^3} \cdot 10^3$

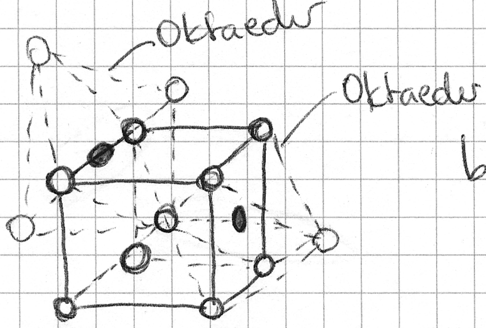
A2

a)



sc: Mitte des Würfels

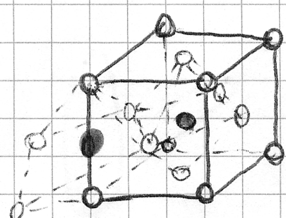
→ Punktsymmetrie, Spiegelsymm., Drehsymm.



bcc: - Mitten der Seitenflächen

- Mitten der Kanten

→ Punktsymmetrie



fcc: - Mitten der Kanten

- Mitte des Würfels

→ Punktsymmetrie

b)

sc: $a = 2r$ Raumdiag.: $\sqrt{3}a = 2r + x$

$\rightarrow x = (\sqrt{3} - 1)a$

bcc: $\sqrt{3}a = 4r \rightarrow r = \frac{\sqrt{3}}{4}a$

Kanten: $a = 2r + x \rightarrow x = \left(1 - \frac{\sqrt{3}}{2}\right)a$

Seitenflächen: $\cancel{\sqrt{2}a = 2r + x \rightarrow x = \left(\sqrt{2} - \frac{\sqrt{3}}{2}\right)a}$

$a = 2r + x \rightarrow x = \left(1 - \frac{\sqrt{3}}{2}\right)a$

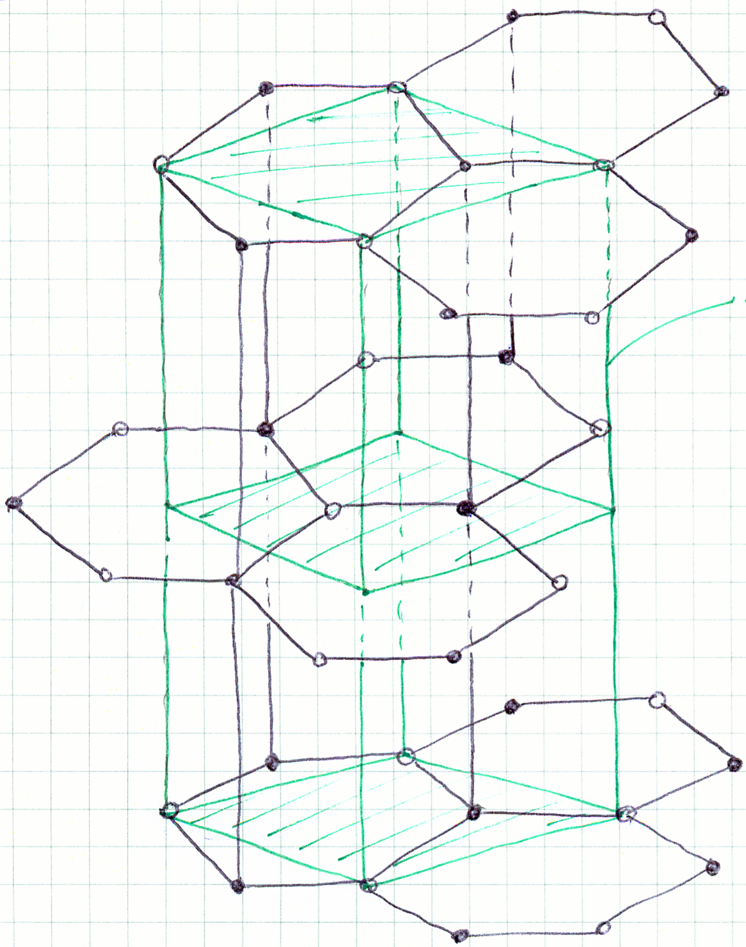
fcc: $\sqrt{2}a = 4r \rightarrow r = \frac{\sqrt{2}}{4}a$

Kanten: $a = 2r + x \rightarrow x = \left(1 - \frac{\sqrt{2}}{2}\right)a$

Mitte: $\cancel{\sqrt{3}a = 2r + x \rightarrow x = \left(\sqrt{3} - \frac{\sqrt{2}}{2}\right)a}$

$a = 2r + x \rightarrow x = \left(1 - \frac{\sqrt{2}}{2}\right)a$

A3



← $4 \cdot \frac{1}{8} + 1 \cdot \frac{1}{2}$ Atome

primitive EZ enthält 4 Atome

← $2 \cdot 1$ Atome

← $4 \cdot \frac{1}{8} + 1 \cdot \frac{1}{2}$ Atome

A4

a) $\vec{G} = h\vec{g}_1 + k\vec{g}_2 + l\vec{g}_3$

(hkl): Ebene, die $\vec{a}_1$ bei $\frac{1}{h}$, $\vec{a}_2$ bei $\frac{1}{k}$, $\vec{a}_3$ bei $\frac{1}{l}$ schneidet

$\rightarrow$ Vektoren $\vec{a} = \frac{1}{h}\vec{a}_1 - \frac{1}{k}\vec{a}_2$ und $\vec{b} = \frac{1}{h}\vec{a}_1 - \frac{1}{l}\vec{a}_3$ spannen Ebene auf

~~$\rightarrow$ Normalenvektor~~ $\cancel{\vec{n} = \left(\frac{1}{h}\vec{a}_1 - \frac{1}{k}\vec{a}_2\right) \times \left(\frac{1}{h}\vec{a}_1 - \frac{1}{l}\vec{a}_3\right)}$

$$\cancel{= \frac{1}{hl}\vec{a}_1 \times \vec{a}_3 - \frac{1}{kh}\vec{a}_2 \times \vec{a}_1 + \frac{1}{kl}\vec{a}_2 \times \vec{a}_3}$$

$$\cancel{\vec{n} = \frac{1}{hl}\vec{a}_2 + \frac{1}{hk}\vec{a}_3 + \frac{1}{kl}\vec{a}_1}$$

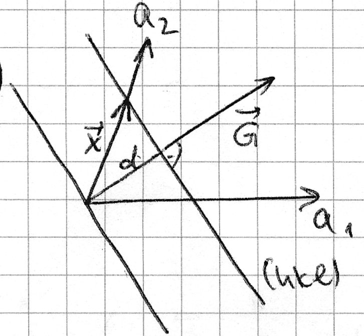
$$\vec{G} \cdot \vec{a} = \left(\frac{1}{h}\vec{a}_1 - \frac{1}{k}\vec{a}_2\right) \cdot \left(h\vec{g}_1 + k\vec{g}_2 + l\vec{g}_3\right)$$

$$\xrightarrow{\vec{a}_i \vec{b}_j = 2\pi\delta_{ij}} \; = 2\pi - 2\pi = 0$$

$$\vec{G} \cdot \vec{b} = \left(\frac{1}{h}\vec{a}_1 - \frac{1}{l}\vec{a}_3\right) \cdot \vec{G} = 2\pi - 2\pi = 0$$

$\rightarrow$ $\vec{G}_{hkl} \perp (hkl)$ ☒

b)

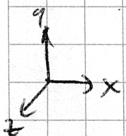


$$d = \vec{x}\cdot\hat{G} = \frac{\vec{x}\cdot\vec{G}}{|\vec{G}|}$$

$$\vec{x} = \frac{1}{k}\vec{a}_2 \,, \quad \vec{G} = h\vec{g}_1 + k\vec{g}_2 + l\vec{g}_3$$

$$\rightarrow \quad d = \frac{2\pi}{|\vec{G}|}$$

c) bcc: $\vec{a}_1 = \begin{pmatrix} -1/2 \\ 1/2 \\ -1/2 \end{pmatrix}, \quad \vec{a}_2 = \begin{pmatrix} 1/2 \\ 1/2 \\ -1/2 \end{pmatrix}, \quad \vec{a}_3 = \begin{pmatrix} -1/2 \\ 1/2 \\ +1/2 \end{pmatrix}$



reziprok: fcc

fcc: $\vec{a}_1 = \begin{pmatrix} 1/2 \\ 0 \\ 1/2 \end{pmatrix}, \quad \vec{a}_2 = \begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix}, \quad \vec{a}_3 = \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}$

reziprok: bcc

d) $\vec{G}_1\cdot\vec{G}_2 = |\vec{G}_1||\vec{G}_2|\cos\phi$

$$d_1 = \frac{2\pi}{|\vec{G}_1|}\,, \quad d_2 = \frac{2\pi}{|\vec{G}_2|}$$