Exercises Physics VI (Nuclei and Particles)
Summer Semester 2009

Exercise sheet Nr. 6 Work out until 18.06.2008

Exercise 1: Structure function (Points: 2)

Sketch the structure function $F_2(x)$ of a proton for fixed Q^2 under the assumption that it is:

a) a single point-like elementary particle

b) composed of three non-interacting valence quarks

c) composed of three valence quarks interacting with each other

d) composed of valence and see quarks and gluons.

For each of the assumptions, draw a Feynman diagram of ep scattering.

Exercise 2: Breit-Wigner resonance (Points: 2)

From mesons, only the neutral vector mesons (ρ^0, ω, ϕ, J/ψ, Υ) can be directly produced in e^+e^- colliders. After production, vector mesons decay into leptons or hadrons. The cross section can be parametrised using the relativistic Breit-Wigner formula:

$$\sigma(e^+e^- \to VM \to f) = \frac{\pi(2J+1)}{W^2} \cdot \frac{4m^2\Gamma_{ee}\Gamma_f}{(W^2-m^2)^2+m^2\Gamma^2}$$

Here VM is a vector meson, f is the final state (e.g. e^+e^- or hadrons), W is the centre of mass energy, m and J are mass and spin of the vector mesons, Γ_f the partial width to the given final state and Γ the total width of the resonance.

For the J/ψ resonance, the total width $\Gamma = \Gamma_{had}+\Gamma_{ee}+\Gamma_{\mu\mu}$ is smaller than the resolution of experiments. This is due to the uncertainties in the energy of electron and positron beams. The total and partial widths can be extracted from the measured cross section integrated over the resonance:

$$\Sigma_f := \int \sigma(e^+e^- \to VM \to f)\, dW$$

First show that around the resonance peak ($W \approx m$), one can make the following approximation:

$$\sigma(e^+e^- \to VM \to f) \approx \frac{\pi(2J+1)\Gamma_{ee}\Gamma_f}{m^2[(W-m)^2+\Gamma^2/4]}$$

Using this approximation, calculate Σ_f, Σ_{ee} and $\Sigma_{tot} = (\Sigma_{had} + \Sigma_{ee} + \Sigma_{\mu\mu})$ and show how to obtain Γ_{ee}, Γ and Γ_f from the measured cross sections.

Exercise 3: Neutrino scattering (Points: 1)

Find out which of the following reactions are possible:

a) $\nu_e + e^- \to \nu_e + e^-$

b) $\nu_\mu + e^- \to \nu_\mu + e^-$

c) $\bar{\nu}_e + e^- \to \bar{\nu}_\mu + e^-$

d) $\nu_e + e^- \to \nu_\mu + \mu^-$

e) $\nu_\mu + e^- \to \nu_e + \mu^-$

Give Feynman diagrams (at leading order) or reasons why the reaction is not possible.

Exercise 4: Kaon decay and Golden rule (Points: 2)

In the book by Povh, exercise 4 of chapter 10, the ratio of partial decay widths of charged pions into an electron or muon and neutrino is calculated. In analogy with this calculation, calculate

$$\frac{\Gamma(K^+ \to e^+\nu_e)}{\Gamma(K^+ \to \mu^+\nu_\mu)}$$

In the calculation, employ Fermi's golden rule: $\Gamma(K^+ \to l^+\nu_l) \propto |M_{Kl}|^2\rho(E_0)$.

Calculate the following quantities as function of kaon mass $m_K = 493.6$ MeV/c^2 and lepton mass m_l:

- momentum and energy of the charged lepton
- the ratio of squares of the matrix elements under the assumption that $|M_{Kl}|^2 \propto 1 - v/c$
- the ratio of the density of states $\rho_e(E_0)/\rho_\mu(E_0)$
- the ratio of partial decay widths

Compare the result with the value which can be obtained from the measured branching fractions $\mathcal{B}(K^+ \to e^+\nu_e) = \Gamma(K^+ \to e^+\nu_e)/\Gamma = (1.55 \pm 0.07) \cdot 10^{-5}$ and $\mathcal{B}(K^+ \to \mu^+\nu_\mu) = (63.43 \pm 0.17)\,\%$.