# Electromagnetics and Numerical Calculation of Fields

Lecture: Prof. Dr. rer nat Olaf Dössel

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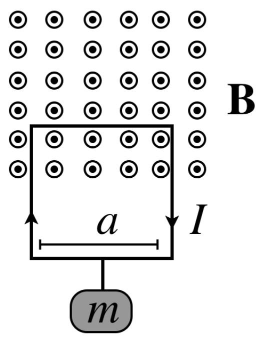
## Examination 2012

1. (a) Write down Maxwell's equations in differential and integral form. [8 pt]
   (b) Write down the materials equations that combine **E** with **D**, **B** with **H** and **J** with **E** for the most general case. [4 pt]

2. What is the force on a moving charge in an E and B field (simultaneously) ? [2 pt]

3. (a) What type of equation has to be solved for finding the electric potential $\phi$ in vacuum ($\epsilon_r = 1$) of an arbitrary charge distribution $\rho(x, y, z)$ ? [2 pt]
   (b) What are Dirichlet boundary conditions and Neumann boundary conditions? [2 pt]

4. A rectangular loop of wire, carrying a mass m, hangs vertically with one side ( length a ) in a uniform magnetic field **B** wich points into the page ( see figure). For what current $I$, the magnetic force upward would be the same as the gravitational force downward ? [4 pt]




5. A magnetic field

$$\mathbf{H} = \begin{pmatrix} 0 \\ 0 \\ a\sin(by)e^{bx} \end{pmatrix} \quad (1)$$

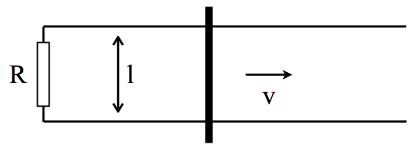
is produced by an electric current ($\frac{d}{dt}\mathbf{D} = 0$). What is the density of that current ? [4 pt]

6. What is the electric field inside and outside a sphere with radius R when the charge distribution with **spherical symmetry** is given by [6 pt]

$$\rho = \begin{cases} \frac{\rho_0 r}{R} & 0 \leq r \leq R \\ 0 & r > R \end{cases} \tag{2}$$

*Hint:* $dV = r^2 sin(\theta) d\phi d\theta dr$ *(spherical coordinates)*

7. A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart (see figure) . A resistor $R$ is connected across the rails an a uniform magnetic field **B** , pointing in to the pages, fills the entire region.




   (a) If the bar moves to the right at speed $v$, what is the current in the resistor [4 pt]
   (b) What is the magnitude of the magnetic force as a function of $v$ on the bar and what is the direction of the magnetic force ? [2 pt]
   (c) If the bar starts with speed $v_0$ at time $t = 0$ and is left to slide, what is the speed at a later time $t$ ? [2 pt]
   *Hint:* $F = m\frac{dv}{dt}$

8. (a) What is the formular to determine the energy density in an **electric** field ? [2 pt]
   (b) What is the formular to determine the energy density in an **magnetic** field ? [2 pt]
   (c) What is the power per volume lost in an ohmic conductor ? [2 pt]

9. (a) What is meaning of the Poynting vector **S** ? [2 pt]
   (b) Give the formula to determine **S** for a given electromagnetic field. [2 pt]

10. (a) What is characteristic for TM waves in a waveguide and what is characteristic for TE waves in a waveguide ? [2 pt]
    (b) What is the meaning (explanation, no formula) of "cut-off frequency" for a specific mode ? [1 pt]

11. Write down the central difference approximation used in the Finite Difference Method (FDM) for: [4 pt]

$$\frac{\partial}{\partial x}\phi \quad \text{and} \quad \frac{\partial^2}{\partial x^2}\phi \tag{3}$$

12. Give a sketch of Yee's lattice (FDTD). [6 pt]

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## Examination 2012: Solutions

1. (a)

$$\nabla \cdot \mathbf{D} = \rho \leftrightarrow \int_S \mathbf{D} d\mathbf{a} = \int \rho dV$$
$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \leftrightarrow \int_C \mathbf{E} d\mathbf{l} = -\frac{\partial}{\partial t}\int_S \mathbf{B} d\mathbf{a}$$
$$\nabla \cdot \mathbf{B} = 0 \leftrightarrow \int_S \mathbf{B} d\mathbf{a} = 0$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D} \leftrightarrow \int_C \mathbf{H} d\mathbf{l} = \int_S (\mathbf{J} + \frac{\partial}{\partial t}\mathbf{D}) d\mathbf{a}$$

(b)

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$
$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

2.

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

3. (a) Poisson equation:

$$\Delta \phi = -\frac{\rho}{\epsilon_0}$$

or Coulomb integral

$$\phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dv'$$

(b) Dirichlet: The potential is given on the boundaries:

$$\phi|_{\text{boundary}} = \hat{\phi}$$

Neumann: The normal derivative of the potential is given on the boundaries:

$$\frac{\partial}{\partial n}\phi|_{\text{boundary}} = \hat{\phi}_n$$

4.

$$\mathbf{F} = \int I(d\mathbf{l} \times \mathbf{B}) \Rightarrow F = IBa$$
$$IBa \stackrel{!}{=} mg \Rightarrow I = \frac{mg}{Ba}$$

5.

$$\begin{aligned}
\nabla \times \mathbf{H} &= \mathbf{J} \\
\nabla \times \mathbf{H} &= \begin{pmatrix} \frac{\partial H_z}{\partial y} \\ -\frac{\partial H_z}{\partial x} \\ 0 \end{pmatrix} \\
\Rightarrow \mathbf{J} &= abe^{bx} \begin{pmatrix} \cos(by) \\ -\sin(by) \\ 0 \end{pmatrix}
\end{aligned}$$

6. Since symmetry exists, we can apply Gauss's law to find **E**.

$$\epsilon_0 \oint \mathbf{E} d\mathbf{S} = Q_{enc} = \int \rho dv$$

For $r < R$ :

$$\begin{aligned}
\epsilon_0 E_r 4\pi r^2 &= Q_{enc} = \int_0^r \int_0^\pi \int_0^{2\pi} \rho r'^2 sin\theta d\phi d\theta dr' \\
&= \int_0^r 4\pi r^2 \frac{\rho_0 r'}{R} dr' = \frac{\rho_0 \pi r^4}{R} \\
\Rightarrow \mathbf{E} = \frac{\rho_0 r^2}{4\epsilon_0 R} \mathbf{a}_r
\end{aligned}$$

For $r > R$ :

$$\begin{aligned}
\epsilon_0 E_r 4\pi r^2 &= Q_{enc} = \int_0^r \int_0^\pi \int_0^{2\pi} \rho r'^2 sin\theta d\phi d\theta dr' \\
&= \int_0^R 4\pi r^2 \frac{\rho_0 r'}{R} dr' + \int_R^r 4\pi r^2 dr' = \pi \rho_0 R^3 \\
\Rightarrow \mathbf{E} = \frac{\rho_0 R^3}{4\epsilon_0 r^2} \mathbf{a}_r
\end{aligned}$$

7. (a)

$$U = -\frac{d}{dt} \int_0^t \mathbf{B} d\mathbf{a} = -\frac{d}{dt}(Blvt) = -Blv$$
$$U = RI \Rightarrow |I| = \frac{Blv}{R}$$

(b)

$$\mathbf{F} = \int I(dl \times B) \Rightarrow |\mathbf{F}| = IlB = \frac{B^2 l^2}{R} v$$

Direction: To the left (Lenz rule)

(c)

$$F = ma = m\frac{dv}{dt} = -\frac{B^2 l^2}{R} v$$

Minus since the force acts in opposite direction to the velocity !

$$\Rightarrow \frac{dv}{dt} = -\frac{B^2 l^2}{Rm} v$$

Ansatz for this kind of differential equation:

$$v(t) = ae^{bt}$$

Since $v(0) = v_0$ we find $a = v_0$ and furthermore:

$$\frac{dv}{dt} = v_0 b e^{bt} = -\frac{B^2 l^2}{Rm} v_0 e^{bt}$$
$$\Rightarrow b = -\frac{B^2 l^2}{Rm}$$
$$\Rightarrow v(t) = v_0 e^{-\frac{B^2 l^2}{Rm} t}$$

8. (a)

$$w_e = \frac{1}{2}\mathbf{E} \cdot \mathbf{D}$$

(b) What is the formular to determine the energy density in an **magnetic** field ? [2 pt]

$$w_e = \frac{1}{2}\mathbf{H} \cdot \mathbf{B}$$

(c) What is the power per volume lost in an ohmic conductor ? [2 pt]

$$\frac{\partial w_j}{\partial t} = \mathbf{J} \cdot \mathbf{E}$$

9. (a) The Poynting vector is the power per area that is transmitted through a given area via electromagnetic fields.

(b)

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \quad (1)$$

10. (a) For TE waves the electric field is completely transverse to the direction of propagation, $E_z = 0$. For TM waves the magnetic field is completely transverse to the direction of propagation, $H_z = 0$.

(b) For every mode, the cut-off frequency is the lowest frequency that allows undamped propagation.

11.

$$\left.\frac{\partial \phi}{\partial x}\right|_{ij} \approx \frac{\phi(i+1,j) - \phi(i-1,j)}{2\Delta x}$$

$$\left.\frac{\partial^2 \phi}{\partial x^2}\right|_{ij} \approx \frac{\phi(i+1,j) - 2\phi(i,j) + \phi(i-1,j)}{(\Delta x)^2}$$

12. Sketch of the Yee's lattice