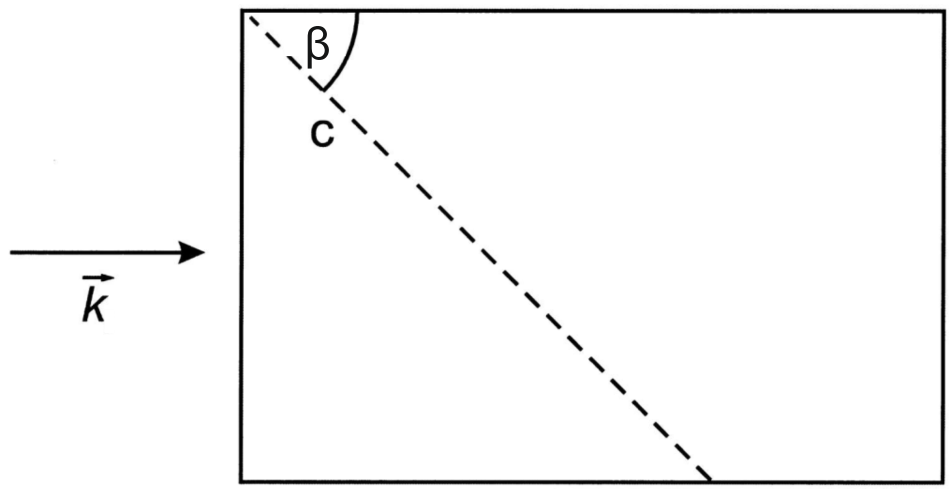
Family name	First name	Matriculation number

☐ KSOP, M.Sc. Optics and Photonics
☐ other (please specify) ....

- 90 min to work on the questions
- Use **only** the sheets provided for your calcuations and answers
- Put your name on every sheet
- Use separate sheets for each question
- No calculators, no mobile phones, no electronic devices allowed
- No books, no personal notes, or other documents allowed
- Generally, derive first a final equation before plugging in values.

No	Points possible	Points achieved
1	6	
2	6	
3	6	
Sum	**18**	

1. **Birefringence** (6 points)
a) An unpolarized, plane wave (propagating from the left to the right side) with wave vector $\vec{k}$ impinges under normal incidence on a birefringent crystal. Complete this sketch by drawing the ordinary and extraordinary beam inside the crystal and behind the crystal (it is not necessary to take into account the correct deflection direction, yet).



Add the directions of the vectors listed below for the ordinary and for the extraordinary beam for both inside and behind the crystal:

- wave vector $\vec{k}$
- dielectric displacement $\vec{D}$
- electric field vector $\vec{E}$
- Poynting vector $\vec{S}$

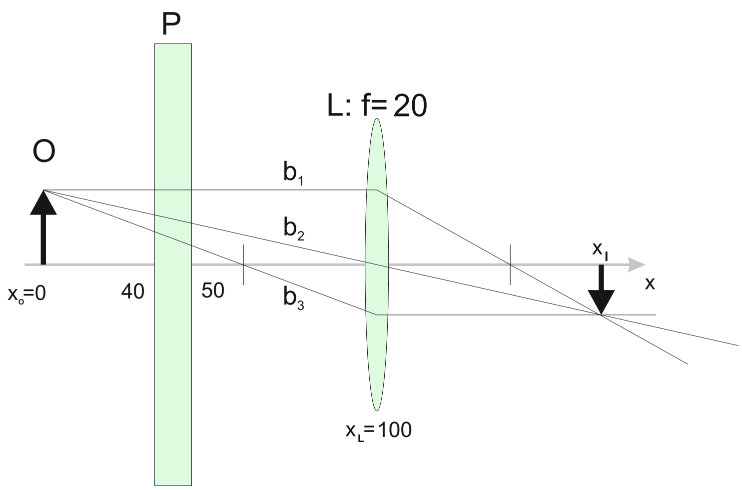
b) Justify briefly for **each** vector of the above list why you choose its specific direction for both the ordinary and extraordinary beam (only inside the crystal).

c) Deduce qualitatively, for the case of Calcite ($CaCO_3$,$n_\perp = 1.5$, $n_{||}$ =1.4 and $\beta = 45°$) in which direction the extraordinary beam is actually deflected.

Hint: Consider the parallel and orthogonal components of $\vec{E}$ and $\vec{D}$ with respect to the optical axis c.

d) Visualize the dependence of the refractive index on the angle Θ between the wavevector $\vec{k}$ and the optical axis c for a negative uniaxial and a positive uniaxial crystal.

2. **Geometrical and matrix optics** (6 points)
Consider the setup sketched below, which is used to image the object O by the lens L. The object O positioned at $x_O = 0$ is 30 units tall. The lens is considered as a thin lens. It is positioned at $x_L = 100$ and has a focal length of $f = 20$.
For problems 1a), 1b) and 1c) the plate P is not present, it is introduced in problem 1d).



a) First, neglect the plate P. Calculate at which position x_I the image of object O is situated.

b) Deduce the three ray vectors $\begin{pmatrix} r \\ \alpha \end{pmatrix}$ of the rays b_1, b_2, b_3 at $x_O = 0$ to use them for matrix optics (small angle approximation).

c) Find the distance of the 3 rays with respect to the optical axis at position x_I using matrix optics[1] (neglect the thickness of the lense L and neglect P)!

d) Now, a thick plate P (with refractive index $n = 3$) is introduced between $x = 40$ and $x = 50$. Calculate the distance to the optical axis of the three rays at x_I starting from the same ray vectors at $x = 0$ calculated in 1b).
(Hint: only calculate the necessary matrix elements).
Give a short argument why in this case there is no sharp image of the object O at the position x_I.

[1]possible matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$: $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ -\frac{1}{R}\frac{n_1-n_2}{n_2} & \frac{n_1}{n_2} \end{pmatrix}$

3. **Miscellaneous (6 points)**

   1. **Spatial Filter**
      Explain how a spatial filter works in the context of lasers. What is typically used as a spatial filter? How does a typical setup look like?

   2. **Helmholtz equation**
      Derive the Helmholtz equation starting from the 1d wave equation

      $$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

      using a harmonic ansatz of the form $E(x)e^{-i\omega t}$ and eliminate time dependence. What is the big advantage of this equation?

   3. **Beam phases**
      If you consider cylindrical symmetry for the Helmholtz equation you will end up with the paraxial Helmholtz equation. One solution of it is the Gaussian Principal mode. Explain briefly, what additional phase factors occur compared to a plane wave and what meaning they have.

   4. **Beam optics**
      A frequency doubled Nd:YAG laser ($\lambda = 532\,\mathrm{nm}$) of beam waist w_0 is focused using a lens of focal length f=100 mm to a beam waist of w_0'. What would be the beam waist w_0^* compared to w_0' if instead you used a frequency tripled Nd:YAG laser ($\lambda = 355\,\mathrm{nm}$) and a lens of f=150 mm?