1.

a) Why run moving clocks slower than clocks at rest?
b) What is length contraction?
c) The muon is an unstable elementary particle, which is generated in the atmosphere by cosmic radiation. Its mean lifetime is $\tau = 2.2 \cdot 10^{-6}$ s. The muon number decays according to the following law $N(t) = N_0 \exp(-t/\tau)$. The number of muons with the velocity 0.98·c is measured at an altitude of 2000 m and at sea level. How large is the ratio of the muon numbers, if the effect of time dilation is ignored?
d) How large is the ratio of muons really?

2.

a) Write down the Galilei transformation for two frames of reference moving with the relative velocity v!
b) Write down the corresponding Lorentz transformation!
c) Write down the Lorentz transformation for energy and momentum!
d) How transforms the energy of a photon?

3. Test of the relativistic Doppler effect with neon atoms. Neon atoms are moving with the velocity v throught a resonator formed by two mirrors. The light of a Laser with the frequency ν_0 is reflected back and forth between the mirrors of the resonator and the neon atoms and photons are moving along the same axis.

a) How large is in the rest frame of the neon atoms the frequency of the photons moving opposite to the direction of the neon atoms?
b) How large is in the rest frame of the neon atoms the frequency of the photons moving in the same direction as the neon atoms?

Three energy levels of the neon atom with $\Delta E_{12} = h\nu_1$, $\Delta E_{23} = h\nu_2$ and $\Delta E_{13} = h(\nu_1 + \nu_2)$ are used for the experiment. The transition frequencies are $\nu_1 = 5.0451 \cdot 10^{14}$ Hz and $\nu_2 = 5.0811 \cdot 10^{14}$ Hz.

c) How large is the frequency ν_0 of the Laser, when both transitions can be excited within the resonator?
d) How large is the velocity of the neon atoms, when this happens?

4. Photo effect

a) How would depend the kinetic energy of the photo-electrons on the frequency of the light, if the laws of classical physics could be applied?
b) How depends the kinetic energy of the photo-electrons on the frequency of light?
c) For nickel atoms the work function is 5 eV. How large is the threshold frequency for the emission of electrons?
d) How large is the smallest kinetic energy of the photo-electrons emitted from graphit (nuclear charge Z=6), when x-rays (Al-K_α-radiation) with an energy of 1486,6 eV are used?

5. The lattice parameter of NaCl is $d = 561 \cdot 10^{-12}$ m.

a) A beam of X-rays (wavelength $\lambda = 4 \cdot 10^{-10}$ m) strikes a NaCl crystal. What are the angles of the diffracted beams with respect to the incident beam?
b) How large is the energy of the photons forming the beam?
c) How large has to be the acceleration voltage, when the same experiment is carried out with an electron beam?

d) When the experiment is carried out with neutrons:
   i) How large is the kinetic energy of the neutrons?
   ii) What is the velocity of the neutrons?
   iii) The neutrons are produced in a fission reaction with a mean kinetic energy of ~2 MeV. How large is the velocity of these neutrons and how is it possible to slow down efficiently these neutrons?

6. A particle with the mass m_0 is captured in a potential well $E_{pot} = \frac{1}{2} D x^2$. The wavefunction of the groundstate is $\psi_1(x,t) \propto \exp\left(-\frac{x^2}{a_1^2}\right) \exp\left(-i\frac{E_1 t}{\hbar}\right)$.

a) Calculate the oscillation frequency of a classical particle in the potential well!
b) Write down the Schrödinger equation of a particle in the potential well!
c) Determine the relation between the energy E_1 and the parameter a_1 with the constant D and the mass m_0 !
d) Write down the wavefunction of the first excited state $\psi_2(x,t)$ und determine the corresponding energy E_2!

7. The distance between the two protons of the hydrogen molecule is d = 75 pm. Both protons rotate around the common centre of gravity.
a) Calculate the angular momentum and the kinetic energy for the classical motion of the two protons!
b) Write down the Schrödinger equation for the rotating hydrogen molecule! Neglect oscillations of the protons and excitations of the electrons.
c) Give the wavefunctions and energy eigenvalues of the Schrödinger equation!
d) Calculate the energy of the first excited state! How many wavefunctions exist for the first excited state?

8. The rare earth ion Ho^{3+} is characterized by ten 4f valence electrons.
a) Determine the groundstate of the $4f^{10}$ configuration.
b) Calculate the *g*-factor of the groundstate of the $4f^{10}$ configuration.
c) Determine the groundstate of the $4f^4$ configuration (Pm^{3+}).
d) Discuss the spin-orbit splitting of the groundstate of the $4f^4$ and $4f^{10}$ configuration.

Required physical constants:

Planck's constant:	$h = 4.14 \cdot 10^{-15}$ eVs
Velocity of light:	$c = 3 \cdot 10^{+8}$ m/s
Rest energy of the electron:	$m_e c^2 = 0.5$ MeV
Rest energy of the neutron:	$m_n c^2 = 939.6$ MeV
Rest energy of the proton:	$m_p c^2 = 938.3$ MeV
Rydberg constant:	$R_y = 13.6$ eV

Problem	1	2	3	4	5	6	7	8
Points	4	4	4	4	4	4	4	4

1.

a) Why run moving clocks slower than clocks at rest?
b) What is length contraction?
c) The muon is an unstable elementary particle, which is generated in the atmosphere by cosmic radiation. Its mean lifetime is $\tau = 2.2 \cdot 10^{-6}$ s. The muon number decays according to the following law $N(t) = N_0 \exp(-t/\tau)$. The number of muons with the velocity 0.98·c is measured at an altitude of 2000 m and at sea level. How large is the ratio of the muon numbers, if the effect of time dilation is ignored?
d) How large is the ratio of muons really?

a) The velocity of light is a physical constant and does not depend on the velocity of the observer. Therefore, more time is necessary for light to travel between to points in a moving frame of reference than in a frame of reference at rest.

b) A moving scale becomes shorter along the direction of motion.

c) For the muon the time of flight is $t = \dfrac{2000 \text{ m}}{0{,}98 \cdot 3 \cdot 10^{8} \text{ m/s}} = \dfrac{2}{3} \cdot 10^{-5} \text{ s}$.

$$\frac{N(t)}{N_0} = \exp\left(-\frac{\frac{2}{3} \cdot 10^{-5} \text{ s}}{2{,}2 \cdot 10^{-6} \text{ s}}\right) = 9{,}4 \cdot 10^{-4}.$$

d) With length contraction to path of the muons has the length

$$\ell(v) = \ell_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} = \ell_0 \sqrt{1 - 0{,}98^2} = 2000 \text{ m} \cdot 0{,}2 = 400 \text{ m}.$$

The time of flight is $t = \dfrac{400 \text{ m}}{0{,}98 \cdot 3 \cdot 10^{8} \text{ m/s}} = \dfrac{4}{3} \cdot 10^{-6} \text{ s}$.

$$\frac{N(t)}{N_0} = \exp\left(-\frac{\frac{4}{3} \cdot 10^{-6} \text{ s}}{2{,}2 \cdot 10^{-6} \text{ s}}\right) = 0{,}42.$$

One can alternatively consider the time dilation effect. The mean life-time $\tau = 2{,}2 \cdot 10^{-6}$ s is measured in the rest frame of the muon. The mean life-time of the moving muon is

$$\tau(v) = \frac{\tau}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \tau \cdot \frac{1}{\sqrt{1 - 0{,}98^2}} = \tau \cdot 5,$$

and

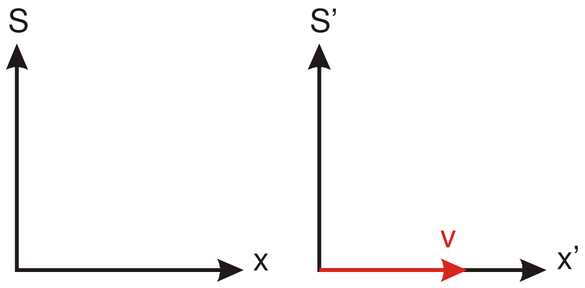
$$\frac{N(t)}{N_0} = \exp\left(-\frac{\frac{2}{3}\cdot 10^{-5}\ \text{s}}{5\cdot 2{,}2\cdot 10^{-6}\ \text{s}}\right) = \exp\left(-\frac{\frac{4}{3}\cdot 10^{-5}\ \text{s}}{2{,}2\cdot 10^{-5}\ \text{s}}\right) = 0{,}42$$

The time dilation effect can be verified in detail by observing muons with different velocities.

2.

a) Write down the Galilei transformation for two frames of reference moving with the relative velocity v!
b) Write down the corresponding Lorentz transformation!
c) Write down the Lorentz transformation for energy and momentum!
d) How transforms the energy of a photon?

a)



Galilei transformation

$$x = x' + vt$$
$$t = t'$$

b) Lorentz transformation

$$x = \gamma\left(x' + \frac{v}{c}(ct')\right)$$
$$ct = \gamma\left(ct' + \frac{v}{c}x'\right)$$

with $\gamma = \dfrac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$.

c) Energy and momentum transform according to

$x \to cP_x$ und $ct \to E$:

$$cP_x = \gamma\left(cP_x' + \frac{v}{c}E'\right)$$
$$E = \gamma\left(E' + vP_x'\right)$$

d) With $E' = h\nu'$ and $P_x' = \pm\dfrac{E'}{c}$ (+: the photon moves in direction of v, –: the photon moves opposite to the direction of v).

$$E = \gamma\left(E' + vP_x'\right) = \gamma\left(E' \pm v\frac{E'}{c}\right) = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\left(1 \pm \frac{v}{c}\right)E' = E'\sqrt{\frac{1 \pm \frac{v}{c}}{1 \mp \frac{v}{c}}}$$ (Doppler effect).

3. Measurement of the relativistic Doppler shift in Neon. Neon atoms are moving with the velocity v throught a resonator formed by two mirrors. The light of a Laser with the frequency ν_0 is reflected back and forth between the mirrors of the resonator and the neon atoms and photons are moving along the same axis.

a) How large is in the rest frame of the neon atoms the frequency of the photons approaching the neon atoms?

b) How large is the frequency ot the photons moving in the direction of the Neon atoms?

Three energy levels of the Neon with $\Delta E_{12} = h\nu_1$, $\Delta E_{23} = h\nu_2$ and $\Delta E_{13} = h(\nu_1 + \nu_2)$ are used for the experiment. The transition frequencies are $\nu_1 = 5.0451 \cdot 10^{14}$ Hz and $\nu_2 = 5.0811 \cdot 10^{14}$ Hz .

c) is the frequency ν_0 of the Laser, so that both transitions can be excited within the resonator?

d) How large is the velocity of the Neon atoms, when this happens?

a) The frequency of the approaching photons is enhanced (Doppler-effect)

$$\nu_+ = \nu_0 \sqrt{\frac{c+v}{c-v}} \, .$$

b) The frequency of photons moving in the direction of the Neon atoms is reduced

$$\nu_- = \nu_0 \sqrt{\frac{c-v}{c+v}}$$

c) In order to excite both transitions one has

$$\nu_+ = \nu_2 \qquad \text{and} \qquad \nu_- = \nu_1$$

i.e. $\dfrac{\nu_+}{\nu_0} = \sqrt{\dfrac{c+v}{c-v}} = \dfrac{\nu_0}{\nu_-} \rightarrow \nu_0 = \sqrt{\nu_- \cdot \nu_+} = \sqrt{5{,}0451 \cdot 5{,}0811} \cdot 10^{14} \text{ Hz} = 5{,}0631 \cdot 10^{14} \text{ Hz}$.

Alternatively on can excite the transitions $\Delta E_{12} = h\nu_1$ or $\Delta E_{23} = h\nu_2$ and $\Delta E_{13} = h(\nu_1 + \nu_2)$

then: $\nu_+ = \nu_1 + \nu_2$ and $\nu_- = \nu_1$ or $\nu_- = \nu_2$ since $\nu_1 \approx \nu_2$ and $\nu_0 = 7{,}15 \cdot 10^{14}$ Hz

d) The velocity of the Ne atoms:

$$\nu_+ = \sqrt{\nu_- \cdot \nu_+} \sqrt{\frac{c+v}{c-v}}$$

$$\frac{\nu_+}{\nu_-} = \frac{c+v}{c-v} \rightarrow \frac{\nu_+}{\nu_-}(c-v) = c+v \rightarrow c\left(\frac{\nu_+}{\nu_-} - 1\right) = v\left(\frac{\nu_+}{\nu_-} + 1\right) \rightarrow \frac{v}{c} = \frac{\left(\frac{\nu_+}{\nu_-} - 1\right)}{\left(\frac{\nu_+}{\nu_-} + 1\right)}$$

with $\nu_+ = \nu_2$ and $\nu_- = \nu_1$

$$\frac{v}{c} = \frac{\left(\frac{\nu_+}{\nu_-} - 1\right)}{\left(\frac{\nu_+}{\nu_-} + 1\right)} = \frac{3{,}55 \cdot 10^{-3}}{2{,}00355} = 1{,}77 \cdot 10^{-3} \rightarrow v = 5{,}32 \cdot 10^{5} \text{ m/s}$$

Remark: The mean thermal velocity of Neon is ~400 m/s. With v ~ 500 km/s is the deviation from classical behaviour still small. The experiment is possible due to the high accuracy of frequency measurements.

or with $\nu_+ = \nu_1 + \nu_2$ and $\nu_- = \nu_1$ or $\nu_- = \nu_2$: $\frac{\nu_+}{\nu_-} = 2$

$$\frac{v}{c} = \frac{(2-1)}{(2+1)} = \frac{1}{3} \rightarrow v = \frac{1}{3}c$$

4. Emission of electrons from a surface by light.

a) How would depend the kinetic energy of the photo-electrons on the freqeuncy of the light, if the laws of classical physics could be applied?

c) How depends the kinetic energy of the photo-electrons on the frequency of light?

c) For Nickel atoms the work function is 5 eV. How large is the threshold frequency?

d) How large is the smallest kinetic energy of the photo-electrons emitted from graphit, when x-rays (Al-K_α-radiation) with an energy of 1486,6 eV are used?

a) $\vec{E}(\vec{r},t) = \vec{E}_0 \exp\left(i\left(\vec{k}\vec{r} - \omega t\right)\right)$

$$\rightarrow m \cdot a = qE \rightarrow v = \int a dt = \frac{q}{m} E_0 \frac{1}{i\omega} \exp\left(i\left(\vec{k}\vec{r} - \omega t\right)\right) \rightarrow |v|^2 = \left(\frac{q}{m}\frac{E_0}{\omega}\right)^2 \rightarrow E_{kin} \propto \frac{1}{\omega^2}$$

b) $E_{kin} \propto \nu \qquad (E_{kin} = h \cdot \nu - W_A)$

c) $h \cdot \nu_G = W_A \rightarrow \nu_G = \dfrac{5 \text{ eV}}{4{,}14 \cdot 10^{-15} \text{ eVs}} = 12{,}08 \cdot 10^{14} \text{ Hz}$

d) 1s electron have the highest binding energy. With Moseley's law and the nuclear charge $Z = 6$ one gets the binding energy

$$E_{1s} = 13{,}6 \cdot \text{eV} \cdot (Z-1)^2$$

$$E_{1s} = 13{,}6 \cdot 5^2 \text{ eV} = 340 \text{ eV}$$

and

$$\rightarrow E_{kin} = 1486{,}6 \text{ eV} - 340 \text{ eV} = 1146{,}6 \text{ eV}$$

5. The lattice parameter of NaCl is $d = 561 \cdot 10^{-12}$ m.

a) A beam of X-rays (wavelength $\lambda = 4 \cdot 10^{-10}$ m) strikes a NaCl crystal. What are the angles of the diffracted beams with respect to the incident beam?

b) How large is the energy of the photons forming the beam?

c) How large is the acceleration voltage, when the same experiment is carried out with electrons?

d) Finally the experiment is carried out with neutrons.
   i) How large is the kinetic energy of the neutrons?
   ii) What is the velocity of the neutrons?
   iii) The neutrons are produced in a fission reaction with a mean kinetic energy of ~2 MeV. How large is the velocity of these neutrons and how is it possible to slow down efficiently these neutrons?

a) With Bragg's law $n\lambda = 2d\sin\theta_n$

$$\sin\theta_n = \frac{n\lambda}{2d} = n\frac{4 \cdot 10^{-10}\ \text{m}}{2 \cdot 561 \cdot 10^{-12}\ \text{m}} = n \cdot 0{,}36.$$

Only the 1st and 2nd order can be observed.
The angles of deflection are $2\theta_1 = 41{,}8°$ und $2\theta_2 = 91°$.

b) $E_\gamma = h \cdot \nu = \dfrac{h \cdot c}{\lambda} = \dfrac{4{,}14 \cdot 10^{-15}\ \text{eVs} \cdot 3 \cdot 10^8\ \text{m/s}}{4 \cdot 10^{-10}\ \text{m}} = 3{,}1\ \text{keV}$

c) The momentum is

$$\lambda = \frac{h}{P} \rightarrow P = \frac{h}{\lambda}$$

$$\rightarrow E_{kin} = \frac{P^2}{2m_e} = \frac{h^2 \cdot c^2}{2\lambda^2 m_e \cdot c^2} = \frac{1}{2}\left(\frac{4{,}14 \cdot 10^{-15}\ \text{eVs} \cdot 3 \cdot 10^8\ \text{m/s}}{4 \cdot 10^{-10}\ \text{m}}\right)^2 \frac{1}{500\ \text{keV}} = 9{,}64\ \text{eV}$$

The acceleration voltage is 9,64 V.

d)
i)

$$E_{kin} = \frac{P^2}{2m} = \frac{h^2 \cdot c^2}{2\lambda^2 m \cdot c^2}$$

$$= \frac{1}{2}\left(\frac{4{,}14 \cdot 10^{-15}\ \text{eVs} \cdot 3 \cdot 10^8\ \text{m/s}}{4 \cdot 10^{-10}\ \text{m}}\right)^2 \frac{1}{939{,}6\ \text{MeV}} = 5{,}1\ \text{meV} = 8{,}17 \cdot 10^{-22}\ \text{J}$$

ii) $v = \sqrt{\dfrac{2E_{kin}}{m}} = c\sqrt{\dfrac{2 \cdot 5{,}1 \cdot 10^{-3}\ \text{eV}}{939{,}6 \cdot 10^6\ \text{eV}}} = c \cdot 0{,}33 \cdot 10^{-5} = 991{,}3\ \text{m/s}$

iii) $v = \sqrt{\dfrac{2E_{kin}}{m}} = c\sqrt{\dfrac{2 \cdot 2 \cdot 10^6\ \text{eV}}{939{,}6 \cdot 10^6\ \text{eV}}} = c \cdot 0{,}065 = 19574\ \text{km/s}$

Neutrons can be slowed down by collisions with atoms. E.g., the masses of protons and neutrons are nearly equal. A beam of high energy neutrons can be guided through water, in order to slow down these neutrons efficiently.

6. A particle with the mass m_0 is captured in a potential well $E_{pot} = \frac{1}{2} D x^2$. The wavefunction of the groundstate is $\psi_1(x,t) \propto \exp\left(-\frac{x^2}{a_1^2}\right) \exp\left(-i\frac{E_1 t}{\hbar}\right)$.

a) Calculate the oscillation frequency of a classical particle in the potential well!
b) Write down the Schrödinger equation of a particle in the potential well!
c) Determine the relation between the energy E_1 and the parameter a_1 with the constant D and the mass m_0 !
d) Write down the wavefunction of the first excited state $\psi_2(x,t)$ und determine the corresponding energy E_2!

a) Newton's 2nd law: $m_0 a = -Dx$. With $x = x_0 \sin \omega t$ and $a = -\omega^2 x_0 \sin \omega t$ one gets

$$m_0 \omega^2 = D \rightarrow \omega = \sqrt{\frac{D}{m_0}}$$

b) Schrödinger equation: $i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m_0} \frac{\partial^2 \psi}{\partial x^2} + \frac{1}{2} D x^2 \psi$

c) $i\hbar \frac{\partial \psi_1(x,t)}{\partial t} = E_1 \cdot \psi_1(x,t)$

$$\frac{\partial \psi_1}{\partial x} = -\frac{2x}{a_1^2} \psi_1, \quad \frac{\partial^2 \psi_1}{\partial x^2} = \frac{4x^2}{a_1^4} \psi_1 - \frac{2}{a_1^2} \psi_1$$

With Schrödinger's equation one gets

$$E_1 \cdot \psi_1 = -\frac{\hbar^2}{2m_0} \left(\frac{4x^2}{a_1^4} - \frac{2}{a_1^2} \right) \psi_1 + \frac{1}{2} D x^2 \psi_1$$

$$E_1 = \frac{\hbar^2}{m_0 a_1^2} \quad \text{and} \quad \frac{4\hbar^2}{m_0 a_1^4} = D \rightarrow a_1^2 = \frac{2\hbar}{\sqrt{m_0 D}}$$

and

$$\rightarrow E_1 = \frac{\hbar^2}{m_0 2\hbar} \sqrt{D m_0} = \frac{1}{2} \hbar \sqrt{\frac{D}{m_0}} = \frac{1}{2} \hbar \omega$$

d) The wavefunction of the 1st excited state has one node, i.e.

$$\psi_2(x,t) = A_2 \cdot x \exp\left(-\frac{x^2}{a_2{}^2}\right)\exp\left(-i\frac{E_2 t}{\hbar}\right).$$

A_2 is the amplitude of the wavefunction.

With

$$\frac{\partial \psi_2}{\partial x} = A_2 \cdot \exp\left(-\frac{x^2}{a_2{}^2}\right)\exp\left(-i\frac{E_2 t}{\hbar}\right) - A_2 \cdot \frac{2x^2}{a_2{}^2}\exp\left(-\frac{x^2}{a_2{}^2}\right)\exp\left(-i\frac{E_2 t}{\hbar}\right)$$

$$\begin{aligned}
\frac{\partial^2 \psi_2}{\partial x^2} = &-A_2 \cdot \frac{2x}{a_2{}^2}\exp\left(-\frac{x^2}{a_2{}^2}\right)\exp\left(-i\frac{E_2 t}{\hbar}\right) \\
&- A_2 \cdot \frac{4x}{a_2{}^2}\exp\left(-\frac{x^2}{a_2{}^2}\right)\exp\left(-i\frac{E_2 t}{\hbar}\right) \\
&+ A_2 \cdot \frac{4x^3}{a_2{}^4}\exp\left(-\frac{x^2}{a_2{}^2}\right)\exp\left(-i\frac{E_2 t}{\hbar}\right) \\
= &\, A_2 \cdot \left(-\frac{6x}{a_2{}^2} + \frac{4x^3}{a_2{}^4}\right)\exp\left(-\frac{x^2}{a_2{}^2}\right)\exp\left(-i\frac{E_2 t}{\hbar}\right)
\end{aligned}$$

and the Schrödinger equation one gets

$$E_2 x = -\frac{\hbar^2}{2m_0}\left(\frac{4x^3}{a_2{}^4} - \frac{6x}{a_2{}^2}\right) + \frac{1}{2}Dx^3 \rightarrow E_2 = \frac{3\hbar^2}{m_0 a_2{}^2} \text{ und } \frac{4\hbar^2}{m_0 a_2{}^4} = D \rightarrow a_2{}^2 = \frac{2\hbar}{\sqrt{m_0 D}} = a_1{}^2$$

and

$$E_2 = \frac{3}{2}\hbar\sqrt{\frac{D}{m}} = \frac{3}{2}\hbar\omega$$

7. The distance between the two protons of the hydrogen molecule is d = 75 pm. Both protons rotate around the common centre of gravity.

a) Calculate the angular momentum and the kinetic energy for the classical motion of the two protons!

b) Write down the Schrödinger equation for the rotating hydrogen molecule! Neglect oscillations of the protons and excitations of the electrons.

c) Give the wavefunctions and energy eigenvalues of the Schrödinger equation!

d) Calculate the energy of the first excited state! There are how many wavefunctions of the first excited state?

a) Angular momentum: $L = r \cdot m_p \cdot v$.

With $r = \frac{d}{2}$ one gets $L = 2 \cdot \frac{d}{2} m_p v = d m_p v$.

The kinetic energy is $E_{kin} = 2 \cdot \frac{1}{2} m_p v^2 = m_p v^2 = \frac{d^2 m_p{}^2 v^2}{m_p d^2} = \frac{L^2}{m_p d^2}$

b) Schrödinger equataion $i\hbar \frac{\partial \psi}{\partial t} = \frac{\hat{L}^2}{m_p d^2} \psi$

c) The solution of Schrödinger's equation is

$$\psi_{\ell,m}(\theta,\varphi) = Y_{\ell,m}(\theta,\varphi) \cdot \exp\left(-i \frac{E_\ell \cdot t}{\hbar}\right) \text{ with } E_\ell = \frac{\ell(\ell+1)\hbar^2}{m_p d^2}$$

The $Y_{\ell,m}(\theta,\varphi)$ are spherical harmonics.

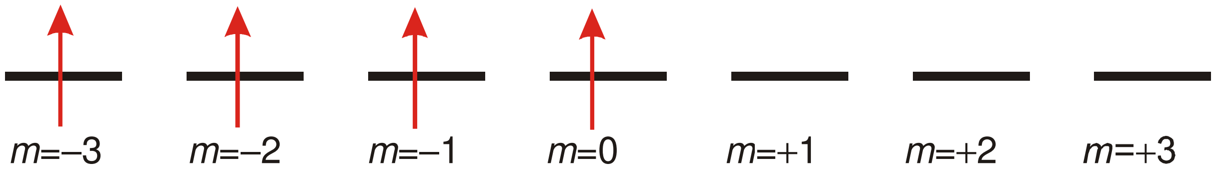
d) The energy of the first excited state is

$$E_{\ell=1} = \frac{2 \cdot \hbar^2}{m_0 d^2} = \frac{2 \cdot \left(4{,}14 \cdot 10^{-15} \text{ eVs}\right)^2 \cdot \left(3 \cdot 10^8 \text{ m/s}\right)^2}{4\pi^2 \cdot 938.3 \cdot 10^6 \text{ eV} \left(75 \cdot 10^{-12} \text{ m}\right)^2} = 14.8 \text{meV}$$

For the first excited state there are three wave functions $Y_{1,-1}$, $Y_{1,0}$ und $Y_{1,+1}$, which determine the orientation of the molecule.

8. The rare earth ion Ho^{3+} is characterized by ten 4f valence electrons.
a) Determine the groundstate of the $4f^{10}$ configuration.
b) Calculate the *g*-factor of the groundstate of the $4f^{10}$ configuration.
c) Determine the groundstate of the $4f^{4}$ configuration (Pm^{3+}).
d) Discuss the spin-orbit splitting of the groundstate of the $4f^{4}$ and $4f^{10}$ configuration.

a) For the Ho^{3+} ion there are 4 electron holes in the 4f shell and the application of Hund's rules yields



$$L = \left|-3-2-1+0\right| = 6 \text{ and } S = 4\cdot\frac{1}{2} = 2$$

Since $4f^{10}$ is an hole configuration, the spin-orbit constant $\xi<0$ so that $J = 8$. The groundstate is: $^5I_8$.

b) The *g*-factor of $^5I_4$ is $g = 1 + \dfrac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$.

$$g = 1 + \frac{8\cdot 9 - 6\cdot 7 + 2\cdot 3}{2\cdot 8\cdot 9} = 1 + \frac{36}{2\cdot 8\cdot 9} = \frac{5}{4} = 1.25$$

c) The $4f^4$ configuration is an electron configuration. Therefore L=6 and S=2 as for the $4f^{10}$ configuration, but $\xi>0$ so that $J = 4$. The groundstate is $^5I_4$.

d) For $4f^4$ the order of states is with increasing energy $^5I_4$, $^5I_5$, $^5I_6$, $^5I_7$, $^5I_8$.

For $4f^{10}$ the order of states is reversed: $^5I_8$, $^5I_7$, $^5I_6$, $^5I_5$, $^5I_4$.

Due to the larger nuclear charge of the Ho^{3+} ion is the spin-orbit splitting larger than the spin-orbit splitting of the Pm^{3+} ion.