Ex

A2

$$N = \int_{440\,Hz}^{880\,Hz} D(\omega)\, d\omega = \frac{V}{2\pi^2} \int \frac{\omega^2}{v^3}\, d\omega$$

Ü7

$$= \frac{V}{2\pi^2 v^3} \cdot \frac{1}{3} \left[\omega^3\right]_{440\,Hz}^{880\,Hz} = 6438{,}8$$

A4

lineare Kette:

$$U = \frac{L}{2\pi} \int_{k=0}^{\infty} \frac{\hbar\omega_s \; dk}{\exp\left(\frac{\hbar\omega_s}{k_B T}\right) - 1} \overset{\omega = v_s k}{=} \frac{L\hbar v}{2\pi} \int \frac{k \; dk}{\exp\left(\frac{\hbar v k}{k_B T}\right) - 1}$$

Subst. $x = \frac{\hbar v k}{k_B T} \rightarrow \frac{dx}{dk} = \frac{\hbar v}{k_B T}$

$$\Rightarrow U = \frac{L\hbar v}{2\pi} \frac{(k_B T)^2}{(\hbar v)^2} \int_0^{\infty} \frac{x}{e^x - 1}\, dx = \frac{L k_B^2 \pi}{12 \hbar v} T^2$$

$$c_V = \frac{1}{m} \frac{\partial U}{\partial T} = \frac{L k_B^2 \pi}{6 \hbar v m} T$$

$T \rightarrow \infty \Rightarrow c_V \rightarrow \infty$

$T \rightarrow 0 \Rightarrow c_V \rightarrow 0$

dreidim. Gitter

$$U = \frac{L^3 \hbar}{(2\pi)^3} \int_{\vec{k}, s} \frac{\omega_s(\vec{k})}{\exp\left(\frac{\hbar\omega_s}{k_B T}\right) - 1}\, d^3k$$

im isotropen Medium: $\omega_s(\vec{k}) = \omega_s(k) = v_s k$

$$U = \frac{L^3 \hbar v_s}{(2\pi)^3} \int_{\mathbb{R}^3} \frac{k}{\exp\left(\frac{\hbar v_s k}{k_B T}\right) - 1}\, d^3k$$

$$= \frac{L^3 \hbar v_s}{(2\pi)^3} \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \frac{k^3 \sin\theta}{\exp\left(\frac{\hbar v_s k}{k_B T}\right) - 1}\, dk\, d\theta\, d\varphi$$

$$= \frac{4\pi L^3 \hbar v_s}{(2\pi)^3} \int_0 \frac{k^3}{\exp\left(\frac{\hbar v_s k}{k_B T}\right)-1}\, dk$$

Subst. $x = \frac{\hbar v_s k}{k_B T} \rightarrow \frac{dx}{dk} = \frac{\hbar v_s}{k_B T}$

$$\Rightarrow U = \frac{4\pi L^3 \hbar v_s}{(2\pi)^3} \cdot \left(\frac{k_B T}{\hbar v_s}\right)^{\cancel{3}4} \int_0^\infty \frac{x^3}{e^x - 1}\, dx$$

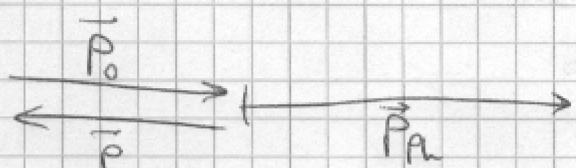
$$= \frac{L^3 k_B^4}{2\pi^2 \hbar^3 v_s^3} T^4$$

$$c_V = \frac{1}{m} \frac{\partial U}{\partial T} = \frac{2 L^3 k_B^4}{\pi^2 \hbar^3 v_s^3 m} T^3$$

$$c_V \xrightarrow{T \to \infty} \infty$$

$$c_V \xrightarrow{T \to 0} 0$$

<u>A1</u>



max. Phononenimpuls bei $-\vec{p}_0 = \vec{p}$

$$\rightarrow \quad p_{Ph} = 2p_0 \quad , \quad p_0 = \hbar k_0 = \frac{2\pi}{\lambda_{0,n}} = \frac{2\pi n}{\lambda_0}$$

$$p_{Ph} = \hbar k_{Ph} = \hbar \frac{2\pi}{\lambda_{Ph}} \quad , \quad v_s = \lambda_{Ph} \nu_{Ph}$$

$$\rightarrow \quad p_{Ph} = \frac{2\pi\hbar \nu_{Ph}}{v_s}$$

$$\Rightarrow \frac{\cancel{2\pi}\hbar \nu_{Ph}}{v_s} = \frac{2\cancel{2\pi} n}{\lambda_0} \Rightarrow \nu_{Ph} = \frac{2 n v_s}{\hbar \lambda_0} = 253 \cdot 10^{42}\ \mathrm{Hz}$$