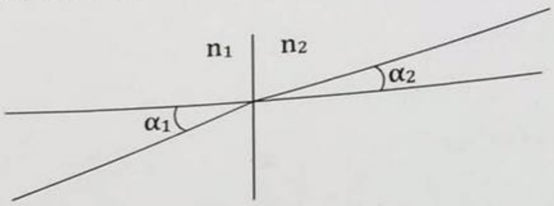
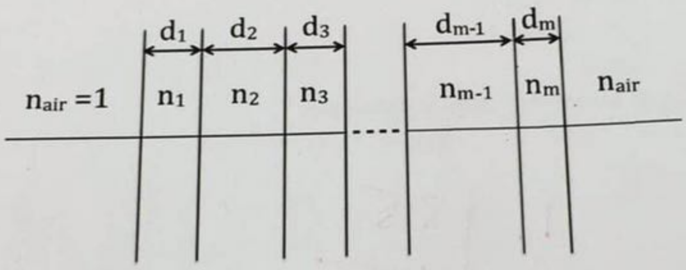
1. **Matrix Optics**

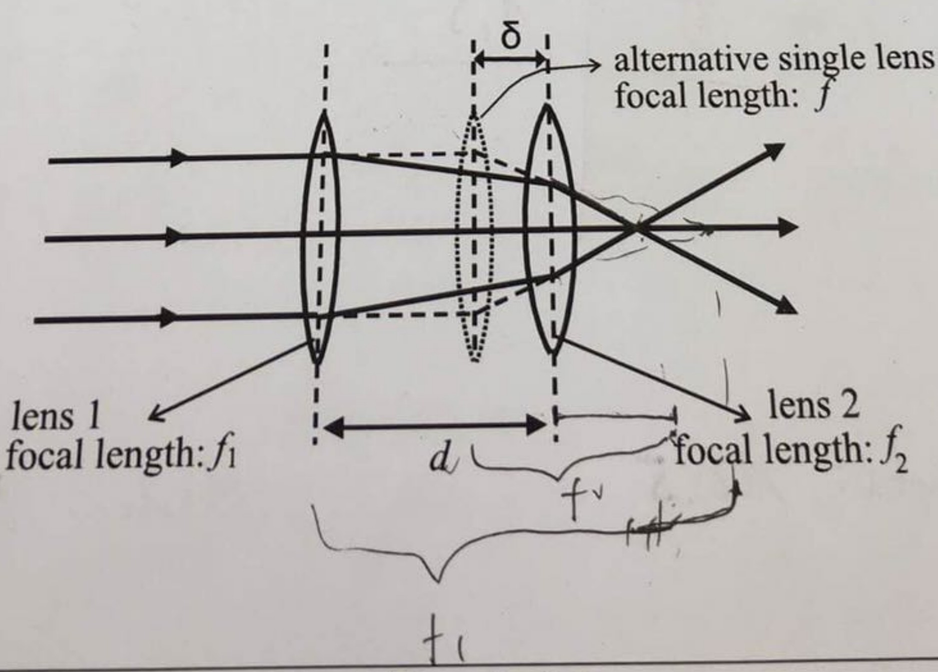
a. Derive the ray transfer matrix M for refraction at a transparent planar interface between two media of refractive indeces n_1 and n_2.



b. Consider a set of m parallel planar transparent plates of refractive indeces and thicknesses n_i and d_i respectively in air. Establish the ray transfer matrix of such system.



c. Consider a two thin bi-convex lenses of focal length f_1 and f_2. They are placed closely together separated by a distance d. Light parallel to the optical axis hits the lens system and is focused behind the lens pair. Derive the expression for the focal length f of an alternative single lens, such that the rays behind lens 2 do not change. At which distance δ does the lens has to be placed relative to the second lens?



2. **Electromagnetic waves**

Consider an electromagnetic plane wave in vacuum with the intensity I.

The electric field in vacuum is given by $\vec{E}(x,t) = \vec{E}_0 \cos(kx - \omega t)$ and the B-field by $\vec{B}(x,t) = \vec{B}_0 \cos(kx - \omega t)$. Note: In a linear, isotropic medium the relation $\frac{E_0}{H_0} = \sqrt{\frac{\mu_0 \mu}{\epsilon_0 \epsilon}}$ is valid.

$\vec{D} = \epsilon \vec{E}$ $\vec{B} = \mu \vec{H}$

a. Give the expressions for the impedance, $|E_0|$ and $|B_0|$ in vacuum. What are the units for these values? E - V/m B - T

b. Now the wave propagates in glass ($\epsilon = 1.44$, $\mu = 1$). (Reflections at the interface neglected). Express the ratio $\frac{E_0}{E_0^{\text{glass}}}$ and $\frac{B_0}{B_0^{\text{glass}}}$.

c. Express the pressure P, which is given by $P = c_0 \cdot |\vec{\pi}|$ ($\vec{\pi}$ is the momentum density) and the force F as a function of the intensity.

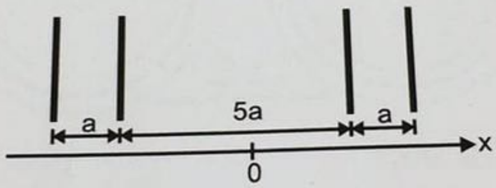
d. If the force exerted on a $10cm$ x $10cm$ square plate that absorbs all the light is $F = 46nN$ evaluate I and P.

e. This intensity is typical for which light source?

3. **Fraunhofer diffraction**

a. Give the Fraunhofer diffraction conditions of illumination and detection.

b. Within the framework of Fraunhofer diffraction establish the transmission function of the following arrangement of slits:



The width of the single slits can be neglected, so that the transmission function can be expressed as sum of δ-functions.

c. Calculate the diffraction pattern intensity by Fourier transformation of the transmission function. Express the resulting profile as product of cosine functions.

d. Sketch the resulting diffraction pattern. belong

e. Answer qualitatively: Describe the light wave coming from each individual slit.

f. Answer qualitatively: If the slit width can no longer be neglected. How is the diffraction intensity profile function affected?

4. **Interference**

4.1) Michelson Interferometer

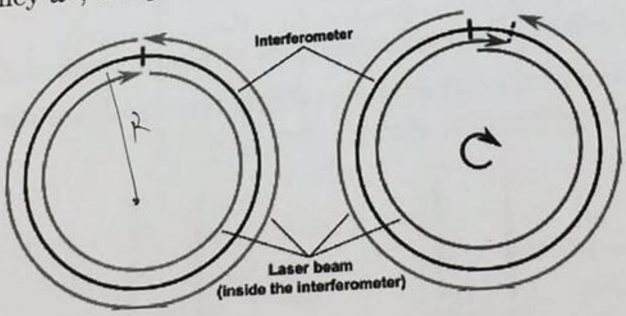
a. If one of the mirrors of the Michelson interferometer is misaligned by a small angle, describe the shape of the interference pattern in the detector plane. What happens to this pattern as the other mirror moves?.

b. How must the path length in a Michelson interferometer be so that constructive or destructive interference occurs?

c. If you move one mirror $30\mu m$ and you counted 100 fringes passing on your screen. What is the wavelength of the laser you are using to illuminate your interferometer?

4.2) Sagnac Interferometer

In a Sagnac interferometer the light can be guided with an optical light guide bent to a circle of radius R. The light needs a time t_1 to travel along the fiber at a velocity c in one direction and t2 in the opposite direction. At rest, $t1 = t2$. If the fiber rotates at an angular frequency ω , the path is enlarged or reduced by ΔL.



a. What is the expression of t_1?

b. What is the expression of ΔL?

c. Deduce from a. and b. an expression for t_1.

d. Formulate an analogous expression for t_2 and derive $Deltat$.

e. What is the phase shift condition as a function of $Deltat$ when they are brought in overlap?

f. Show that the phase shift between the two beams is then $\delta\phi = 8\pi^2 R^2 \omega / \lambda c$

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Prof. Dr. H. Kalt
22.02.2016 09:00-11:00

5. **Short answers**

   a. Write down the Helmholtz equation. For which type of waves can it be used?

   b. Give the definition of the "spectral resolution" of a spectrometer and describe its meaning in words.

   c. Explain why it is not possible to have a two level laser system?

   d. What is a phonon? What is an optical phonon? What is required for an optical phonon to couple to light?

   e. Give the expression for the Rayleigh criterion? The Rayleigh criterion describes that the performance of optical instruments is: .... Sketch the images of two point light sources and describe the condition for which the above formula is derived.

   f. A diffraction grating produces for a light of wavelength $\lambda = 500nm$ fourth-order pattern at an angle $\theta = \frac{\pi}{6}$. What is the number of slits for a grading of width $10mm$.

Helpful information:

x	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$
sin(x)	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
cos(x)	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$

$$\begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}, \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$$