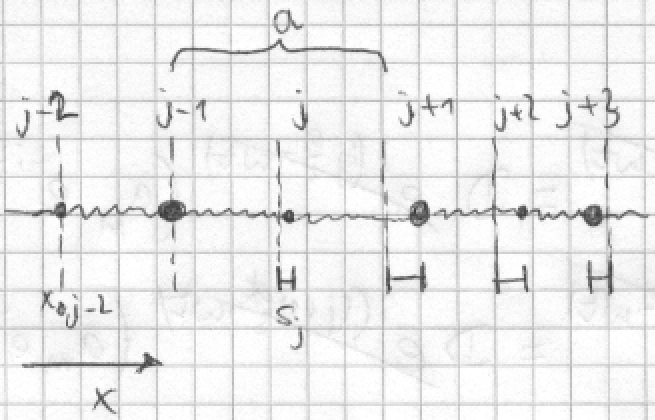
Ex5

A1

Üb

b)



$$x_j(t) = x_{0j} + s_j(t)$$

Ruhelage des j-ten Teilchens: $x_{0,j}$

Auslenkung aus Ruhelage: s_j

$\Rightarrow$ Kraft auf j-tes Teilchen

$$F_j = D\left(s_{j+1} - s_j + s_{j-1} - s_j\right)$$
$$= D\left(s_{j-1} + s_{j+1} - 2s_j\right)$$

$$m_j \ddot{s}_j = D\left(s_{j-1} + s_{j+1} - 2s_j\right)$$

bei j sei ~~klein~~ Teilchen der Masse m,

bei j+1 ~ M

$$\Rightarrow \boxed{\begin{aligned} m\ddot{s}_j &= D\left(s_{j-1} + s_{j+1} - 2s_j\right) \quad (1) \\ M\ddot{s}_{j+1} &= D\left(s_j + s_{j+2} - 2s_{j+1}\right) \quad (2) \end{aligned}}$$

<u>Ansatz</u>: Ebene Wellen an den Ruhelagen $x_{0,j}$:

$$s_j(t) = e^{i(kx_{0,j} - \omega t)}$$

mit $x_{0,j} = j \cdot \frac{a}{2}$

$$\Rightarrow \quad s_{j-1}(t) = a_M\, e^{i(k(j-1)\frac{a}{2} - \omega t)}$$
$$s_j(t) = a_m\, e^{i(kj\frac{a}{2} - \omega t)}$$
$$s_{j+1}(t) = a_M\, e^{i(k(j+1)\frac{a}{2} - \omega t)}$$
$$s_{j+2}(t) = a_m\, e^{i((j+2)k\frac{a}{2} - \omega t)}$$

in DGL:

$$(1)\ -m\omega^2 a_m \cancel{e^{i(k_j\frac{a}{2}-\omega t)}} = D\, \cancel{e^{i(k_j\frac{a}{2}-\omega t)}} \left(a_M e^{-i\frac{ak}{2}} + a_M e^{i\frac{ak}{2}} - 2a_m\right)$$

$$(2)\ -M\omega^2 a_M \cancel{e^{i((j+1)\frac{ak}{2}-\omega t)}} = D\, \cancel{e^{i((j+1)\frac{ak}{2}-\omega t)}} \left(a_m e^{-i\frac{ak}{2}} + a_m e^{i\frac{ak}{2}} - 2a_M\right)$$

$$\Leftrightarrow \quad (1)\quad -m\omega^2 a_m = 2D\left(a_M \cos\left(\tfrac{ka}{2}\right) - a_m\right)$$

$$(2)\quad -M\omega^2 a_M = 2D\left(a_m \cos\left(\tfrac{ka}{2}\right) - a_M\right)$$

$$\cancel{(m\omega^2 + 2D)\, a_m}$$

$$\Leftrightarrow \quad (-m\omega^2 + 2D)\, a_m - 2D\cos\tfrac{ka}{2}\, a_M = 0$$

$$(-M\omega^2 + 2D)\, a_M - 2D\cos\tfrac{ka}{2}\, a_m = 0$$

$$\Leftrightarrow \quad \begin{pmatrix} -m\omega^2 + 2D & -2D\cos\frac{ka}{2} \\ -2D\cos\frac{ka}{2} & -M\omega^2 + 2D \end{pmatrix} \begin{pmatrix} a_m \\ a_M \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

nichttriviale Lösg. für $\begin{pmatrix} a_m \\ a_M \end{pmatrix}$ erfordert $\det|\cdot| \overset{!}{=} 0$

$$(-m\omega^2 + 2D)(-M\omega^2 + 2D) - 4D^2\cos^2\frac{ka}{2} = 0$$

$$mM\omega^4 - 2D\omega^2(m+M) + 4D^2 - 4D^2\cos^2\frac{ka}{2} = 0$$

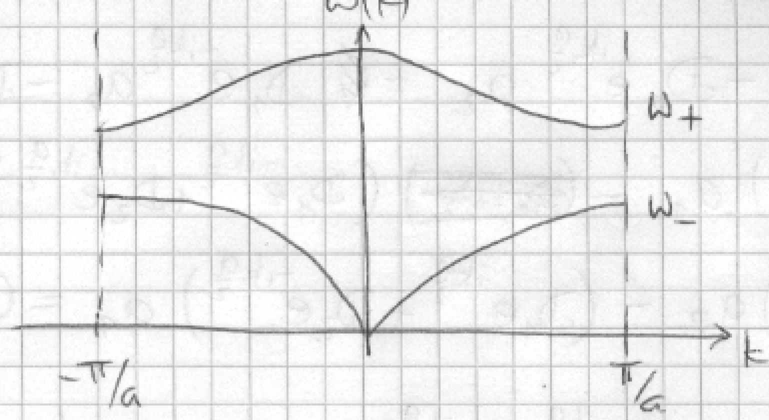
$$\omega^4 + \frac{-2D(m+M)}{mM}\omega^2 + \frac{4D^2}{mM}\left(1-\cos^2\frac{ka}{2}\right) = 0$$

$$\Rightarrow \quad \omega_\pm^2 = +\frac{D(m+M)}{mM} \pm \sqrt{\frac{D^2(m+M)^2}{m^2M^2} - \frac{4D^2}{mM}\left(1-\cos^2\frac{ka}{2}\right)}$$

$$\mu = \frac{mM}{m+M} \qquad \boxed{\omega_\pm^2 = +\frac{D}{\mu} \pm D\sqrt{\frac{1}{\mu^2} - \frac{4}{mM}\sin^2\left(\frac{ka}{2}\right)}}$$

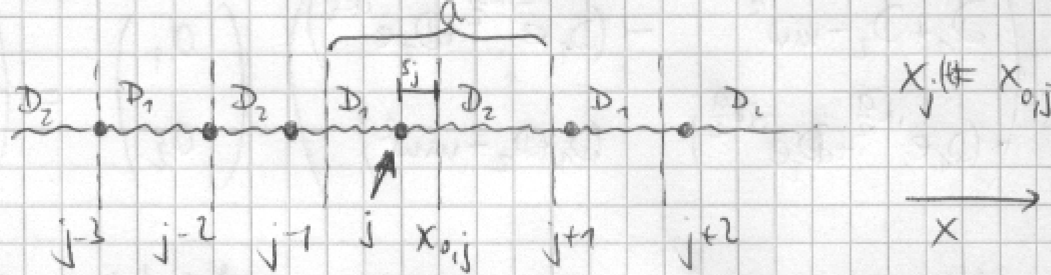
ExS

Ü6



A1

a)



$x_j(t) = x_{0,j} + s_j(t)$

Kraft auf j-tes Teilchen: (D_1 links, D_2 rechts)

$$\boxed{\begin{aligned} m\ddot{s}_j &= F_j = D_2\left(s_{j+1} - s_j\right) + D_1\left(s_{j-1} - s_j\right) \quad (1) \\ m\ddot{s}_{j+1} &= F_{j+1} = D_1\left(s_{j+2} - s_{j+1}\right) + D_2\left(s_j - s_{j+1}\right) \quad (2) \end{aligned}}$$

Ansatz: ebene Wellen bei $x_{0,j} = j \cdot \frac{a}{2}$

$$s_j(t) = a_1 e^{i(kj\frac{a}{2} - \omega t)}$$

$$s_{j+1}(t) = a_2 e^{i(k(j+1)\frac{a}{2} - \omega t)}$$

$$s_{j+2}(t) = a_1 e^{i(k(j+2)\frac{a}{2} - \omega t)}$$

$$s_{j-1}(t) = a_2 e^{i(k(j-1)\frac{a}{2} - \omega t)}$$

in DGL:

$$-m\omega^2 a_1 \cancel{e^{i(kj\frac{a}{2}-\omega t)}} = D_2 \cancel{e^{i(kj\frac{a}{2}-\omega t)}}\left(a_2 e^{ik\frac{a}{2}} - a_1\right)$$

$$+ D_1 \cancel{e^{i(kj\frac{a}{2}-\omega t)}}\left(a_2 e^{-ik\frac{a}{2}} - a_1\right)$$

$$-m\omega^2 a_2 = D_1\left(a_1 e^{ik\frac{a}{2}} - a_2\right) + D_2\left(a_1 e^{-ik\frac{a}{2}} - a_2\right)$$

$\cancel{(1) + (2)}$

$\cancel{\rightarrow -m\omega^2(a_1+a_2) =}$

$$\rightarrow (1) \quad (D_2 - m\omega^2)\, a_1 - D_2 e^{ik\frac{a}{2}} a_2 = \cancel{0}\; D_1 e^{-ik\frac{a}{2}} a_2 - D_1 a_1$$

$$(D_1 + D_2 - m\omega^2)\, a_1 - \cancel{(D_1+D_2)} \left(D_1 e^{-ik\frac{a}{2}} + D_2 e^{ik\frac{a}{2}}\right) a_2 = 0$$

$$(2) \quad (D_1 + D_2 - m\omega^2)\, a_2 - \left(D_1 e^{ik\frac{a}{2}} + D_2 e^{-ik\frac{a}{2}}\right) a_1 = 0$$

$$\Leftrightarrow \begin{pmatrix} D_1 + D_2 - m\omega^2 & -\left(D_1 e^{-ik\frac{a}{2}} + D_2 e^{ik\frac{a}{2}}\right) \\ -\left(D_1 e^{ik\frac{a}{2}} + D_2 e^{-ik\frac{a}{2}}\right) & (D_1 + D_2 - m\omega^2) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

nichttriviale Lösg. nur wenn $\det|\cdot| = 0$

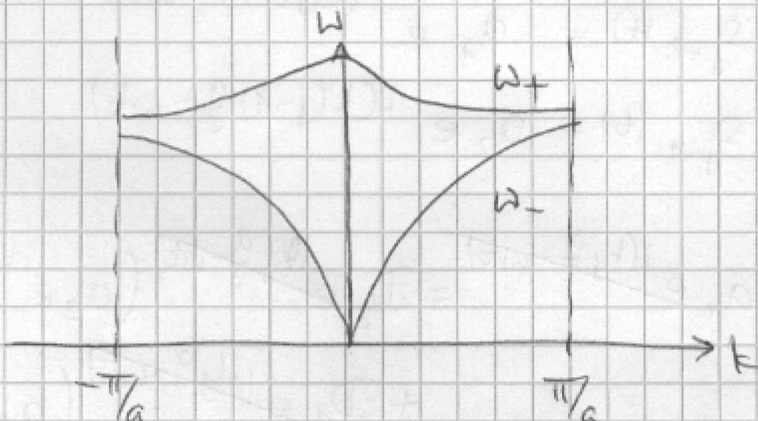
$$\rightarrow (D_1 + D_2 - m\omega^2)^2 - \left(D_1 e^{-ik\frac{a}{2}} + D_2 e^{ik\frac{a}{2}}\right)\left(D_1 e^{ik\frac{a}{2}} + D_2 e^{-ik\frac{a}{2}}\right) = 0$$

$$(D_1 + D_2)^2 + m^2\omega^4 - 2(D_1+D_2)m\omega^2 - \cancel{D_1^2} - \cancel{D_2^2} - D_1 D_2 \left(e^{-ika} + e^{ika}\right) = 0$$

$$2D_1D_2 + m^2\omega^4 - 2(D_1+D_2)m\omega^2 - 2D_1D_2\cos(ka) = 0$$

$$\omega^4 - \frac{2}{m}(D_1+D_2)\,\omega^2 + \frac{2D_1D_2}{m^2}\left(1 - \cos(ka)\right) = 0$$

$$\Rightarrow \omega_\pm^2 = \frac{D_1+D_2}{m} \pm \sqrt{\left(\frac{D_1+D_2}{m}\right)^2 - \frac{2D_1D_2}{m^2}\underbrace{\left(1-\cos(ka)\right)}_{\sim \sin^2\frac{ka}{2}}}$$



Ex5 Ü6

A2

$$m\ddot{u}_n = -D(2u_n - u_{n-1} - u_{n+1})$$

$\lambda >> a \Rightarrow$ ~~a~~ Teilchen n und $n\pm1$ liegen nahe beeinander

$$\Rightarrow u_n = u(x)$$

$$u_{n\pm1} = u(x \pm \Delta x) \quad , \quad \Delta x \text{ klein}$$

$\Rightarrow$ Taylorentwicklung

$$u_n = u(x)$$

$$u_{n\pm1} = u(x \pm \Delta x) = u(x) \pm \frac{\partial u}{\partial x}\Big|_x \cdot \Delta x + \frac{\partial^2 u}{\partial x^2}\Big|_x (\Delta x)^2 + \mathcal{O}(\Delta x$$

DGL

$$\Rightarrow m\ddot{u} = -D\Big(\cancel{2u} - \big(\cancel{u} - \cancel{\frac{\partial u}{\partial x}\Big|_x \Delta x} + \frac{\partial^2 u}{\partial x^2}\Big|_x (\Delta x)^2\big) - \big(\cancel{u} + \cancel{\frac{\partial u}{\partial x}\Big|_x \Delta x} + \frac{\partial^2 u}{\partial x^2}\Big|_x (\Delta x)^2\big)\Big)$$

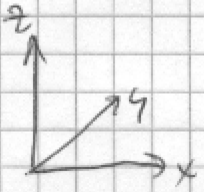
$$m\ddot{u} = 2D \frac{\partial^2 u}{\partial x^2}\Big|_x (\Delta x)^2$$

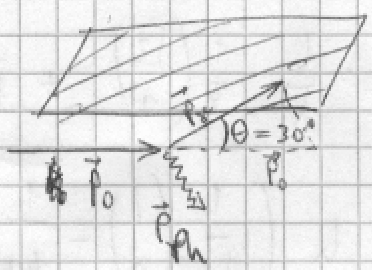
$$\frac{\partial^2 u}{\partial t^2} = \left(\sqrt{\frac{2D}{m}}(\Delta x)\right)^2 \frac{\partial^2 u}{\partial x^2} = c_{Schall}^2 \frac{\partial^2 u}{\partial x^2}$$

Ex5

Tu6

A4





a) Neutronenstrahl fällt parallel zur Streuebene ein → Streuung nur mit Phononen möglich

b) Impulserhaltung → Phonon wird erzeugt

c) $\vec p_0 = \vec p_\gamma + \vec p_{Ph}$

$\uparrow$ ~~Neutron~~ Neutron vorher $\uparrow$ Neutron nachher $\uparrow$ Phonon

$h\nu_0 = h\nu_\gamma + h\nu_{Ph}$

mit $E = \frac{p^2}{2m} = \hbar\omega$

$\Rightarrow \frac{(\hbar k)^2}{2m} = \hbar\omega$

$\Rightarrow \hbar^2 \left(\frac{2\pi}{\lambda}\right)^2 = \hbar\, 2\pi\nu$

$\Rightarrow \nu = \frac{h}{2m\lambda^2}$

$\Rightarrow \nu_{Ph} = \frac{h}{2m_n\lambda_0^2} - \frac{h}{2m_n\lambda^2} = \frac{h}{2m_n}\left(\frac{1}{\lambda_0^2} - \frac{1}{\lambda^2}\right)$

$= 13{,}4 \cdot 10^{45}\,\text{Hz} \cdot h = 8{,}85 \cdot 10^{12}\,\text{Hz}$