Ex A3

Ü9 a) $c_e = \frac{\partial U}{\partial T} = \frac{\pi^2}{3} k_B^2 T \, D(E_F)$

mit $D(E_F) = \frac{3}{2} \frac{n}{E_F}$

$\Rightarrow c_e = \frac{\pi^2}{\cancel{3}} k_B^2 T \frac{\cancel{3}}{2} \frac{n}{E_F} = \frac{\pi^2 k_B^2 n}{2 E_F} T = \gamma T$

$\rightarrow \gamma = \frac{\pi^2 k_B^2 n}{2 E_F} = 70{,}8 \frac{J}{K^2 m^3} \hat{=} 0{,}79 \cdot 10^{-5} \frac{J}{g K^2}$

Cu: $n = 8{,}49 \cdot 10^{28} \, m^{-3}$

$E_F = 7{,}04 \, eV$

$\rho = 8920 \frac{kg}{m^3}$

tiefe Temp.: Gitterbeitrag $c_g = \frac{12\pi^4}{5} n k_B \left(\frac{T}{\theta_D}\right)^3$

$c_e \overset{!}{=} c_g$

$\Rightarrow \gamma T = \frac{12\pi^4}{5} n k_B \left(\frac{T}{\theta_D}\right)^3$

$\Leftrightarrow T_{exp} = \sqrt{\frac{5 \gamma \theta_D^3}{12\pi^4 n k_B}} = \cancel{6{,}9 \cdot 10^{-2} \, K} \; 40{,}1 \cdot 10^{-3} \, K$

b) hohe Temp.: $c_g = 3 n k_B$

$c_e \overset{!}{=} 0{,}1 \, c_g$

$\Rightarrow T = \frac{0{,}1}{\gamma} \cdot 3 n k_B = 32{,}2 \cdot 10^6 \, K$

A2

a) $E = \frac{\hbar^2 k^2}{2m}$, $v_g = \frac{\partial E}{\partial(\hbar k)} = \frac{\hbar k}{m} = \frac{1}{\hbar} \frac{dE}{dk_\perp}$

$D(E)\,dE = \rho_k \int_E^{E+dE} d^3k = \rho_k \int_E^{E+dE} dk_\perp \, dS_E$

$= \frac{\rho_k}{\hbar} dE \int_{E=\text{const}} \frac{dS_E}{v_g}$

v_g hängt beim freien e^--Gas nicht von der Richtg. ab.

3-dim.: Oberfläche mit $E=$const. ist Kugel

$$\Rightarrow \int_E dS_E = 4\pi k^2$$

$$\varrho_k = \frac{V}{(2\pi)^3}$$

$$\Rightarrow D^{(3)}(E) = \frac{\varrho_k}{\hbar}\frac{4\pi k^2}{v_g} = \frac{V\,4\pi k^2 m}{(2\pi)^3 \hbar^2 k} = \frac{Vkm}{2\pi^2\hbar^2}$$

$$= \frac{Vm}{2\pi^2\hbar^3}\sqrt{2mE} \quad ; \quad \underline{D(E)} = \frac{2D(E)}{V}$$

Spinfreiheitsgrade

$$T=0: \quad n = \int_0^{E_F} \underline{D(E)}\,dE = \frac{(2m)^{3/2}}{2\pi^2\hbar^3}\frac{2E_F^{3/2}}{3}$$

$$\Rightarrow E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$$k_F = (3\pi^2 n)^{1/3}$$

2-dim.: Oberflächenintegral $\longrightarrow$ Linienintegral

$$\Rightarrow \int dS_E = 2\pi k$$

Fläche

$$\Rightarrow D^{(2)}(E) = \frac{\varrho_k\, 2\pi k}{\hbar v_g} = \frac{V}{4\pi^2}\frac{2\pi k m}{\hbar^2 k} = \frac{Vm}{2\pi\hbar^2}$$

$$\underline{D^{(2)}(E)} = \frac{2D^{(2)}(E)}{V} = \frac{m}{\pi\hbar^2}$$

$$T=0: \quad n = \int_0^{E_F} \underline{D(E)}\,dE = \frac{mE_F}{\pi\hbar^2} \Rightarrow E_F = \frac{\pi\hbar^2}{m}n$$

$$k_F = \sqrt{\pi n \cdot 2}$$

1-dim.:

$$\int dS_E = 2$$

$$D^{(1)}(E) = \frac{2\varrho_k}{\hbar v_g} = \frac{2V}{2\pi}\frac{\cancel{\hbar} m}{\hbar^2 k}$$

$$\underline{D^{(1)}(E)} = \frac{2m}{\pi\hbar^2\sqrt{2mE}} = \frac{\sqrt{2m}}{\pi\hbar^2}\cdot\frac{1}{\sqrt{E}}$$

$$T=0: \quad n = \int_0^{E_F} \underline{D(E)}\,dE = 2\frac{\sqrt{2m}}{\pi\hbar^2}\sqrt{E_F} \Rightarrow E_F = \left(\frac{\pi\hbar^2}{2^{3/2}\sqrt{m}}\right)^2\cdot n^2$$

$$k_F = \frac{\pi\hbar n}{4}$$

Ex A2 b)

Ü9

$$E_F = \frac{\hbar^2}{2m_e}\left(3\pi^2 n\right)^{\cancel{3/2}\, 2/3}$$

$$n = \frac{N}{V}\,,\quad \rho = \frac{m}{V} = \frac{N\cdot m_{Atom}}{V} = \frac{N\,\overbrace{N_A\, m_{Atom}}^{M}}{N_A\, V}$$

$$= \frac{N}{V}\cdot\frac{M}{N_A} = n\,\frac{M}{N_A}$$

$$\Rightarrow E_F = \frac{\hbar^2}{2m_e}\left(3\pi^2\,\frac{\rho N_A}{M}\right)^{\cancel{3/2}\, 2/3} = [\text{struck out}]\ 1{,}185\ \mathrm{eV}$$

$$k_F = \left(3\pi^2\frac{\rho N_A}{M}\right)^{1/3} = 5{,}58\cdot 10^9\ \frac{1}{m}$$

$$T_F = \frac{E_F}{k_B} = 13{,}8\ \mathrm{kK}$$

$$v_F = \frac{\hbar k_F}{m_e} = 645{,}8\ \frac{\mathrm{km}}{\mathrm{s}}$$

$$j = \rho_{el} v_D = \frac{Ne}{m}v_D = ne\frac{V}{m}v_D = \frac{ne}{\rho}v_D = \frac{N_A e}{M}v_D$$

$$\Rightarrow v_D = \frac{M}{eN_A}\,j = [\text{struck out}]\ 4{,}47\ \frac{\mathrm{m}}{\mathrm{s}}$$