Exercises Physics VI (Nuclei and Particles)
Summer Semester 2009

Exercise sheet Nr. 9 Work out until 09.07.2009

Exercise 1: K^0 Production (Points: 2)

Using a π^- beam shot onto a stational proton target one can produce K mesons through strong interaction. In a part of the allowed region for the pion momentum it is possible to create only K^0 mesons and no $\bar{K}^0$ mesons.

a) What is the minimal pion momentum to produce K^0 mesons? How does the reaction look in this case?

b) From which pion momentum can one produce also $\bar{K}^0$ mesons, and through which reaction?

Exercise 2: K^0 Oscillation (Points: 2)

At the time $t = 0$ and place $x = 0$, $N_0 = 10000$ K^0 mesons are produced. They propagate through the vacuum with a momentum of $p = p_x = 1$ GeV/c. Through the second order weak interaction processing the originally K^0 beam is a mixture of K^0 and $\bar{K}^0$ mesons at $t > 0$. In the following, assume that the CP-parity is conserved, e.g. $|K_S^0\rangle \equiv |K_1^0\rangle$ and $|K_L^0\rangle \equiv |K_2^0\rangle$.

a) Show that the number of K_S^0, K_L^0, K^0 and $\bar{K}^0$ mesons as function of time t in the kaon rest frame is given by ($\hbar = c = 1$):

$$\begin{aligned}
N_{K_S^0}(t) &= \frac{N_0}{2}\, e^{-\Gamma_S t} \\
N_{K_L^0}(t) &= \frac{N_0}{2}\, e^{-\Gamma_L t} \\
N_{K^0}(t) &= \frac{N_0}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2\cos(\Delta m\, t)\, e^{-\Gamma t}\right] \\
N_{\bar{K}^0}(t) &= \frac{N_0}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\cos(\Delta m\, t)\, e^{-\Gamma t}\right]
\end{aligned}$$

with

$$\Delta m = |m_S - m_L| \quad , \quad \Gamma_{S/L} = \frac{1}{\tau_{S/L}} \quad , \quad \Gamma = \frac{\Gamma_S + \Gamma_L}{2}$$

Here $m_{S/L}$ is the mass and $\tau_{S/L}$ the lifetime of K_S^0 and K_L^0 mesons.

Apply the formalism to obtain the wave function of K_S^0 and K_L^0 mesons

$$\left|K_{S/L}^0\right\rangle(t) = A_{S/L} \cdot e^{-im_{S/L}t} \cdot e^{-\Gamma_{S/L}t/2}$$

with $A_{S/L}$ as normalisation constant and phase space factor.

b) Make a graphical representation of the number of K_S^0, K_L^0, K^0 and $\bar{K}^0$ mesons on time scale from 0 to $2 \cdot 10^{-9}$ s. What is the corresponding flight path lenght? Use here $\Delta m = 5.3 \cdot 10^9$ s^{-1}.

Exercise 3: K^0 Regeneration (Points: 2)

The Kaon beam described in exercise 2 propagates over 100 cm and hits a target with a density of 10^{26} nucleons/cm^2. The absorbtion cross section of the kaons at 1 GeV/c momentum is on average $\sigma(K^0N) = 20$ mb and $\sigma(\bar{K}^0N) = 50$ mb. Calculate the number of K_S^0, K_L^0, K^0 and $\bar{K}^0$ in front of the target. What approximations can be made here? How many K_S^0, K_L^0, K^0 and $\bar{K}^0$ do we expect directly behind target.

Exercise 4: B^0 Oscillation (Points: 3)

a) Similar to K^0 mesons, also B^0 and B_s^0 mesons can change to their antiparticle through the weak interaction. Sketch the Feynman diagrams for the B^0 and B_s^0 oscillations.

b) For the B^0 and B_s^0 mesons the approximation $\tau := \tau_S = \tau_L = 1.5 \cdot 10^{-12}$ s can be made. Give the number of B^0 and $\bar{B}^0$ and B_s^0 and $\bar{B}_s^0$ mesons as well the asymmetry

$$A(t) = \frac{N_{B^0}(t) - N_{\bar{B}^0}(t)}{N_{B^0}(t) + N_{\bar{B}^0}(t)}$$

as a function of time t. Use $N_0 = 10000$ B^0 or B_s^0 respectively at $t = 0$.

c) In the B^0 system, Δm is $0.5 \cdot 10^{12}$ s^{-1}. For B_s^0 the measured value of Δm (usually denoted as Δm_s) is $17.8 \cdot 10^{12}$ s^{-1}. Plot the number of B^0, $\bar{B}^0$, B_s^0 and $\bar{B}_s^0$ mesons in both system, in the time range $0 \leq t \leq 10^{-11}$ s. To what distance does the oscillation period of a B^0 and B_s^0 with a momentum of 5 GeV/c correspond?