Problem 1 (4 Points)

A space ship with a length of 20 m is flying with a velocity of 0.9 *c* directly towards an observer. Radio signals are emitted on both ends of the space ship.

a) How long is the space ship in the frame of reference of the observer?
b) How long is the time interval between the two radio signals in the frame of the space ship, when both signals reach the observer at the same time?
c) How long is the time interval for the emission of the two radio signals in the frame of the observer?
d) How large is the distance between the places of emission in the frame of the observer?

Problem 2 (4 Points)

a) What is a relativistic four vector?
b) Write up the four vector of frequency and wave number of an electromagnetic wave in vacuum.
c) Write up the Lorentz transformation of frequency and wave number.
d) An observer passes with the velocity 0.9 *c* a light source which emits light at frequency $5.1 \cdot 10^{14}$ Hz. Calculate the observed frequency, when the light source is seen by the observer under an angle of 90° with respect to the velocity.

Problem 3 (4 Points)

a) Describe the properties of a photon.
b) In which processes leading to thermal radiation is the photon involved?
c) Why is it possible to describe the spectrum of thermal radiation by an universal law with the temperature T as the only parameter?
d) How can the temperature of thermal radiation be determined?

Problem 4 (4 Points)

a) What are the basic assumptions of Bohr's model of the atom?
b) What is the de Broglie wave length?
c) Show that the de Broglie wave length justifies a basic assumption of Bohr's atomic model.
d) Calculate for an electron bound to a proton the radius of the circular orbit with the smallest energy using the assumptions of Bohr's model.

Problem 5 (4 Points)

a) Give the wave function for a beam of particles with momentum $\vec{P}$ and energy E.
b) Write up the quantum mechanical operators of energy and momentum.
c) Write up the eigenvalue equations for energy and momentum.
d) Write up the Schrödinger equation for a particle of charge q moving in the electric potential $\Phi(\vec{r})$.

Please note reverse side.

Problem 6 (4 Points)

Consider an electron moving in the Coulomb potential of a nucleus. The motion is characterized by the spherical coordinates r, θ and φ.

a) Why are the electron orbits not planar, whereas the corresponding motion of planets around the sun is planar?
b) Give for the coordinates θ and φ the corresponding quantum numbers and the range of their values.
c) What is a π-orbital? Make a sketch of a π-orbital.
d) Sketch the radial part of the wave function corresponding to a π-orbital of smallest energy.

Problem 7 (4 Points)

The magnetic moment of an electron orbiting around a nucleus is $\vec{\mu} = -\mu_B \vec{L}/\hbar$.

a) Calculate for a p-electron the potential energy due to the magnetic moment in a field of $B = 1$ T and sketch the corresponding energy level scheme.
b) Write up the intrinsic magnetic moment of an electron at rest. Which values take its projection on the z-axis?
c) For $B = 0$, sketch the energy level scheme of a p-orbital due to the intrinsic magnetic moment and explain the corresponding quantum numbers.
d) Give the values of all quantum numbers characterizing a p-electron.

Problem 8 (4 Points)

a) Explain the Pauli principle.
b) Two electrons occupy the one particle quantum states $|a\rangle$ and $|b\rangle$. Write up the corresponding two-particle wave function.
c) Explain why the atomic exchange interaction favors a maximal value of the electron spin.
d) Explain why the exchange interaction can bind to atoms together in a chemical bond.

Required physical constants:

Electric field constant:	$\varepsilon_0 = 8.86 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$
Velocity of light:	$c = 3 \cdot 10^8$ m/s
Planck's constant:	$h = 4.14 \cdot 10^{-15}$ eVs
Elementary charge:	$e = 1.6 \cdot 10^{-19}$ As
Mass of the electron:	$m_e = 500$ keV$/c^2$
Bohr's magneton:	$\mu_B = 5.8 \cdot 10^{-5}$ eV/T