1.
Two frames of reference A and B are moving with the relative velocity $v = 0.5\,c$ with respect to each other.

a) Give the Lorentz transformation between A and B.
b) Electrons are accelerated by a voltage of 1 MV in frame A. Calculate the energy and the momentum of these electrons.
c) Give the Lorentz transformation connecting energy and momentum in frame A and B.
d) Calculate the energy and momentum of the accelerated electrons measured by an observer in frame B, when the electrons are moving in the direction of *v*.

2.

a) Calculate the energy level scheme of the He^+ ion (nuclear charge $Z = 2$). How large is the energy difference between the ground state and the first excited state? How large is the energy difference between the 1st and 2nd excited state?
b) The He^+ ion approaches a light source of frequency ν_0 with the velocity *v*. How large is the frequency of the light source in the reference frame of the He^+ ion? How large is the frequency, when the He^+ ion recedes from the light source?
c) Which frequency ν_0 is necessary, to allow the absorption of the photons of the light source
   i) by the transition between the ground state and the 1st excited state in the case of the approaching He^+ ion and
   ii) by the transition between the 1st and 2nd excited state in the case of the receding He^+ ion?
d) Calculate the velocity of the He^+ ion, for which these transitions become possible.

3. In the photoelectric effect, electrons are emitted from materials when they absorb energy from light.

a) Give the relation between the kinetic energy of the photo electrons and the frequency of the light wave.
b) Which features of the photoelectric effect show that the energy of electromagnetic waves is quantized?
c) The photo electrons have an energy of 1.2 eV, when a material is exposed to UV light with the wavelength 200 nm. How large is the work function of the material?
d) What is the threshold wavelength for the emission of photo electrons for this material?

4.

a) Write down the time-dependent Schrödinger equation.
b) Write down the time-independent Schrödinger equation.
c) Write down the time-dependent Schrödinger equation for an electron in the electric field of the proton ignoring the spin of the electron.
d) Write down the time-independent Schrödinger equation for an electron in the electric field of the proton, when a magnetic field is applied. Include the influence of the spin of the electron.

5.

a) Give the definition of the angular momentum in classical physics.
b) Write down the eigenvalue equations of the angular momentum.
c) Give the possible eigenvalues of the angular momentum.
d) Which are the rules for the addition of angular momenta.
e) The distance between the two protons of the hydrogen molecule is $d = 75$ pm. How large is the velocity of the two protons when they rotate in the first excited angular momentum state around the common center of mass and the classical approximation is applied?

6. The lattice parameter of a NaCl-crystal is $d = 561 \cdot 10^{-12}$ m.

a) A beam of X-rays (wavelength $\lambda = 4 \cdot 10^{-10}$ m) strikes a NaCl-crystal. Calculate the angles of the diffracted beams with respect to the incident beam.
b) Give the condition defining the reciprocal lattice and calculate the lattice parameter of the reciprocal lattice of NaCl.
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7. The rare earth ion Eu^{3+} is characterized by six 4f valence electrons.

a) Determine the orbital angular momentum, the spin and the total angular momentum of the ground state of Eu^{3+}.
b) The spin-orbit coupling is $\lambda / hc = 240\ \text{cm}^{-1}$. Calculate the energy difference between the energetically lowest and highest state of the ground state multiplet.
c) How many angular momentum states are in the ground state multiplet of Eu^{3+}.
d) The rare earth ion Tb^{3+} is characterized by six electron holes in the 4f shell. Describe the ground state of Tb^{3+}.

8.

a) Four fundamental forces are known. Name these four forces.
b) These forces are mediated via exchange particles. Which are the exchange particles of these four forces?
c) Denote the elementary particles forming the proton and neutron. How large is the charge of these particles?
d) The nuclear force is independent of the electric charge of the nucleons (protons and neutrons). Why protons exist in stable nuclei, although they cause Coulomb repulsion?

Required physical constants:

Planck's constant: $h = 4.14 \cdot 10^{-15}$ eVs
Velocity of light: $c = 3 \cdot 10^{8}$ m/s
Energy of the electron at rest: $m_e c^2 = 0.5$ MeV
Energy of the proton at rest: $m_p c^2 = 938.3$ MeV
Rydberg constant: $R_y = 13.6$ eV

Problem	1	2	3	4	5	6	7	8
Points	4	4	4	4	4	4	4	4

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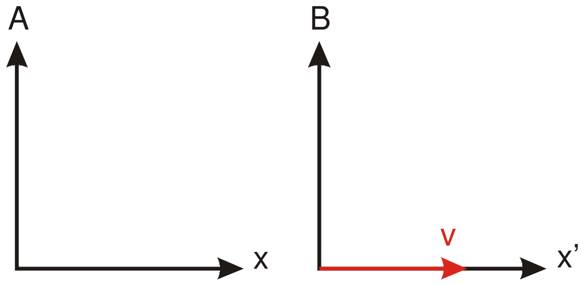
a)



Lorentz-transformation

$$x = \gamma\left(x' + \frac{v}{c}(ct')\right) \qquad \text{and} \qquad x' = \gamma\left(x - \frac{v}{c}(ct)\right)$$

$$ct = \gamma\left(ct' + \frac{v}{c}x'\right) \qquad\qquad ct' = \gamma\left(ct - \frac{v}{c}x\right)$$

with $\gamma = \dfrac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$.

b) The energy of the accelerated electrons is $E = E_{kin} + m_e c^2 = eU + m_e c^2 = 1{,}5$ MeV
The momentum of the electrons is

$$E^2 = c^2P^2 + \left(m_e c^2\right)^2 \rightarrow P = \frac{1}{c}\sqrt{E^2 - \left(m_e c^2\right)^2} = \sqrt{1{,}5^2 - 0{,}5^2}\ \text{MeV/c} = \sqrt{2}\ \text{MeV/c}$$

c) The transformation of energy and momentum:
relativistic four vectors transform according to the Lorentz transformation. The four vectors of time and space and of energy and momentum are $(ct, \vec{r})$ and $(E, c\vec{P})$. Therefore one gets with

$$x' = \gamma\left(x - \frac{v}{c}(ct)\right) \qquad cP_x' = \gamma\left(cP_x - \frac{v}{c}E\right)$$

$$ct' = \gamma\left(ct - \frac{v}{c}x\right) \qquad E' = \gamma\left(E - vP_x\right)$$

the Lorentz transformation for *E* and *P*:

d) The momentum is with $\gamma = \dfrac{1}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}} = \dfrac{1}{\sqrt{1-0,5^2}} = \dfrac{2}{\sqrt{3}}$

$$cP'_x = \frac{2}{\sqrt{3}}\left(\sqrt{2} - 0,5 \cdot 1,5\right)\ \text{MeV} = 0,77\ \text{MeV}\ .$$

The energy becomes

$$E = \frac{2}{\sqrt{3}}\left(1,5 - 0,5 \cdot \sqrt{2}\right)\ \text{MeV} = 0,91\ \text{MeV}$$

2.

a) Calculate the energy level scheme of the He^+ ion (nuclear charge $Z = 2$). How large is the energy difference between the ground state and the first excited state? How large is the energy difference between the 1st and 2nd excited state.

b) The He^+ ion approaches a light source of frequency ν_0 with the velocity v. How large is the frequency of the light source in the reference frame of the He^+ ion? How large is the frequency, when the He^+ ion recedes from the light source?

c) Which frequency ν_0 is necessary, to allow the absorption of the photons of the light source
   - iii) by the transition between the ground state and the 1st excited state in the case of the approaching He^+ ion and
   - iv) by the transition between the 1st and 2nd excited state in the case of the receding He^+ ion?

d) Calculate the velocity of the He^+ ion, for which these transitions become possible.

a) The energy levels of the He^+ ion are: $E_n = -13{,}6\ \text{eV}\dfrac{Z^2}{n^2} = -13{,}6\ \text{eV}\dfrac{2^2}{n^2} = -54{,}4\ \text{eV}\dfrac{1}{n^2}$

The energy difference between the 1st excited state and the ground state is

$$\Delta E_{12} = -54{,}4\ \text{eV}\left(\frac{1}{2^2} - 1\right) = 40{,}8\ \text{eV}\,,$$

and between the 2nd and 1st excited state

$$\Delta E_{23} = -54{,}4\ \text{eV}\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 7{,}6\ \text{eV}$$

b) The relativistic Doppler-effect for the approaching ion

$$\nu_+ = \nu_0\sqrt{\frac{c+v}{c-v}}\,,$$

and the Doppler-effect for the receding ion

$$\nu_- = \nu_0\sqrt{\frac{c-v}{c+v}}$$

c)

$$\Delta E_{12} = h\cdot\nu_0\sqrt{\frac{c+v}{c-v}}$$

$$\Delta E_{23} = h\cdot\nu_0\sqrt{\frac{c-v}{c+v}} = \frac{(h\cdot\nu_0)^2}{\Delta E_{12}}$$

$$\rightarrow \nu_0 = \frac{1}{h}\sqrt{\Delta E_{12}\cdot\Delta E_{23}} = \frac{1}{h}\sqrt{40{,}8\cdot 7{,}6}\ \text{eV} = \frac{17{,}6\ \text{eV}}{h} = 4{,}25\cdot 10^{15}\ \text{Hz}$$

d) Velocity of the He^{+} ion:

$$\frac{\Delta E_{12}}{h\cdot\nu_0}=\sqrt{\frac{c+v}{c-v}}\rightarrow\left(\frac{\Delta E_{12}}{h\cdot\nu_0}\right)^2=\frac{c+v}{c-v}$$

$$\rightarrow\left(\frac{\Delta E_{12}}{h\cdot\nu_0}\right)^2(c-v)=c+v\rightarrow\left(\left(\frac{\Delta E_{12}}{h\cdot\nu_0}\right)^2-1\right)c=v\left(\left(\frac{\Delta E_{12}}{h\cdot\nu_0}\right)^2+1\right)$$

$$\rightarrow v=c\frac{\left(\left(\frac{\Delta E_{12}}{h\cdot\nu_0}\right)^2-1\right)}{\left(\left(\frac{\Delta E_{12}}{h\cdot\nu_0}\right)^2+1\right)}=c\frac{\left(\frac{40,8}{17,6}\right)^2-1}{\left(\frac{40,8}{17,6}\right)^2+1}=0,69\cdot c$$

3. In the photoelectric effect, electrons are emitted from materials when they absorb energy from light.

a) Give the relation between the kinetic energy of the photo electrons and the frequency of the light wave.

b) Which features of the photoelectric effect show that the energy of electromagnetic waves is quantized?

c) The photo electrons have an energy of 1.2 eV, when a material is exposed to UV light with the wavelength 200 nm. How large is the work function of the material?

d) What is the threshold wavelength for the emission of photo electrons for this material?

a) $E_{kin} \propto \nu \quad (E_{kin} = h \cdot \nu - W_A)$

b) The kinetic energy of the photo electrons is independent of the intensity of the light wave.
No photo electrons can be emitted above a certain threshold wave length.

c) $E = h \cdot \dfrac{c}{\lambda} = \dfrac{4{,}14 \cdot 10^{-15}\ \text{eVs} \cdot 3 \cdot 10^{8}\ \text{m/s}}{200\ \text{nm}} = 6{,}2\ \text{eV}$ →the work function is 5 eV

d) The threshold wavelength is: $\lambda_0 = \dfrac{h \cdot c}{E} = \dfrac{4{,}14 \cdot 10^{-15}\ \text{eVs} \cdot 3 \cdot 10^{8}\ \text{m/s}}{5\ \text{eV}} = 248\ \text{nm}$

4.

a) Write down the time-dependent Schrödinger equation.
c) Write down the time-independent Schrödinger equation.
c) Write down the time-dependent Schrödinger equation for an electron in the electric field of the proton ignoring the spin of the electron.
d) Write down the time-independent Schrödinger equation for an electron in the electric field of the proton, when a magnetic field is applied. Include the influence of the spin of the electron.

a) The time-dependent Schrödinger equation is

$$i\hbar\frac{\partial\psi}{\partial t}=\hat{H}\psi$$

with the Hamilton operator

$$\hat{H}=-\frac{\hat{\vec{P}}^2}{2m_0}+V(\vec{r})=-\frac{\hbar^2\nabla^2}{2m_0}+V(\vec{r})$$

b) Die time-independent Schrödinger equation is

$$E\psi=\hat{H}\psi$$

c) The Schrödinger equation for an electron in the Coulomb-potential is

$$i\hbar\frac{\partial\psi}{\partial t}=\left(-\frac{\hbar^2\nabla^2}{2m_0}-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right)\psi$$

(with $\frac{1}{m_0}=\frac{1}{m_{\text{Elektron}}}+\frac{1}{m_{\text{Proton}}}$)

d) The Schrödinger equation is

$$E\psi=\left(-\frac{\hbar^2\nabla^2}{2m_0}-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}-\frac{\lambda}{\hbar^2}\hat{\vec{\ell}}\cdot\hat{\vec{s}}+\frac{\mu_B}{\hbar}\left(\hat{\vec{\ell}}+2\hat{\vec{s}}\right)\vec{B}\right)\psi$$

The Schrödinger equation additionally includes the spin-orbit and the Zeeman interaction

5.

a) Give the definition of the angular momentum in classical physics.
b) Write down the eigenvalue equations of the angular momentum.
c) Give the possible eigenvalues of the angular momentum.
d) Which are the rules for the addition of angular momenta.
e) The distance between the two protons of the hydrogen molecule is $d = 75$ pm. How large is the velocity of the two protons when they rotate in the first excited angular momentum state around the common center of mass and the classical approximation is applied?

a) The angular momentum in classical physics is defined as $\vec{L} = \vec{r} \times \vec{P}$ or simply $L = r \cdot m \cdot v$.

b) In quantum mechanics the angular momentum is defined by the eigenvalue equations for the orbital angular momentum

$$\hat{L}^2\, Y_{\ell,m}(\theta,\varphi) = \ell(\ell+1)\hbar^2\, Y_{\ell,m}(\theta,\varphi)$$
$$\hat{L}_z\, Y_{\ell,m}(\theta,\varphi) = m\hbar\, Y_{\ell,m}(\theta,\varphi)$$

or more general for the total angular momentum

$$\hat{J}^2 \left|J,M\right\rangle = J(J+1)\hbar^2 \left|J,M\right\rangle$$
$$\hat{J}_z \left|J,M\right\rangle = M\hbar \left|J,M\right\rangle$$

c) There are integer angular momentum values $\ell = 0,1,2,....$ and $|m| = 0,1 \ldots \leq \ell$ e.g. for the orbital angular momentum and for half integer angular momentum values

$$J = \frac{1}{2},\frac{3}{2},\frac{5}{2},.... \text{ and } |M| = \frac{1}{2},\frac{3}{2} \ldots \leq J .$$

d) e.g. for $\hat{\vec{J}} = \hat{\vec{L}} + \hat{\vec{S}}$: J takes the values $|L-S|, |L-S|+1, ..., L+S$

e) With the classical angular momentum $L = r \cdot m_p \cdot v$ and $r = \frac{d}{2}$ one gets

$L = 2 \cdot \frac{d}{2} m_p v = d m_p v$.

With $\ell = 1$: $v = \dfrac{\hbar\sqrt{\ell(\ell+1)}}{dm_p} = \dfrac{4{,}14 \cdot 10^{-15}\text{ eVs} \cdot \sqrt{2} \cdot \left(3 \cdot 10^8\text{ m/s}\right)^2}{2 \cdot \pi \cdot 75 \cdot 10^{-12}\text{ m} \cdot 938{,}3 \cdot 10^6\text{ eV}} = 1192\text{ m/s}$

or with $m = 1$: $v = \dfrac{\hbar}{dm_p} = \dfrac{4{,}14 \cdot 10^{-15}\text{ eVs} \cdot \left(3 \cdot 10^8\text{ m/s}\right)^2}{2 \cdot \pi \cdot 75 \cdot 10^{-12}\text{ m} \cdot 938{,}3 \cdot 10^6\text{ eV}} = \dfrac{1192\text{ m/s}}{\sqrt{2}} = 843\text{ m/s}$

6. The lattice parameter of a NaCl-crystal is $d = 561 \cdot 10^{-12}$ m.

a) A beam of X-rays (wavelength $\lambda = 4 \cdot 10^{-10}$ m) strikes a NaCl-crystal. Calculate the angles of the diffracted beams with respect to the incident beam.

b) Give the condition defining the reciprocal lattice and calculate the lattice parameter of the reciprocal lattice of NaCl.

c) What is the Laue condition for X-ray diffraction?

d) What is the meaning of Ewald's sphere for constructing the diffraction pattern?

a) With Bragg's law $n\lambda = 2d\sin\theta_n$

$$\sin\theta_n = \frac{n\lambda}{2d} = n\frac{4 \cdot 10^{-10}\ \text{m}}{2 \cdot 561 \cdot 10^{-12}\ \text{m}} = n \cdot 0{,}36.$$

Only the 1st and 2nd order can be observed.
The angles of deflection are $2\theta_1 = 41{,}8°$ und $2\theta_2 = 91°$.

b) The points of the crystal lattice is defined by $\vec{R} = n_1\vec{a}_1 + n_2\vec{a}_2 + n_3\vec{a}_3$

The points of the reciprocal lattice are $\vec{K} = h\vec{b}_1 + k\vec{b}_2 + \ell\vec{b}_3$ and defined be the condition $\exp\left(i\vec{K}\vec{R}\right) = 1$ or $\vec{K}\vec{R} = 2\pi \cdot m$.

Since NaCl is cubic the lattice parameter of the reciprocal lattice is

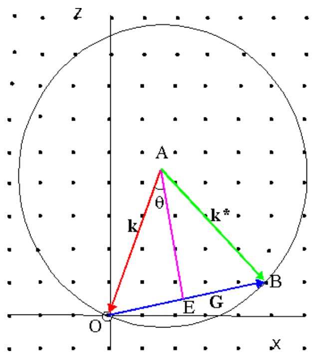
$$b = \frac{2\pi}{a} = \frac{2\pi}{561 \cdot 10^{-12}\text{m}} = 1{,}12 \cdot 10^{10}\ \text{m}^{-1}$$

c) The Laue condition for scattering is

$$\vec{k} - \vec{k}' = \vec{K}$$

$\vec{k}$: wave vector of the incoming wave and $\vec{k}'$ wave vector of the scattered wave.

d) The center of the Ewald sphere is the starting point of vector $\vec{k}$. $\vec{k}$ points to a point of the reciprocal lattice. All points of the reciprocal lattice on the Ewald sphere connected with the center of the sphere fulfill the Laue condition.



7. The rare earth ion Eu^{3+} is characterized by six 4f valence electrons.

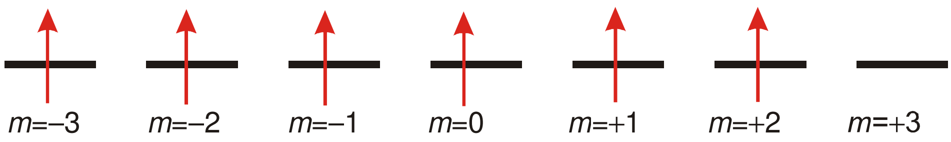
a) Determine the orbital angular momentum, the spin and the total angular momentum of the ground state of Eu^{3+}.

b) The spin-orbit coupling is $\lambda/hc = 240\ \mathrm{cm}^{-1}$. Calculate the energy difference between the energetically lowest and highest state of the ground state multiplet.

c) How many angular momentum states are in the ground state multiplet of Eu^{3+}.

d) The rare earth ion Tb^{3+} is characterized by six electron holes in the 4f shell. Describe the ground state of Tb^{3+}.

a) With Hund's rule one gets



$L = |-3-2-1+0+1+2| = 3$ and $S = 6 \cdot \frac{1}{2} = 3$

Since the spin-orbit coupling is positive the total angular momentum of the ground state is $J = 0$.

b) The spin-orbit coupling is $\hat{H}_{LS} = \frac{\lambda}{\hbar^2} \hat{\vec{L}} \cdot \hat{\vec{S}}$. With $\vec{J} = \vec{L} + \vec{S}$ and $\vec{J}^2 = \vec{L}^2 + 2\vec{L} \cdot \vec{S} + \vec{S}^2$ one gets

$$\left\langle \hat{H}_{LS} \right\rangle = \frac{\lambda}{2}\left(J(J+1) - L(L+1) - S(S+1)\right)$$
$$\rightarrow E_J = 120\ \mathrm{cm}^{-1}\left(J(J+1) - 3 \cdot 4 \cdot 2\right)$$
$$\rightarrow E_6 - E_0 = 120\ \mathrm{cm}^{-1}(6 \cdot 7) = 5040\ \mathrm{cm}^{-1}$$

c) According to the rule for the coupling of angular momenta $|L-S| \le J \le L+S$, J can take the values 0,1,2,3,4,5,6 and there are 7 J states. Each J state has $2J+1$ m states, so that there is a total of 1+3+5+7+9+11+13=49 angular momentum states.

b) L and S are the same as for the electron configuration, but the spin-orbit coupling is negative i.e. $\lambda < 0$. The total angular momentum of the ground state is therefore $J = 6$.

8.

a) Today four fundamental forces are known. Name these four forces.
b) These forces are mediated via exchange particles. Which are the exchange particles of these four forces?
c) Denote the elementary particles forming the proton and neutron. How large is the charge of these particles?
d) The nuclear force is independent of the electric charge. Why nuclei are formed by protons and neutrons in spite of the Coulomb repulsion?

a)
Gravitation, electromagnetic interaction, weak force and strong force.

b) The exchange particles are
Gravitation: Graviton (not detected)
Electromagnetic interaction: Photons
Weak interaction: $Z^0, W^\pm -$Bosons
Strong interaction: Gluons

c) The elementary particles forming the proton and the neutron are the up- and down-quarks. The electric charge is $+\frac{2}{3}$ for the up-quark and $-\frac{1}{3}$ for the down-quark.

d) The nuclear force depends on the orientation of the nucleon spin. A parallel orientation leads to a strong nuclear force whereas an anti parallel orientation leads to a weak nuclear force. Since the nucleons are Fermi particles they can occupy a nuclear quantum state only once. The mixing of protons and neutrons enables the parallel orientation of nucleons in the same nuclear state. This leads to a strong nuclear force and stable nuclei.