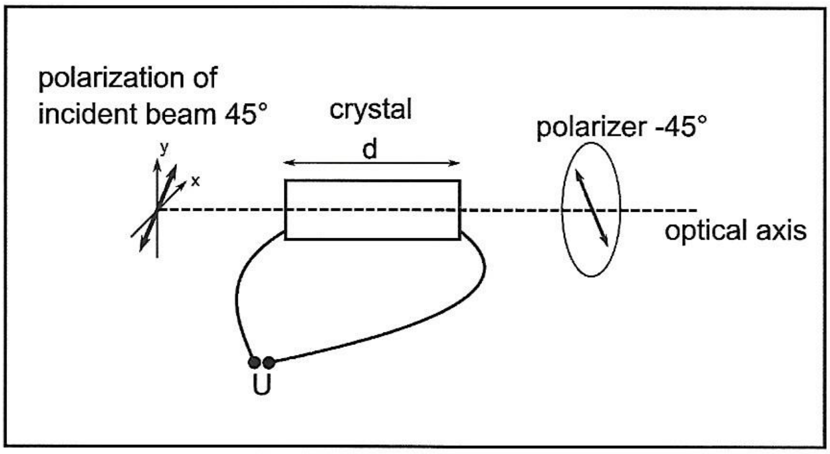


1. **Pockels effect**

The Pockels effect is linear in the applied electrical field E and can be described by

$$\Delta n = -\frac{1}{2} r n^3 E$$

A laser of wavelength $\lambda = 800\,\text{nm}$ is impinging on a Pockels cell. The setup is shown in the figure above. The electrical field of the laser beam is given by $\vec{\mathcal{E}} = \left|\vec{\mathcal{E}}_0\right| \vec{J}$ with $\vec{J}$ being the Jones vector. The linear polarization of the laser is 45° with respect to the x-axis and can be described by $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. The crystal (length d) is followed by a polarizer at an angle of $\theta = -45°$ with respect to the x-axis.

a) Derive a specific Jones matrix for the whole optical system (crystal and polarizer). Use the matrices provided below[1] and the trigonometric table on page 6.

b) Derive an expression for the phase shift Γ between the two components of the polarization vector of the laser beam depending on the applied voltage.

c) For a specific crystal a voltage of U=1000 V is applied. Determine the phase shift of the two polarization components of the laser beam after the crystal. What kind of polarization do you get directly behind the crystal? Furthermore, what is the outgoing intensity after the polarizer compared to the incoming intensity I_0 before the crystal? Use $rn^3 = 4 \cdot 10^{-10} \frac{m}{V}$.

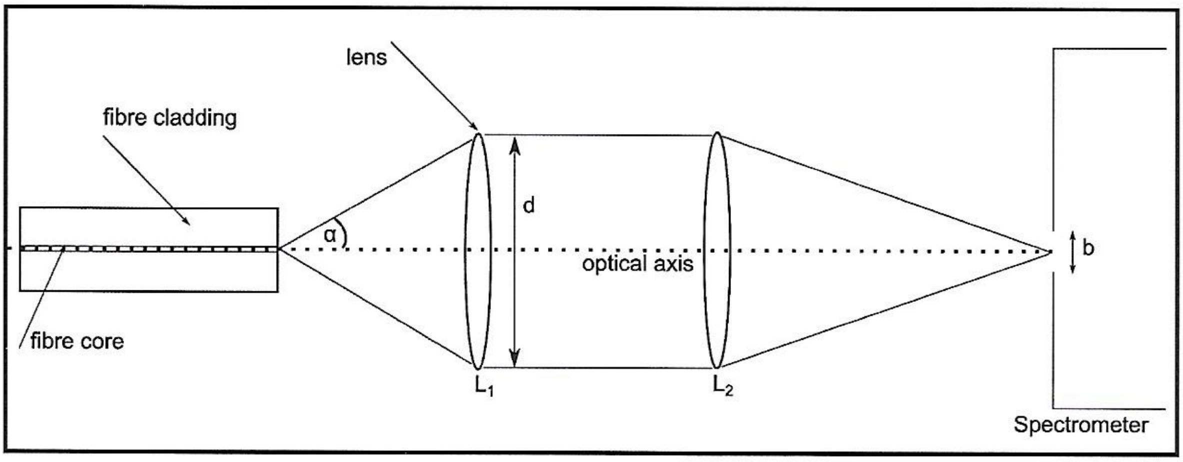
d) Sketch qualitatively the refractive index of a Pockels crystal as a function of the applied voltage and the resulting phase shift Γ. The Pockels cell is now replaced by a nonlinear Kerr medium. Do another sketch which shows the dependence of refractive index and phase shift on the applied voltage.

[1] possible matrices $\begin{pmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{pmatrix}$, $\begin{pmatrix} \cos^2(\theta) & \sin(\theta)\cos(\theta) \\ \sin(\theta)\cos(\theta) & \sin^2(\theta) \end{pmatrix}$

2. **Optical fibre and spectrometer (geometrical optics)**
An optical fibre is guiding light by the principle of total internal reflection. In a simplified picture such a fibre consists of a fibre core with a refractive index n_1 and a fibre cladding with a slightly lower refractive index n_2 so that total internal reflection can occur. At the end of such a fibre light is diverging and one can define the numerical aperture of the fibre as

$$NA = n_{air} \cdot \sin(\alpha)$$

analogously to the NA of a lens. The core diameter of the fibre is neglected for this

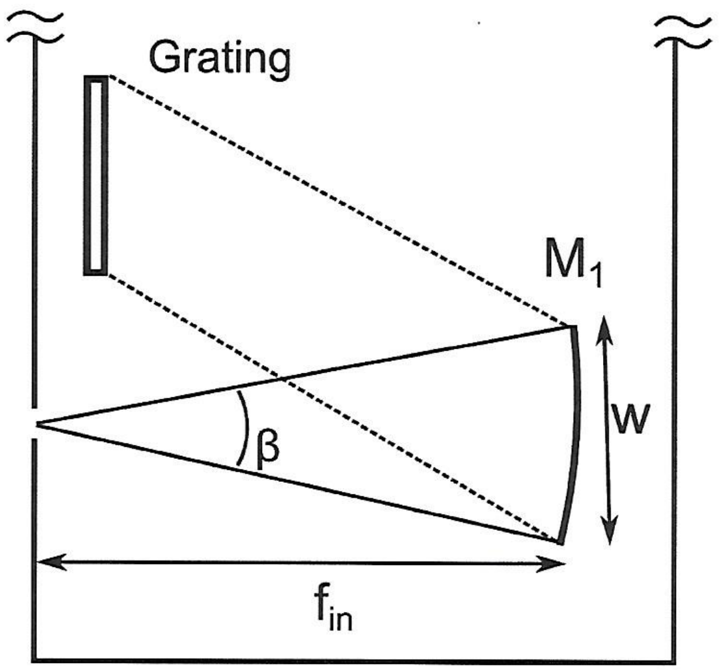


formula.
a) Light is emerging from such an optical fibre of NA=0.25. What lens diameter d would be necessary if you were to collimate the light without losing intensity with a lens L_1 of focal length f_1=100 mm? Use the small angle approximation.

b) Now we consider the fibre as an extended light source. You want to focus the light emerging from the fibre core (diameter 50 μm) onto the entrance slit of a spectrometer and you have to use a lens L_2 of focal length f_2=300 mm. To what size b can you close the entrance slit without losing light? Please derive first an expression for the magnification of the lens system. (Assume that no diffraction occurs at the entrance slit and thus geometrical optics apply)

Problem 2c) see next page

c) The focal length f_{in} of a Czerny-Turner spectrometer is 0.6 m and the width w of the first mirror M_1 is 0.314 m ($\approx \frac{\pi}{10}$ m). Calculate the acceptance angle β of the spectrometer. Calculate the NA of the entrance of the spectrometer and compare it to the NA of L_2 (see task b)). What can you say qualitatively about the illumination of mirror M_1?



3. **Miscellaneous**

   (a) **Beam phases**
   Explain briefly, what additional phase factors occur for a Gaussian principal mode compared to a plane wave and what meanings they have.

   (b) **True or False?**
   Answer the following questions by simply stating whether they are TRUE or FALSE. You don't need to elaborate your answer.

   A focused Gaussion beam has a Gaussian intensity distribution in both lateral and axial direction.

   When damping is present the Eigenfrequency of a harmonic oscillator is shifted relative to the undamped Eigenfrequency.

   The Q factor of a harmonic oscillator is proportional to the ratio of its decay time and oscillation period.

   Due to the Kramers-Kronig relations a dichroic crystal also exhibits birefringence.

   (c) **Focus spot size of a laser**
   You want to focus the output of a Nd:YVO_4 laser after frequency-doubling or after frequency-tripling using the same biconvex lens. The radii of curvature of the lens are R_1=R_2=24 mm. The refractive index is different for the two wavelengths. The exact values are $n_{355\,nm}$=1.4 and $n_{532\,nm}$=1.3. Calculate the focal length of the lens for both given wavelengths using the lensmaker's formula:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

   Calculate the ratio of the beam waist radii for the two wavelengths assuming Gaussian beam optics is valid.