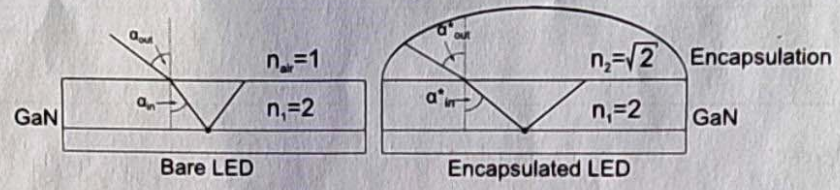
Family name	First name	Matriculation number
		1510454

- ☐ KSOP, M.Sc. Optics and Photonics
- ☐ other (please specify) ....

- 90 min to work on the questions
- Use **only** the sheets provided for your calcuations and answers
- Put your name on every sheet
- Use separate sheets for each question
- No calculators, no mobile phones, no electronic devices allowed
- No books, no personal notes, or other documents allowed
- Generally, derive first a final equation before plugging in values.

No	Points possible	Points achieved
1	6	1.5
2	6	0.5
3	6	3.5
Sum	**18**	5.5



1. **Useful encapsulation**
Consider a GaN (Gallium Nitride) light emitting diode (LED) with a wavelength of $\lambda_0 = \frac{4}{3}\pi \cdot 10^{-7}\,\mathrm{m}$ ($\approx 418\,\mathrm{nm}$) in a normal atmosphere ($n_{\mathrm{air}} = 1$). Light is generated within a pn-junction. For simplicity reasons assume that light is generated at a single point in GaN which has a refractive index of $n_1 = 2$.
a) Calculate the angle of the light cone $\alpha_{\mathrm{in,\,max}}$ from which the light can escape the GaN and is not trapped inside the GaN due to total internal reflection (left picture). Light which is generated at larger angles than $\alpha_{\mathrm{in,\,max}}$ is trapped within the volume and is lost due to reabsorption.
b) Now assume the LED is covered by a polymer encapsulation as it is common in commercial LEDs (right picture). The encapsulation has a refractive index of $n_2 = \sqrt{2}$. Calculate the new angle of the exit cone $\alpha^*_{\mathrm{in,\,max}}$ for the transition from the LED to the encapsulation. Is it larger or smaller than $\alpha_{\mathrm{in,\,max}}$? Sketch the reflectance R as a function of the angle α_{in} for the bare LED and for the LED with encapsulation for parallel and perpendicular polarization (compared to the plane of incidence).
c) The encapsulation also absorbs a part of the generated light. To consider this the material is described by its complex refractive index $\tilde{n} = n + i\kappa$. Derive an expression for the absorption coefficient α as a function of the wavelength λ by plugging the complex refractive index $\tilde{n}$ into a plane wave ansatz. Calculate the penetration depth z_{p} for the encapsulation when $\tilde{n} = \sqrt{2} + i \cdot 10^{-8}$ and the emission wavelength in vacuum is $\lambda_0 = \frac{4}{3}\pi \cdot 10^{-7}\,\mathrm{m}$. Explain in one sentence why the encapsulation does not strongly reduce the intensity of the emitted light due to absorption (when you assume a thickness of 5 mm)?
Hint: The penetration depth is defined by the distance at which the intensity drops to 1/e of the initial intensity.

2. **Polarisation and birefringence**

a) Consider linearly polarised light with an arbitrary polarisation angle θ (measured to the x-axis). The light is passing a half-wave plate. Show that transmitted light is polarised in $-\theta$ direction, so the polarisation has been rotated by an angle of 2θ. Explain briefly the difference to a polarization rotator concerning the effective rotation angle and the mechanism which is responsible for the rotation of polarisation. Use the matrices provided below[1].

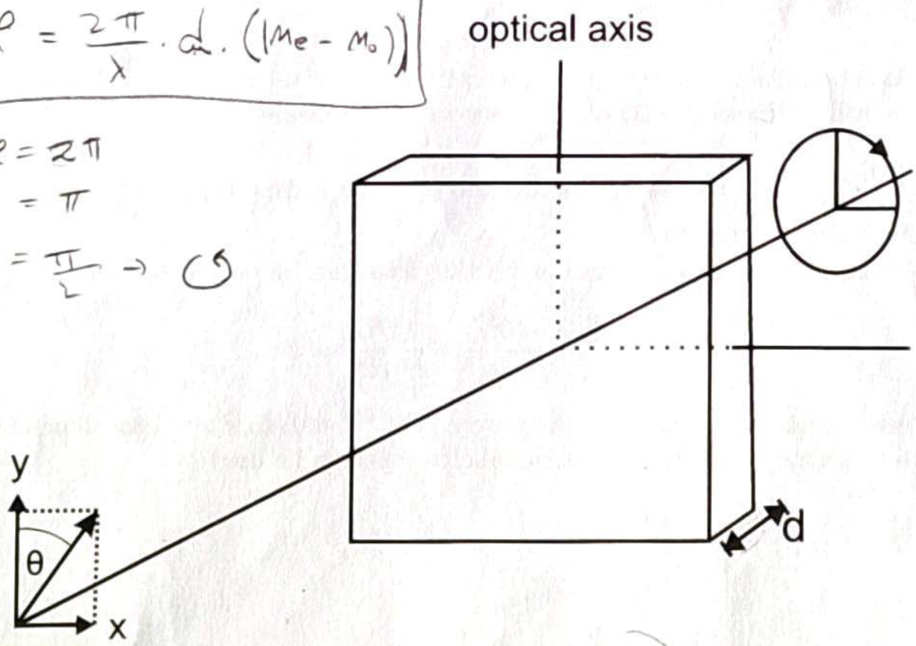
b) In potassium phosphate the refractive index depends on the E-field orientation. Consider a beam ($\lambda = 600\,\text{nm}$) propagating in z-direction linearly polarized at $\theta = 45°$ to the optical axis of the crystal (see figure). The refractive index for light polarized perpendicular to the optical axis is $n_2 = 1.5095$ and for light polarised parallel to the optical axis is $n_1 = 1.5085$. What is the minimum thickness of the crystal in order to obtain circularly polarized light behind the crystal.

$\frac{\pi}{2} = \boxed{\Delta\varphi = \frac{2\pi}{\lambda} \cdot d \cdot (|n_e - n_o|)}$

Full wave $\Rightarrow \Delta\varphi = 2\pi$

1/2 " $\Rightarrow \Delta\varphi = \pi$

1/4 " $\Rightarrow \Delta\varphi = \frac{\pi}{2} \rightarrow$



$\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$ $\begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}$

[1]possible matrices: $\begin{pmatrix}1 & 0 \\ 0 & e^{-i\Gamma}\end{pmatrix}$, $\begin{pmatrix}\cos^2(\theta) & \sin(\theta)\cos(\theta) \\ \sin(\theta)\cos(\theta) & \sin^2(\theta)\end{pmatrix}$, $\begin{pmatrix}\cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta)\end{pmatrix}$

3. **Miscellaneous**

(a) **Beam optics**

The laser beam from a frequency doubled Nd:YAG laser ($\lambda = 532\,\text{nm}$) of beam waist w_0 is focused using a lens of focal length f=100 mm to a beam waist of w_0'. What would be the beam waist w_0^* compared to w_0' if you used a frequency tripled Nd:YAG laser ($\lambda = 355\,\text{nm}$) and a lens of f=150 mm instead?

(b) **True or False?**

Answer the following questions by simply stating whether they are TRUE or FALSE. You don't need to elaborate your answer.

A focused Gaussion beam has a Gaussian intensity distribution in both lateral and axial direction.

The Q factor of a harmonic oscillator is proportional to the ratio of its decay time and oscillation period.

At a boundary between two media with different refractive indices the wavelength as well as the frequency of the electro-magnetic wave change.

The distance between anti-nodes in a standing wave pattern is $\frac{\lambda}{2}$.

(c) **Helmholtz equation**

Derive the Helmholtz equation starting from the 1d wave equation

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

using a harmonic ansatz of the form $E(z)e^{-i\omega t}$ and eliminate time dependence. For which type of waves can the Helmholtz equation be used?

x	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$
sin(x)	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
cos(x)	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$