Exercises Physics VI (Nuclei and Particles)
Summer Semester 2009

Exercise sheet Nr. 5 Work out until 04.06.2008

Exercise 1: Phase space of β-Decay (Points: 2)

The momentum distribution of electrons or positrons from the β-decay is given by Fermis golden rule for decay:

$$N(p_e)dp_e = \frac{2\pi}{\hbar} \cdot |M_{fi}|^2 \cdot \frac{dn_e \cdot dn_\nu}{dE}$$

Here, p_e is the four-momentum of electron or positron, M_{fi} is the matrix element and $dn_e dn_\nu/dE$ is the density of states in the phase space interval dE. The matrix element depends on the energy only very weakly and therefore the momentum distribution is essentially given by the phase space factor. Calculate dependence of the phase space factor on the energy of electron or positron for massless and massive neutrino. Neglect the recoil energy of the daughter nucleus.

How can one use a measurement of the energy distribution of electrons or positrons to infer mass of the (anti-)neutrino?

Exercise 2: HERA Kinematics (Points: 5)

At HERA collider electrons with an energy of 27.5 GeV collide with protons of energy 920 GeV in head-on collisions. Lets denote electron four-momenta by $\mathbf{k}$, the proton four-momenta $\mathbf{P}$ and the four-momentum of scattered electron by $\mathbf{k}'$. The four-momentum exchanged between electron and proton is denoted by $\mathbf{q}$. In following neglect the masses of the involved particles.

a) In order to determine the kinematics of the ep-scattering the energy E'_e and the angle θ of the scattered electron is measured. The angle θ is measured with respect to the proton beam. Using those two quantities, express the virtuality $Q^2 = -\mathbf{q}^2$ and inelasticity $y = \frac{\mathbf{P}\cdot\mathbf{q}}{\mathbf{P}\cdot\mathbf{k}}$ of the interaction. What is the meaning of the inelasticity y?

b) Express the W^2 and the Bjorken scale variable x in terms of Q^2, y and centre of mass energy s.

c) What are the requirements for a detector which one would use to measure Q^2 and y?

d) Not in all interactions an electron is observed in the detector. What are the reasons for not seeing an electron in the detector?

e) What possibilities one has to estimate kinematics of *ep*-scattering in cases where no electron is detected? Derive the corresponding formula to calculate y.

Exercise 3: Bjorken-x and Parton momentum (Points: 1)

Approximately the Bjorken scale variable x gives the fraction of the momentum of the nucleon which is carried by the given parton. Show that the fraction ξ of the nucleon momentum carried by a scattered parton in case of nucleus with mass M and parton mass m is ($c = 1$):

$$\xi = x\left(1 + \frac{m^2 - M^2x^2}{Q^2}\right)$$

Apply the approximation where $\sqrt{1+\epsilon(1+\epsilon')} \approx 1 + \frac{\epsilon}{2}\left(1 + \epsilon' - \frac{\epsilon}{4}\right)$ for small ϵ and ϵ'. When is this approximation applicable?