1.
A spaceship with the length L_{R0} moves with a velocity of $0.3 \cdot c$. An observer at rest sees how two flashes hit the spaceship at both ends at the same time.

a) Where is the observer relative to the spaceship when the flashes happen and where is the observer relative to the spaceship when he notes the two flashes?
b) Write down the Lorentz transformations.
c) At which times do the flashes occur in the frame of the spaceship?
d) Where is the pilot of the spaceship when he also notes the flashes at the same time?

2.
An ambulance drives with a velocity of 100 km/h over a crossroad. The traffic light is red ($\lambda_{\text{rot}} = 650$ nm) and the siren of the ambulance has a frequency of 1 kHz. Nearby the crossroad is placed a radar trap emitting electromagnetic waves with the frequency 200 MHz.

a) How large is the frequency of the siren heard by a pedestrian waiting at the crossroad after the ambulance has passed? The velocity of sound is $c_S = 340$ m/s.
b) The electromagnetic wave of the radar trap is reflected by the approaching ambulance. Calculate the frequency shift of the reflected wave recorded by the detector of the radar trap.
c) How large is the velocity of the ambulance when the traffic light appears to be green ($\lambda_{\text{grün}} = 525$ nm) for the driver of the ambulance?
d) Explain the essential difference between the classical Doppler effect and the Doppler effect of electromagnetic waves.

3. Consider the photo-electric effect.

a) Sketch an experimental set-up for the measurement of the photo-current as a function of the applied potential.
b) Explain why the photo-current saturates for a high value of the potential and why there is a stopping potential, which is independent of the intensity of the light source.
c) The photo-current starts immediately after the photo-cathode is illuminated. Why is the observation in contradiction to classical physics?
d) The work function of gold is 5.3 eV. Determine the lowest frequency for which a photo-current can be detected and explain why no photo-current can be measured with visible light.

4.
In Bohr's model of the atom, the orbit of an electron moving around a nucleus with the charge number Z is calculated according to the laws of classical physics but the electron itself is regarded as a matter wave according to de Broglie, which obeys the condition $2\pi r = n\lambda$ for standing waves.

a) Calculate the possible radii of the orbits as a function of the nuclear charge Z and the quantum number n.
b) Calculate the allowed velocities as a function of the nuclear charge Z and the quantum number n. Determine the nuclear charge for which the electron in the groundstate ($n = 1$) becomes faster than the velocity of light in vacuum?
c) Explain the generation of x-rays with an x-ray tube and sketch the x-ray spectrum.
d) Explain the changes of the x-ray spectrum when (i) the heating current of the tube or (ii) the acceleration voltage is increased.

Please note reverse side.

5.

a) Write down the Schrödinger equation for an electron in the electric field of the proton (neglect the spin of the electron).

b) The solution of the Schrödinger equation is $\psi = R_{n\ell}(r) Y_{\ell m}(\theta,\varphi)$. What is the name of the quantum numbers n, ℓ and m and write down which values they can take.

c) What is the meaning of the quantum numbers ℓ and m?

d) Sketch the functions $R_{n\ell}(r)$ for $n = 2$ and justify the shape of the curve in each case.

6. The crystal lattice of Co is hexagonal with the lattice parameters $a = 251$ pm and $c = 407$ pm. Remark: The hexagonal plane is formed by two vectors of length a and an angle of 120° in between. The lattice vector of length c is perpendicular to the hexagonal plane.

a) Give the condition defining the reciprocal lattice and calculate the lattice parameters of the reciprocal lattice.

b) Indicate the directions of the reciprocal lattice vectors in the hexagonal plane of the crystal lattice.

c) What is the Laue condition for X-ray diffraction?

d) What is the smallest scattering angle for an x-ray beam with the wavelength 217 pm within the hexagonal plane of Co?

7.

a) What is the spin of the electron and how was the spin discovered?

b) Explain why the first excited state of the hydrogen atom $n = 2$ is splitted into three states. Make a sketch of the splitting and give the quantum numbers of these states.

c) How can the influence of a homogeneous static magnetic field be included in the Schrödinger equation?

d) What is the Paschen-Back effect?

8. The density of aluminium is $\rho = 2.7$ g/cm^3. The atomic mass is 27 g/mol and the electronic configuration is [Ne]3s^{2}3p^1.

a) Calculate the Fermi energy of Al. Remark: All valence electrons contribute to the electric conductivity.

b) What is the difference between the specific heat capacity of a metal and an insulator?

c) What is an energy band?

d) Why are most of the elements in the solid state metals?

Required physical constants:

Planck's constant:	$h = 4.14 \cdot 10^{-15}$ eVs
Velocity of light:	$c = 3 \cdot 10^{8}$ m/s
Electric field constant:	$\varepsilon_0 = 8.85 \cdot 10^{-12}$ As/Vm
Elementary charge:	$e = 1.6 \cdot 10^{-19}$ As
Mass of the electron:	$m_e = 9.1 \cdot 10^{-31}$ kg
Avogardo's number:	$N_A = 6 \cdot 10^{23}$ mol^{-1}

Problem	1	2	3	4	5	6	7	8
Points	4	4	4	4	4	4	4	4

1.

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b) Write down the Lorentz transformations.
c) At which times do the flashes occur in the frame of the spaceship?
d) Where is the pilot of the spaceship when he also notes the flashes at the same time?

a) The flashes occur, when the center of the spaceship has reached the observer, since the light of each flash need the same time to reach the observer.

The length of the spaceship is reduced in the frame of the observer

$L_R(v) = L_{R0}\sqrt{1-\left(\frac{v}{c}\right)^2} = L_{R0}\sqrt{1-0{,}3^2} = 0.954 \cdot L_{R0}$. Each flash needs the time

$t_F = \frac{L_R(v)}{2c}$ to reach the observer. The spaceship moves during that time

$\Delta\ell = v \cdot t_F = \frac{v}{c} L_R(v)$. The distance to the stern of the spaceship is

$L_S = \frac{L_R(v)}{2} - \Delta\ell = L_R(v)(0.5-0.3) = 0.2 \cdot L_R(v)$ and correspondingly the distance to the front $L_F = 0.8 \cdot L_R(v)$

(1)

b) Lorentz transformation. The frame S' is moving with the velocity v to the right with respect to frame S. Then

$$x = \gamma\left(x' + \frac{v}{c}(ct')\right) \qquad x' = \gamma\left(x - \frac{v}{c}(ct)\right)$$
$$ct = \gamma\left(ct' + \frac{v}{c}x'\right) \quad \text{and} \quad ct' = \gamma\left(ct - \frac{v}{c}x\right)$$

with $\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$.

(1)

c) When the stern of the spaceship has reached the coordinate $x_1 = 0$ at the moment when the flash occurs at $t = 0$ and the front of the spaceship the coordinate $x_2 = L_R(v) = 0.954 \cdot L_{R0}$, then the times of the flashes in the frame of the spaceship are

$$t_1' = 0 \;\text{ und }\; ct_2' = \gamma\left(-\frac{v}{c}\right)x_2 = -\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}0.3\cdot L_{R0}\sqrt{1-\left(\frac{v}{c}\right)^2} \rightarrow t_2' = -0{,}3\frac{L_{R0}}{c} \;.$$

(1)

d) When the pilot is at position x_P in the spaceship, then the light from the stern of the spaceship needs the time

$$t_s = \frac{x_P}{c},$$

and the light from the front

$$t_v = \frac{L_{R0} - x_P}{c} + t_2'$$

to reach the pilot.
With

$$t_s = t_v$$

$$\frac{x_P}{c} = \frac{L_{R0} - x_P}{c} - 0.3\frac{L_{R0}}{c}$$

$$2\frac{x_P}{c} = 0.7\frac{L_{R0}}{c}$$

one get for the position of the pilot

$$x_P = 0.35L_{R0}$$

(1)

2.

An ambulance drives with a velocity of 100 km/h over a crossroad. The traffic light is red ($\lambda_{\text{rot}} = 650\ \text{nm}$) and the siren of the ambulance has a frequency of 1 kHz. Nearby the crossroad is placed a radar trap emitting electromagnetic waves with the frequency 200 MHz.

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c) How large is the velocity of the ambulance when the traffic light appears to be green ($\lambda_{\text{grün}} = 525\ \text{nm}$) for the driver of the ambulance?

d) Explain the essential difference between the classical Doppler effect and the Doppler effect of electromagnetic waves.

a) The wavelength increases due to the motion of the ambulance. The wavelength noted by an observer at the crossroad is

$\lambda_B = (c_S + v)\dfrac{1}{f_H}$ and the frequency

$$f_B = \frac{c_S}{\lambda_B} = f_H \frac{c_S}{c_S + v} = 1\ \text{kHz}\frac{340\ \text{m/s}}{340\ \text{m/s}\ +\ 100 \cdot 10^3\ \text{m/3600 s}} = 924\ \text{Hz}$$

(1)

b) The relativistic Dopper-effect for an approaching source is $\nu_+ = \nu_0\sqrt{\dfrac{c+v}{c-v}}$. This is the frequency received by the ambulance. The ambulance acts as a source when the wave is reflected. The frequency of the reflected wave is therefore

$$\nu_R = \nu_+\sqrt{\frac{c+v}{c-v}} = \nu_0 \frac{c+v}{c-v} \ .$$

Since $\dfrac{v}{c} << 1$ it is possible to make the following approximation

$$\nu_R = \nu_0\left(1+\frac{v}{c}\right)\left(1+\frac{v}{c}+...\right) = \nu_0\left(1+2\left(\frac{v}{c}\right)+...\right).$$

The frequency shift is

$$\nu_R = \nu_0 + 200\cdot 10^6\ \text{Hz}\cdot 2\cdot\frac{10^5\ \text{m/3600 s}}{3\cdot 10^8\ \text{m/s}} = \nu_0 + 37\ \text{Hz}$$

(1)

c)

With $\nu\cdot\lambda = c \rightarrow \lambda_+ = \lambda_0\sqrt{\dfrac{c-v}{c+v}}$, d.h. $\dfrac{525\ \text{nm}}{650\ \text{nm}} = \sqrt{\dfrac{c-v}{c+v}}$.

$$0.652\left(1+\frac{v}{c}\right) = \left(1-\frac{v}{c}\right) \rightarrow 1.652\frac{v}{c} = 0.348 \rightarrow v = 0.21\cdot c$$

(1)

d) For electromagnetic waves is only the relative motion of source and receiver important. In the case of sound waves is also the motion relative to the air important. In addition, the time dilatation effect is negligible.

(1)

3. Consider the photo-electric effect.

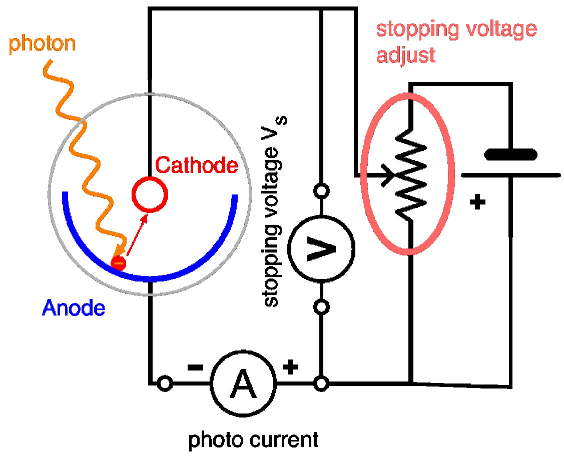
a) Sketch an experimental set-up for the measurement of the photo-current as a function of the applied potential.

b) Explain why the photo-current saturates for a high value of the potential and why there is a stopping potential, which is independent of the intensity of the light source.

c) The photo-current starts immediately after the photo-cathode is illuminated. Why is the observation in contradiction to classical physics?

d) The work function of gold is 5.3 eV. Determine the lowest frequency for which a photo-current can be detected and explain why no photo-current can be measured with visible light.

a)



(1)

b) The saturation of the photo-current is determined by the intensity of the light source, i.e. by the number of photons.
The stopping voltage is determined by the energy of the photon according to $eU_B = hf - W_A$.

(1)

c) In classical physics is the energy of the photo-electron proportional to the intensity of the light wave. A certain time after the start of the illumination is necessary to collect enough energy so that a photo-electron can be emitted by the photo-cathode.

(1)

d) With $eU_B = hf - W_A$ and $0 = hf_k - W_A \rightarrow f_k = \dfrac{W_A}{h} = \dfrac{5.3\text{ eV}}{4.14 \cdot 10^{-15}\text{ eVs}} = 1.28 \cdot 10^{15}\text{ Hz}$

The wave length is

$\lambda = \dfrac{c}{f_k} = \dfrac{3 \cdot 10^{8}\text{ m/s}}{1.28 \cdot 10^{15}\text{ 1/s}} = 2.34 \cdot 10^{-7}\text{ m} = 234\text{ nm} < 400\text{ nm (blue)}$. The photoelectric effect starts in the UV range of the electromagnetic spectrum.

(1)

4.

In Bohr's model of the atom, the orbit of an electron moving around a nucleus with the charge number Z is calculated according to the laws of classical physics but the electron itself is regarded as a matter wave according to de Broglie, which obeys the condition $2\pi r = n\lambda$ for standing waves.

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b) Calculate the allowed velocities as a function of the nuclear charge Z and the quantum number n. Determine the nuclear charge for which the electron in the groundstate ($n = 1$) becomes faster than the velocity of light in vacuum?

c) Explain the generation of x-rays with an x-ray tube and sketch the x-ray spectrum.

d) Explain the changes of the x-ray spectrum when (i) the heating current of the tube or (ii) the acceleration voltage is increased.

a) Centripetal force = Coulomb force

$$\frac{m_e v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

de Broglie $\lambda = \frac{h}{m_e v} \rightarrow \frac{m_e^{\,2} v^2}{r} = \frac{m_e}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} = \frac{h^2}{r\lambda^2} = \frac{h^2 n^2}{r^3\left(2\pi\right)^2}$

$$r = \frac{h^2\varepsilon_0}{\pi m_e e^2}\frac{n^2}{Z} = \frac{\left(4{,}14\cdot 10^{-15}\ \text{eVs}\right)^2 8{,}85\cdot 10^{-12}\ \text{As/Vm}}{\pi\cdot \text{e}^2\ 9{,}1\cdot 10^{-31}\ \text{kg}}\frac{n^2}{Z} = 5{,}3\cdot 10^{-11}\ \text{m}\frac{n^2}{Z}$$

(1)

b) Velocity: $2\pi\cdot r = n\lambda = n\frac{h}{m_e v} \rightarrow v = \frac{nh}{m_e 2\pi r} = \frac{nh}{m_e 2\pi\cdot 0{,}53\cdot 10^{-10}\ \text{m}}\frac{Z}{n^2}$

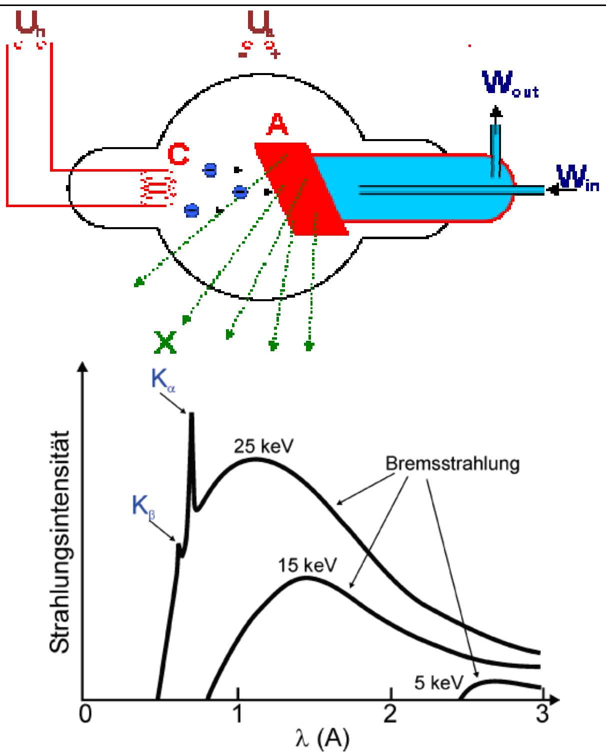
$$v = \frac{4{,}14\cdot 10^{-15}\cdot 1{,}6\cdot 10^{-19}\ \text{AsVs}}{9{,}1\cdot 10^{-31}\ \text{kg}\cdot 2\cdot\pi\cdot 0{,}53\cdot 10^{-12}}\frac{Z}{n} = \left(2{,}19\cdot 10^{8}\ \text{m/s}\right)\frac{Z}{n}$$

Already for Z=2 (Helium) is v larger than c.

(1)

c) The electrons are generated from a hot cathode C and accelerated by the voltage U_a towards the anode A. The electrons are decelerated in the material of the anode and generate the broad bremsspectrum. The shortest wavelength is determined by the maximal kinetic energy of the electrons according to $\lambda_{min} = \frac{hc}{eU_a}$.

In addition to this smooth loss of energy the accelerated electrons can also knock electrons out of the atoms of the anode inducing thereby electronic transitions between the electron shells. The photons generated in these transitions lead to sharp spectral lines. These peaks of the spectrum are called the characteristic lines (e.g. K lines) since they depend on the material used for the anode.



(1)

d) (i) The intensity of the x-ray spectrum increases, when the heating current is increased, since the number of electrons is increased.

(ii)A higher acceleration voltage shifts the starting point of the x-ray spectrum towards smaller wave length as it is shown in the figure above.

(1)

5.

a) Write down the Schrödinger equation for an electron in the electric field of the proton (neglect the spin of the electron).

b) The solution of the Schrödinger equation is $\psi = R_{n\ell}(r)Y_{\ell m}(\theta,\varphi)$. What is the name of the quantum numbers n,ℓ and m and write down which values they can take.

c) What is the meaning of the quantum numbers ℓ and m?

d) Sketch the functions $R_{n\ell}(r)$ for $n = 2$ and justify the shape of the curve in each case.

a) $E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\psi$

(1)

b) n,ℓ and m are integer numbers
$n \geq 1$, $\ell \geq 0$ and $-\ell \leq m \leq \ell$

(1)

c) ℓ and m are the quantum numbers of the angular momentum

They are determined by the eigenvalue equations

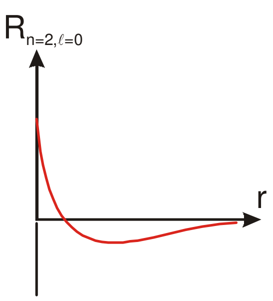
$$\hat{L}^2\, Y_{\ell,m}(\theta,\varphi) = \ell(\ell+1)\hbar^2\, Y_{\ell,m}(\theta,\varphi)$$
$$\hat{L}_z\, Y_{\ell,m}(\theta,\varphi) = m\hbar\, Y_{\ell,m}(\theta,\varphi)$$

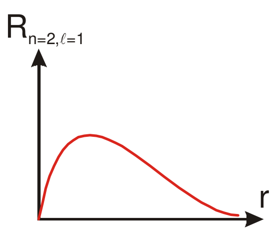
or alternatively:

ℓ determines the angular momentum with $L = \hbar\sqrt{\ell(\ell+1)}$. m determines the projection of L on the axis of quantization with $L_z = m\hbar$.

(1)

d) ℓ can take the values 0 and 1 for n = 2. R reduces exponentially with increasing radius. The function R has a knot for $\ell = 0$ and takes a value $R \neq 0$ for $r \to 0$ since there is no centrifuga lpotential for $\ell = 0$.





There is no knot for $\ell = 1$. Due to the centrifugal potential the electron cannot reach $r = 0$, i.e. $R \to 0$ für $r \to 0$.

(1)

6. The crystal lattice of Co is hexagonal with the lattice parameters $a = 251$ pm and $c = 407$ pm. Remark: The hexagonal plane is formed by two vectors of length a and an angle of $120°$ in between. The lattice vector of length c is perpendicular to the hexagonal plane.

b) Give the condition defining the reciprocal lattice and calculate the lattice parameters of the reciprocal lattice.

b) Indicate the directions of the reciprocal lattice vectors in the hexagonal plane of the crystal lattice.

c) What is the Laue condition for X-ray diffraction?

d) What is the smallest scattering angle for an x-ray beam with the wavelength 217 pm within the hexagonal plane of Co?

a) The crystal lattice is defined by the vectors

$$\vec{R} = n_1\vec{a}_1 + n_2\vec{a}_2 + n_3\vec{a}_3$$

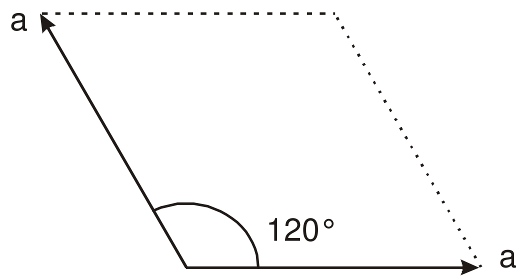
The reciprocal lattice is defined by the vectors

$$\vec{K} = h\vec{b}_1 + k\vec{b}_2 + \ell\vec{b}_3$$

and the condition $\exp\left(i\vec{K}\vec{R}\right) = 1$ or $\vec{K}\vec{R} = 2\pi \cdot m$, i.e.

$$\vec{b}_1 = \frac{2\pi}{V_{Cell}}\vec{a}_2 \times \vec{a}_3,\ \vec{b}_2 = \frac{2\pi}{V_{Cell}}\vec{a}_3 \times \vec{a}_1,\ \vec{b}_3 = \frac{2\pi}{V_{Cell}}\vec{a}_1 \times \vec{a}_2$$

(1/2)



The volume of the unit cell is

$$V_{Cell} = 407\text{ pm} \cdot 251\text{ pm} \cdot 251\text{ pm} \cdot \cos 30° = 22{,}2 \cdot 10^6 \text{pm}^3$$

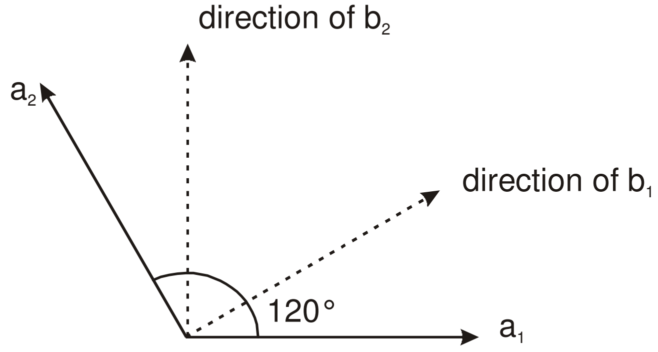
The lattice parameters of the reciprocal lattice are

$$b_1 = b_2 = \frac{2\pi}{22{,}2 \cdot 10^6\ \text{pm}^3} 407\text{ pm} \cdot 251\text{ pm} = 0{,}029\text{ pm}^{-1}$$

$$b_3 = \frac{2\pi}{22{,}2 \cdot 10^6\ \text{pm}^3} 251^2\ \text{pm}^2 \cdot \sin 120° = 0{,}015\text{ pm}^{-1}$$

(1)

b) According to $\vec{b}_1 = \frac{2\pi}{V_{Cell}} \vec{a}_2 \times \vec{a}_3$, $\vec{b}_2 = \frac{2\pi}{V_{Cell}} \vec{a}_3 \times \vec{a}_1$, $\vec{b}_3 = \frac{2\pi}{V_{Cell}} \vec{a}_1 \times \vec{a}_2$ with the vector $\vec{a}_3$ pointing out of the plane:



(1)

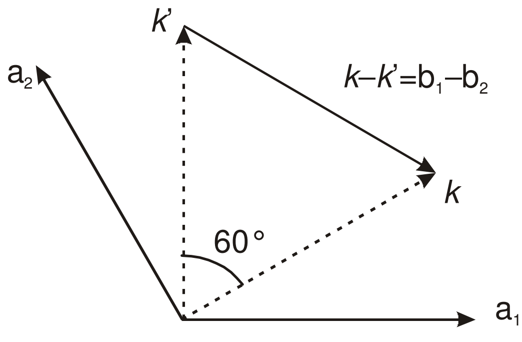
c) The Laue condition for scattering is

$$\vec{k} - \vec{k}' = \vec{K}$$

$\vec{k}$: wave vector of the incoming wave and $\vec{k}'$ wave vector of the scattered wave.

(1/2)

d) $k = \frac{2\pi}{217\ \text{pm}} = 0{,}029\ \text{pm}^{-1}$ equal to the length of the reciprocal lattice vectors in the hexagonal plane, and therefore the smallest angle is 60°.



(1)

7.

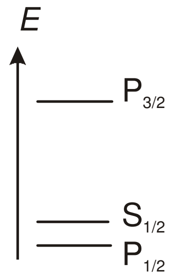
a) What is the spin of the electron and how was the spin discovered?
b) Explain why the first excited state of the hydrogen atom $n = 2$ is splitted into three states. Make a sketch of the splitting and give the quantum numbers of these states.
c) How can the influence of a homogeneous static magnetic field be included in the Schrödinger equation?
d) What is the Paschen-Back effect?

a) The spin is an angular momentum with half integer values of its quantum numbers.

It was discovered by Stern and Gerlach. A beam of Ag-atoms is split in an inhomogenous magnetic field into two beams according to the orientation of the spin ½ of the Ag atom.

(1)

b) For $n = 2$ the orbital angular momentum can take the values $\ell = 0$ or 1.
The states with $\ell = 1$ are splitted due to the spin-orbit interaction into state with j=1/2 and j=3/2. The state with j=1/2 is lower in energy the the state with j=3/2. Due to QED effects there is a splitting between the $\ell = 0$ and the j=1/2. Again the j=1/2 is lower in energy.



(3/2)

c) The magnetic field is included into the Schrödinger equation by the Zeeman-Hamiltonian

$$\hat{H}_{Zee} = \mu_B \hat{\vec{L}}\vec{B} + g\mu_B \hat{\vec{S}}\vec{B}$$

(1/2)

d) The coupling of spin and orbit to the total angular momentum is lifted for a strong magnetic field, i.e.

$\langle H_{Zee} \rangle << \langle H_{SL} \rangle$: $\hat{H}_{Zee} = g(J,L,S)\mu_B \hat{\vec{J}}\vec{B}$

and for

$\langle H_{Zee} \rangle >> \langle H_{SL} \rangle$: $\hat{H}_{Zee} = \mu_B \hat{\vec{L}}\vec{B} + g\mu_B \hat{\vec{S}}\vec{B}$

(1/2)

8. The density of aluminium is $\rho = 2.7\ \text{g/cm}^3$. The atomic mass is 27 g/mol and the electronic configuration is [Ne]$3s^2 3p^1$.

a) Calculate the Fermi energy of Al. Remark: All valence electrons contribute to the electric conductivity.

b) What is the difference between the specific heat capacity of a metal and an insulator?

c) What is an energy band?

d) Why are most of the elements in the solid state metals?

a)

Fermi energy: $E_F = \frac{1}{2}\frac{\left(\hbar k_F\right)^2}{m_e}$

Fermi wavenumber: $N = \frac{2 \cdot \frac{4\pi}{3} k_F{}^3}{\frac{(2\pi)^3}{V}} \rightarrow \frac{N}{V} = \frac{8\pi}{3 \cdot 8\pi^3} k_F{}^3 = \frac{k_F{}^3}{3\pi^3} \rightarrow k_F{}^3 = 3\pi^2 \frac{N}{V}$

$$\rho = \frac{m}{V} = \frac{N_{Al} \cdot m_{mol} / N_A}{V} \rightarrow \frac{N_{Al}}{V} = \rho \frac{N_A}{m_{mol}} = \frac{2.7\ \text{g/cm}^3 \cdot 6 \cdot 10^{23}\ \text{mol}^{-1}}{27\ \text{g/mol}} = 6 \cdot 10^{22}\ \text{cm}^{-3}$$

Since there are three valence electrons: $\frac{N}{V} = 18 \cdot 10^{22}\ \text{cm}^{-3}$

$$k_F = \left(3\pi^2 \cdot 18 \cdot 10^{22}\ \text{cm}^{-3}\right)^{\frac{1}{3}} = 1{,}75 \cdot 10^8\ \text{cm}^{-1}$$

$$E_F = \frac{1}{2}\frac{\left(4.14 \cdot 10^{-15}\text{eVs}\ 1{,}75 \cdot 10^{10}\ \text{m}^{-1}\right)^2}{4\pi^2 \cdot 9{,}1 \cdot 10^{-31}\ \text{kg}} = 11{,}7\ \text{eVs}$$

(2)

b) The specific heat of an insulator varies at low temperature $\propto T^3$. For a metal there is an additional contribution $\propto T$.

(1)

c) The energy levels depend no only on the quantum numbers characterising atoms but in addition also on the wavenumber of the Bloch-waves

(1/2)

d) For an insulator the highest occupied energy band has to be occupied completely. Most often energy bands are only partly occupied. The corresponding elements are therefore metals.

(1/2)