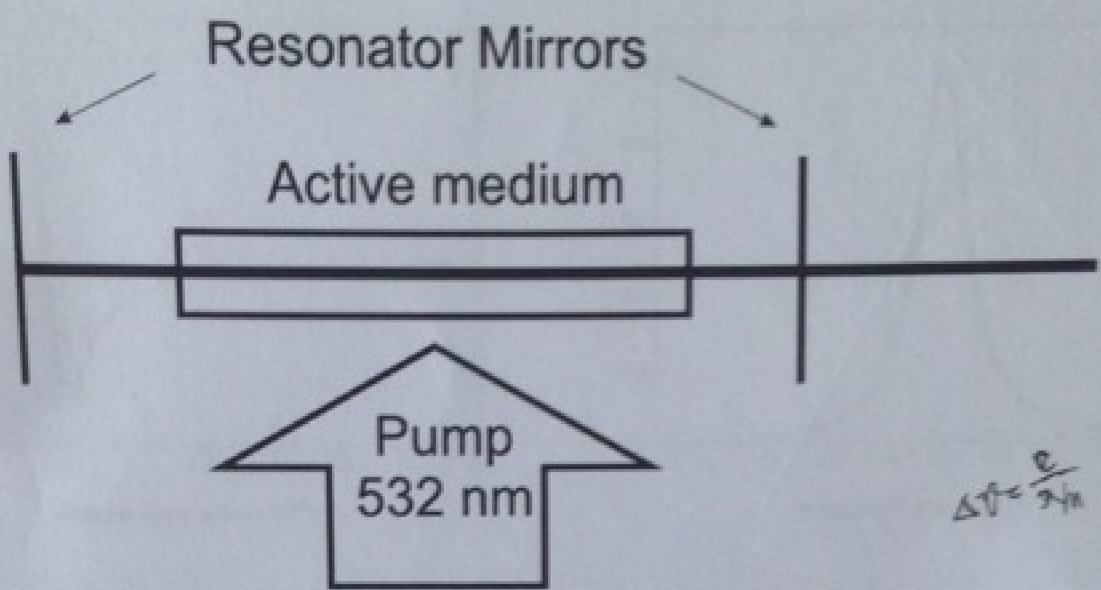
1. **Laser**



$\Delta\nu = \frac{c}{2dn}$ $\quad$ $\nu_{FSR} =$

A solid state laser is realized by putting a Ti^{3+}-doped sapphire (Ti:Sa) crystal as gain material into a Fabry-Pérot cavity of length d=30 cm and a Finesse $\mathcal{F}$. You may neglect the refractive index of the crystal for your calculations.

a) Derive an expression for the free spectral range ν_{FSR} of the cavity and calculate ν_{FSR} for the given resonator parameters and the speed of light $c=3 \cdot 10^8$ m/s.

b) Sketch qualitatively the resulting mode spectrum of the cavity as a function of the frequency ν. Due to dust on one of the mirrors the Finesse drops. Sketch qualitatively the new mode spectrum into the same graph. What happens to the Q-factor and the spectral half width $\delta\nu$ of the cavity when the Finesse drops to half of its original value?

c) The Ti:Sa crystal is now pumped using a frequency-doubled Nd:YAG laser (λ=532 nm). This generates a gain spectrum $\gamma(\lambda)$ within the laser crystal which can be described by

$$\gamma(\lambda) = \gamma_0 \frac{(\Delta\lambda)^2}{(\lambda - \lambda_0)^2 + (\Delta\lambda)^2}$$

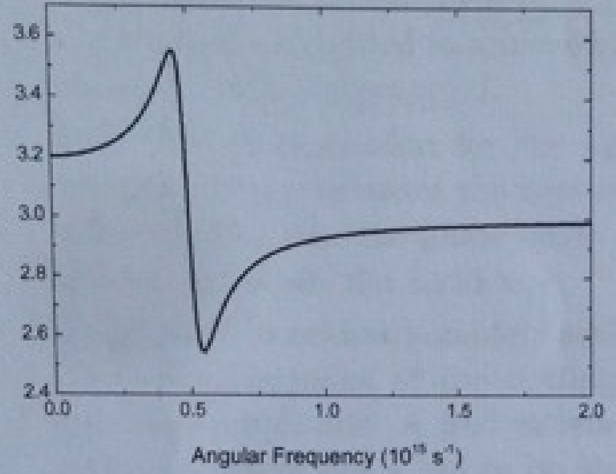
with λ_0=875 nm and $\Delta\lambda$=62.5 nm. The resonator losses can be considered constant with $\alpha_r = \frac{1}{5}\gamma_0$. What are the minimum wavelength and the maximum wavelength for which lasing can occur?

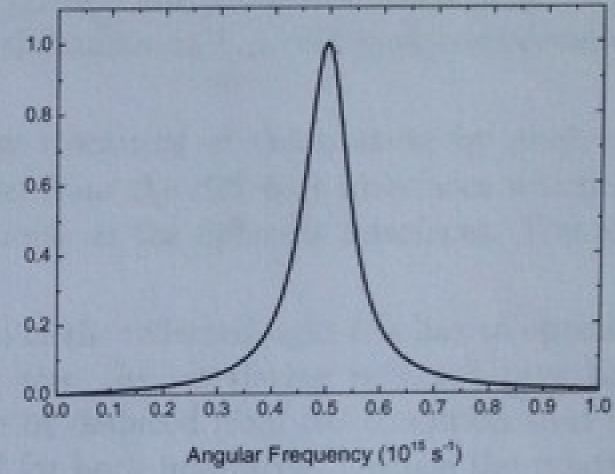
d) How many longitudinal laser modes are supported considering the gain spectrum as well as the cavity spectrum?

125

$5(\Delta\lambda)^2$

2. Lorentz Oscillator





The Lorentz oscillator is a simple classical model to describe the dielectric function $\epsilon(\omega)$ of a given material. This model yields a mathematical expression for $\epsilon(\omega)$ of a form

$$\epsilon(\omega) = \epsilon_b \left(1 + \frac{f}{\omega_0^2 - \omega^2 - i\gamma\omega}\right)$$

where f is the oscillator strength. The figure above shows the real part ϵ_1 and imaginary part ϵ_2 of this function.

a) Derive an expression for $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ and identify which graph belongs to $\epsilon_1(\omega)$ and which one belongs to $\epsilon_2(\omega)$.

b) Estimate from the above graphs the eigenfrequency ω_0, the dielectric background ϵ_b, the static dielectric constant ϵ_s and the broadening parameter γ of the resonance. State in one sentence: what is the reason for the dielectric background ϵ_b to be larger than 1? What is the value of ϵ_s when the oscillator strength f is 0?

c) State briefly what kind of resonances determine the dielectric function of an optical glass such as BK-7 and in which spectral region they lie. What is the approximate value of ϵ_2 in the visible spectral range? How does the dielectric function relate to the complex refractive index mathematically?

d) Sketch qualitatively the real part $\epsilon_1(\omega)$ of the dielectric function of a metal and mark a characteristic frequency. State in one sentence what you can say about the transparency of a metal above and below that frequency.

3. a) **Anti-reflection coating**

In order to optimize the coupling of light into a camera lens (n_{lens}=1.5) the surface is anti-reflection coated by a layer of thickness d and refractive index $n_{coat} < n_{lens}$. The coating is designed to achieve best performance at λ_{opt}. For your considerations you may neglect absorption.

i) Derive an expression for the optimum thickness of the coating by analyzing the interference between the rays reflected from the different interfaces which are involved. Describe the phase shifts that occur at the different interfaces. You may assume perpendicular incidence.

ii) In order to achieve complete extinction of the reflected light one has to optimize the refractive index of the coating such that the interfering reflected rays have the same amplitude. A first estimate can be deduced from the condition that the reflection coefficient ρ has to be identical for both interfaces. Deduce the relation between n_{lens} and n_{coat} of both materials for perpendicular incidence. Use the Fresnel formulas[1].

iii) State in one sentence what has to be modified in the Fresnel formulas if the coating material exhibits a considerable absorption.

b) **TRUE or FALSE?**

State whether the following statements are correct or not. Answer with TRUE or FALSE.

$l = c \cdot t_c$

i) The coherence time t_c corresponds to the reciprocal bandwidth $\frac{1}{\Delta\nu}$ of a wave package.

ii) Solids with a chiral symmetry often exhibit optical activity.

iii) The resolution of an optical grating only depends on the density of grooves.

iv) In Fraunhofer diffraction the intensity pattern of a single slit can be described by a sinc function.

v) The spontaneous emission rate depends on the density of photons with an energy of $E_{21}=E_2-E_1$.

vi) If you can measure the real part of dielectric function at a given wavelength you are also able to calculate the imaginary part of it.

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[1]
$$\rho_{TM} = \frac{n_2\cos(\theta_1) - n_1\cos(\theta_2)}{n_2\cos(\theta_1) + n_1\cos(\theta_2)}, \rho_{TE} = \frac{n_2\cos(\theta_2) - n_1\cos(\theta_1)}{n_2\cos(\theta_2) + n_1\cos(\theta_1)}$$

4. **Kerr cell**

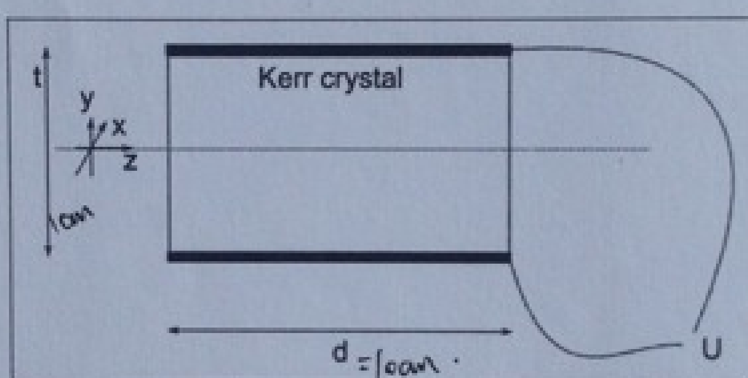
A laser beam of fixed wavelength λ is impinging on different optical systems.

a) The first system consists of 3 components one of which is a Kerr cell of length d. The effect of the first system on the polarization of an incident laser beam can be described by the following product of Jones matrices:

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{1}{2}sn^3E^2d\frac{2\pi}{\lambda}} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Identify the single components of the optical system and their characteristic action and state the order in which light is passing the system. Use the matrices provided below[2] and the trigonometric table on page 8. Derive a formula for the change in the refractive index Δn_y and sketch the dependence of Δn_y on the applied electrical field.

b) The laser of wavelength λ=800,nm is propagating along the direction of the z-axis



and impinges on a single Kerr cell. A transverse electrical field E_y is applied to this Kerr crystal of thickness t=1 cm and length d=10 cm. For a voltage U_1=100 V you observe an additional phase shift of π between the x-component and the y-component of the electrical field of the laser beam. Determine the refractive index n of the crystal for a Kerr coefficient of s=-100·10^{-16}m^2/V^2.

question c) see next page

[2]possible matrices: $\begin{pmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{pmatrix}$, $\begin{pmatrix} \cos^2(\theta) & \sin(\theta)\cos(\theta) \\ \sin(\theta)\cos(\theta) & \sin^2(\theta) \end{pmatrix}$

c) The electrical field of the laser beam (λ=800 nm) is given by $\vec{\mathcal{E}} = \left|\vec{\mathcal{E}_0}\right| \vec{J}$ with $\vec{J}$ being the Jones vector. The laser is right-hand circularly polarized. The laser impinges on another Kerr crystal with the same dimensions as in b) but with $\underline{\underline{s}}n^3 = -200 \cdot 10^{-16} \mathrm{m}^2/\mathrm{V}^2$. Determine the Jones vector of the laser beam after it passed such a Kerr crystal using an appropriate Jones matrix. What voltage do you have to apply to turn the incoming right-hand circularly polarized light of the laser beam into linearly polarized light at an angle of $\theta = 45°$ with respect to the x-axis? Use the Jones vectors provided below.[3]

$\frac{1}{2} \times 200$

[3]linearly polarized light: $\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$ right-hand circularly polarized light: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$
left-hand circularly polarized light: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$

5. **Double slit**

A plane wave with wave vector $\vec{k}$ (coming from the left) parallel to z hits a mask containing two parallel slits along the y-direction as shown below. Each slit is first approximated by a delta-function.

a) Derive an expression for the pattern of the diffracted intensity (in x-direction) as function of $k_x a$ in the Fraunhofer approximation for this double-slit mask using Fourier transformation.

b) Calculate the zeros and the maxima of the diffraction pattern using the expression derived in a). Then, sketch the diffraction pattern as function of $k_x a$ using these points.

c) Now a phase plate is put in front of one slit, retarding the wave by π.
Calculate again the diffraction pattern. How has it changed compared to the situation before (in **a)** + **b)**)?

d) Now the slits have a finite size. Explain briefly what changes in the diffraction pattern qualitatively.