Family name	First name	Matriculation number

☐ KSOP, M.Sc. Optics and Photonics
☐ other (please specify) ....

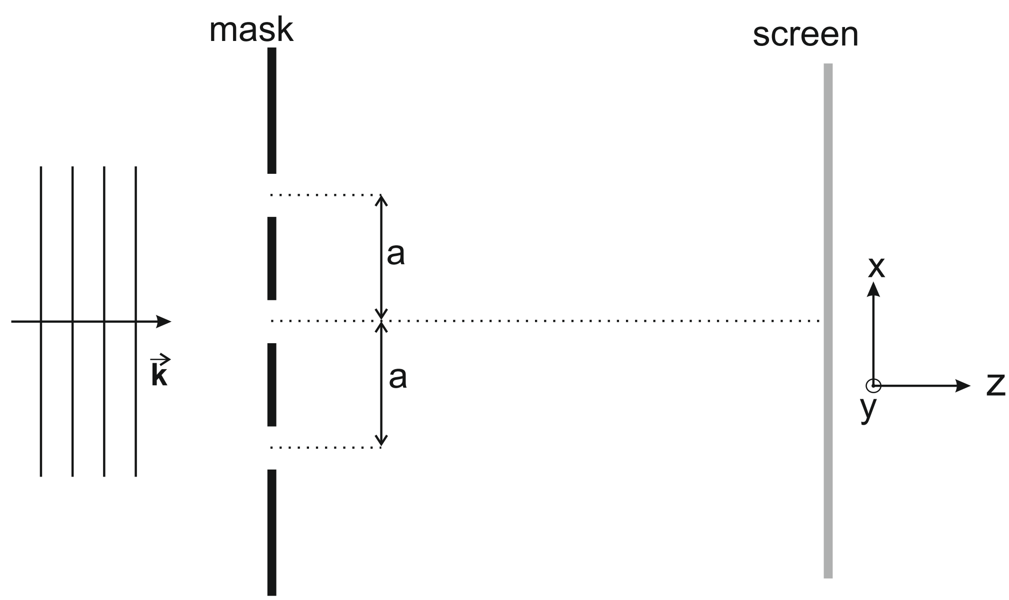
- 120 min to work on the questions
- Use **only** the sheets provided for your calcuations and answers
- Put your name on every sheet
- Use separate sheets for each question
- No calculators, no mobile phones, no electronic devices allowed
- No books, no personal notes, or other documents allowed
- Sine and Cosine function table can be found on the last page
- Generally, derive first a final equation before pluging in values.

No	Points possible	Points achieved
1	10	
2	10	
3	10	
4	10	
5	10	
Sum	**50**	

Mark:	Signature:
Final mark:	Signature:

1. **Three slits** (10 points)

A plane wave with wave vector $\vec{k}$ (coming from left) hits perpendicularly the mask containing three parallel slits along the y-direction as shown below. Each slit can be approximated by a delta-function.
a) Derive an expression for the pattern of the diffracted intensity (in x-direction) as function of $k_x a$ in the Fraunhofer approximation for this three-slit mask.
b) Calculate the zeros and the maxima of the diffraction pattern using the expression derived in a). Then, sketch the diffraction pattern as function of $k_x a$ using these points.
c) Now a phase plate is put in front of the central slit, retarding the wave by π.
Calculate again the diffraction pattern. How has it changed compared to the situation before (in **a)** + **b)**)?



2. **Grating** (10 points)

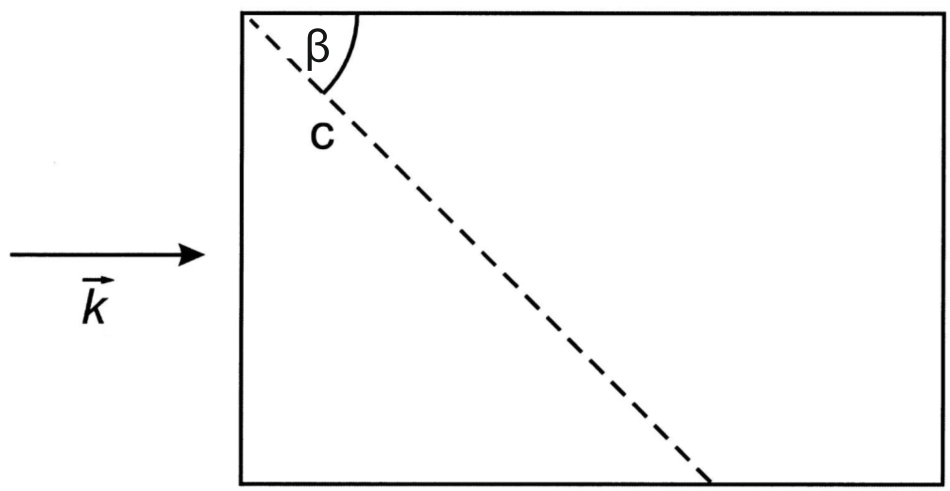
a) Sketch the diffraction pattern resulting from illumination of an optical grating in the Fraunhofer approximation. Discuss qualitatively the three main features of this diffraction pattern.
At which angles are the main maxima located? Use the approximation of small angles.

b) A transmission diffraction grating (width 6 mm) with 100 equidistant slits is homogeneously illuminated by a red laser beam (under normal incidence, Fraunhofer diffraction). The diffraction pattern is viewed on a screen, which is positioned at a distance of $L = 1$ m behind the grating. Two adjacent main maxima on this screen are separated by the distance $\Delta x = 10$ mm.

- Calculate the wavelength λ_0 of the red laser (value in nm).
- In addition to the red laser, a green laser is added to illuminate the grating. The red 5^{th} order main maximum coincides on the screen with the green 6^{th} order main maximum. Calculate the wavelength of the green laser (value in nm).
- The whole setup is now placed inside a liquid. The distance of two adjacent red main maxima is now reduced to $\Delta x_W = 8$ mm (from originally $\Delta x = 10$ mm). Calculate the value of the refractive index n of the liquid.

3. **Birefringence** (10 points)

a) An unpolarized, plane wave (propagating from the left to the right side) with wave vector $\vec{k}$ impinges under normal incidence on a birefringent crystal. Complete this sketch by drawing the ordinary and extraordinary beam inside the crystal and behind the crystal (it is not necessary to take into account the correct deflection direction, yet).



Add the directions of the vectors listed below for the ordinary and for the extraordinary beam for both inside and behind the crystal:

- wave vector $\vec{k}$
- dielectric displacement $\vec{D}$
- electric field vector $\vec{E}$
- Poynting vector $\vec{S}$

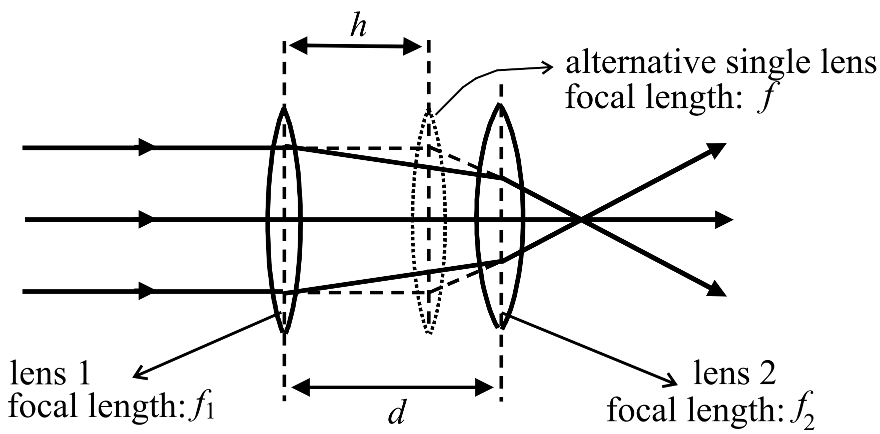
b) Justify briefly for **each** vector of the above list why you choose its specific direction for both the ordinary and extraordinary beam (only inside the crystal).

c) Deduce now, for the case of TiO_2 ($n_\perp = 2.616$, $n_{||}$ =2.903 and $\beta = 45°$) in which direction the extraordinary beam is actually deflected.

Hint: Consider the parallel and orthogonal components of $\vec{E}$ and $\vec{D}$ with respect to the crystal axis c.

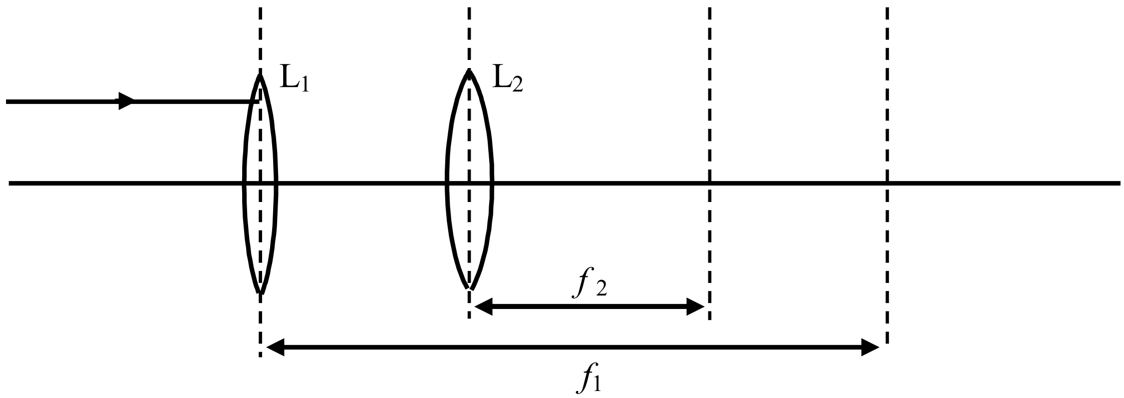
4. **Matrix optics** (10 points)

a) Consider the following optical system: Two thin bi-convex lenses (focal length f_1, f_2) are placed closely together (separated by the distance d). Light parallel to the optical axis hits the lens system and is focused to a point behind the lens pair, as shown by the drawing below.
This lens pair is now replaced by a single (alternative) lens, in a way that the rays behind lens 2 do not change. This condition implies that the focal point remains unchanged.



Using matrix optics, derive an expression for the focal length f of the alternative, single lens. Compute at which distance h the alternative, single lens has to be placed, relative to the position of the original lens 1 (see drawing).
Hint: In order to compare the two systems, use a beam parallel to the optical axis and consider the effect of the different systems on this beam.
The matrix of a thin lens in the basis $\begin{pmatrix} \mathrm{r} \\ \alpha \end{pmatrix}$ is given by $\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$.

b) Construct graphically the optical path of the incident ray for the system displayed below consisting of two thin convex lenses L_1 (focal length f_1) and L_2 (focal length f_2). Complete the drawing. (No calculation, only a graphical derivation required.)
Hint: Do not exclusively consider the optical path of this ray, but make use of other rays which can easily be constructed and help you.



c) Parallel Gaussian beam focussed by a convex lens.

- Sketch the phase planes of a parallel Gaussian beam focussed by a convex lens. Include the phase planes before the lens, between the lens and the focal point, and after the focal point.
- Which parameters define the beam waist (at the focal point)? How do these parameters have to be chosen in order to achieve a minimal beam waist?

5. **Lasers** (10 points)

a) Justify why it is impossible to achieve population inversion in a two-level system by optical pumping. Use Einstein's framework of the rate equations for a two-level system for your argumentation.

b) Discuss qualitatively a four-level system (a drawing helps). What limits the life time of the (upper) laser level before lasing occurs and how do these mechanisms add up?

c) What defines the wavelengths of possible longitudinal modes of a laser? Sketch the frequency dependency of the contributing mechanisms.
How can one achieve single (longitudinal) mode operation of a laser?

d) Derive from an uncertainty relation a condition/requirement for a laser medium, in order to allow for ultra-short (e.g. fs) pulses? Give an example of such a laser medium.

x	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$
sin(x)	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
cos(x)	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$