1.

a) Sketch a Michelson interferometer and denote the various components.
b) Explain, why the Michelson-Morley experiment became famous.
c) Explain, how time dilation shows up in the Michelson-Morley experiment.
d) Explain, how length contraction shows up in the Michelson-Morley experiment.

2.

Two space ships travel with a velocity of 0.9 *c* with respect to each other. Space ship A emits each second a short light flash, which is received by space ship B.

a) How long is the time interval between the light flashes observed in ship B
   i) when the space ships approach,
   ii) when the space ships recede?
b) What is the frequency of the light signals emitted by space ship A, when they are received with a period of 1 s
   i) in the approaching or
   ii) in the receding space space ship B?

3.

In an x-ray tube, the electrons are accelerated by a voltage U_0 and stopped in a target on the anode.

a) Sketch the x-ray spectrum emitted from the anode as a function of the wavelength.
b) Explain the influence of the target material on the x-ray spectrum.
c) Calculate the smallest wavelength of the emitted photons.
d) Explain how the energy of the emitted photons can be measured by means of Bragg reflection.

4.

In the nuclear decay of ^{60}Co (proton number Z = 27) to ^{60}Ni (Z = 28) an electron with an energy up to 318 keV and two photons with the energy 1333 keV and 1173 keV are emitted by the Co nucleus.

a) The photon spectrum of the ^{60}Co decay shows the peaks of the two photons of the decay and a broad spectrum due to the Compton effect. What is the Compton effect?
b) Calculate the smallest possible energy of the photon after the Compton effect.
c) The electrons involved in the Compton effect produce bremsstrahlung. Calculate the highest energies due to the bremsstrahlung.
d) The photon spectrum shows also the so-called K_α line. Explain the reason for this spectral line and calculate the energy of the K_α line.

5.

a) What is the meaning of the acronym LASER?

b) What is the difference between normal light and the light of a laser?

c) Sketch the set-up of the He-Ne laser and explain how the light of the laser is generated.

d) The light of the He-Ne laser is polarised. Why?

6.

a) What is the definition of the angular momentum in classical physics?

b) Write up the equations defining the angular momentum in quantum mechanics.

c) Which quantum numbers characterise a *d*-orbital?

d) Write down the quantum numbers of the angular momentum of the electron in the ground state of the hydrogen atom.

7.

a) Sketch the potential experienced by an electron occupying

   i) a *s*-orbital

   ii) a *p*-orbital

   in the hydrogen atom as a function of the electron-proton distance and explain the difference between the potentials.

b) The difference between the potentials is of crucial importance for the periodic table of elements. Explain the consequences of this difference for the periodic table of elements.

c) A state of the hydrogen atom is denoted by $3^2D_{5/2}$. What is the meaning of $3^2D_{5/2}$?

d) $3^2D_{5/2}$ is one component of a so-called spin-orbit doublet. Write up the second component of the spin-orbit doublet and explain the reason for spin-orbit coupling.

e) Is the energy of the second component of the spin-orbit doublet larger or smaller than the energy of $3^2D_{5/2}$?

8.

a) Sketch the construction of the first and second Brillouin zone of a square lattice.

b) What is a Bloch wave?

c) What is an energy band?

d) What is the Fermi surface?

Required physical constants:

Planck's constant: $h = 4.14 \cdot 10^{-15}$ eVs

Velocity of light: $c = 3 \cdot 10^{8}$ m/s

Compton wavelength: $\lambda_C = 2.24 \cdot 10^{-12}$ m

Elementary charge: $e = 1.6 \cdot 10^{-19}$ As

Mass of the electron: $m_e = 511$ keV/c^2

Rydberg constant: $hcR_\infty = 13{,}6$ eV

Problem	1	2	3	4	5	6	7	8
Points	4	4	4	4	4	4	4	4

1.

a) Sketch a Michelson interferometer and denote the various components.
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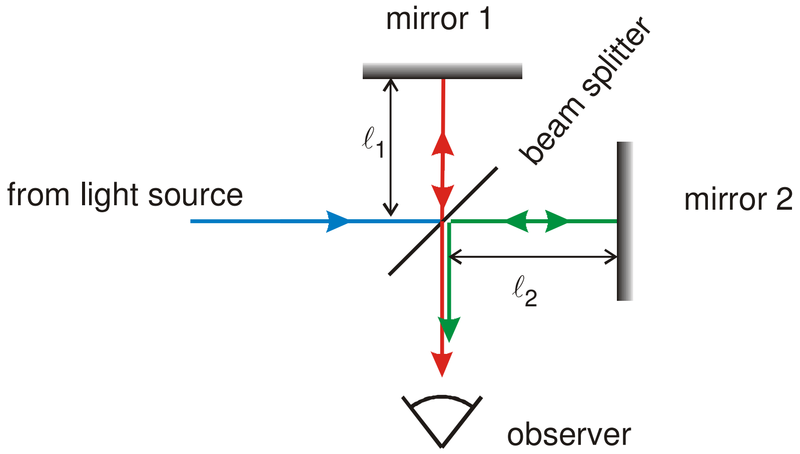
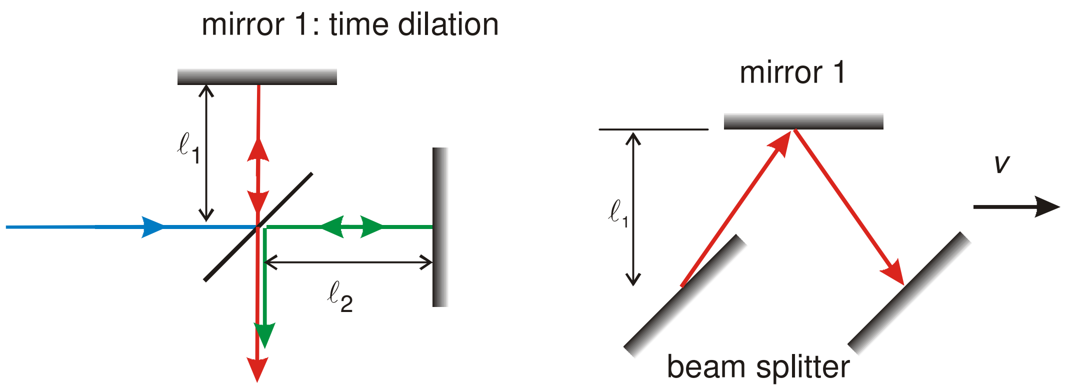
a)



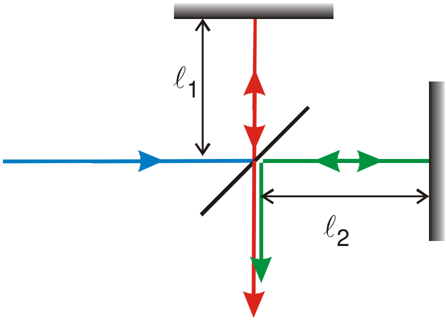
b) The experiment shows that the velocity of light is independent of the velocity of the observer, i.e. the velocity of light is a universal constant.

c)



The interferometer observed at rest and in motion. The path for the light becomes longer for the interferometer in motion, so that the light needs more time for the path between the beam splitter and the mirror: time dilation.

d)



mirror 2: length contraction

For the interferometer in motion the time necessary for a light pulse to travel between the beam splitter and mirror 2 is $t_{\rightarrow} = \frac{\ell_2}{c-v}$ and $t_{\leftarrow} = \frac{\ell_2}{c+v}$ for the way back. The total time is $t_{\rightarrow} + t_{\leftarrow} = \frac{\ell_2}{c-v} + \frac{\ell_2}{c+v} = 2\frac{\ell_2}{c}\frac{1}{1-\left(\frac{v}{c}\right)^2}$.

When $\ell_1 = \ell_2$ the times in way 1 and 2 are only equal when the length ℓ_2 of the moving interferometer becomes $\ell_2(v) = \ell_2\sqrt{1-\left(\frac{v}{c}\right)^2}$, i.e. length contraction.

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a i) Doppler effect for an approaching source

$$\nu_+ = \nu_0\sqrt{\frac{c+v}{c-v}} = \frac{1}{1s}\sqrt{\frac{1.9}{0.1}} = 4.36 s^{-1} \rightarrow T_+ = 0.23\, s$$

a ii) Doppler effect for a receding source

$$\nu_- = \nu_0\sqrt{\frac{c-v}{c+v}} = \frac{1}{1s}\sqrt{\frac{0.1}{1.9}} = 0.23 s^{-1} \rightarrow T_- = 4.36\, s$$

b i) $\nu_+ = 1 \text{ Hz} \rightarrow \nu_0 = \frac{1 \text{ Hz}}{\sqrt{19}} = 0.23 \text{ Hz} \rightarrow T_0 = 4.36 \text{ s}$

b ii) $\nu_- = 1 \text{ Hz} \rightarrow \nu_0 = 1 \text{ Hz}\sqrt{19} = 4.36 \text{ Hz} \rightarrow T_0 = 0.23 \text{ s}$

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In an x-ray tube, the electrons are accelerated by a voltage U_0 and stopped in a target on the anode.

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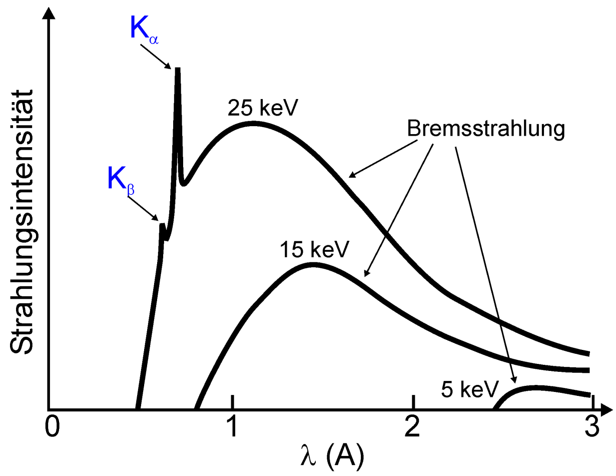
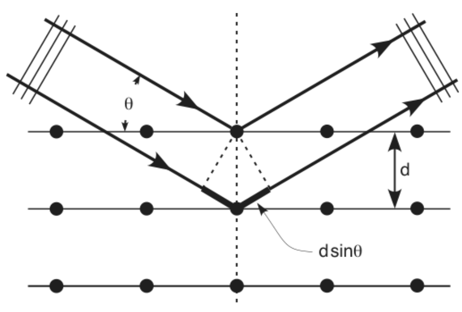


b) The material of the target determines the characteristic lines.

c) The shortest wavelength is determined by the maximal kinetic energy of the electrons according to $\lambda_{\min} = \dfrac{hc}{eU_0}$.

d) Bragg condition for constructive interference $n\lambda = 2d\sin\theta_n$. The energy follows from the scattering angle: $E(\theta) = \dfrac{hc}{\lambda(\theta)}$.



4.

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a) The photon spectrum of the ^{60}Co decay shows the peaks of the two photons of the decay and a broad spectrum due to the Compton effect. What is the Compton effect?

b) Calculate the smallest possible energy of the photon due to the Compton effect.

c) The electrons involved in the Compton effect produce bremsstrahlung. Calculate the highest energies due to the bremsstrahlung.

d) The photon spectrum shows also the so-called K_α line. Explain the reason for this spectral line and calculate the energy of the K_α line.

a) The Compton effect describes the scattering of photons on free (or quasi free) electrons.

b) Compton formula $\lambda' - \lambda = \lambda_C (1 - \cos\theta)$. θ is the angle between the incoming and the scattered photon.
Upper boundaries for forward scattering $E_u = 1333$ keV or 1173 keV
Lower boundaries for backward scattering:

$$\lambda = \frac{hc}{E_\gamma} = \frac{4.14 \cdot 10^{-15}\,\text{eVs} \cdot 3 \cdot 10^8 \text{ m/s}}{1333 \cdot 10^3 \text{ eV}} = 9.32 \cdot 10^{-13} \text{ m}$$

$$E_l = \frac{h \cdot c}{\lambda'} = \frac{h \cdot c}{(0.932 + 2 \cdot 2.24) \cdot 10^{-12} \text{ m}} = 229 \text{ keV}$$

and

$$\lambda = \frac{hc}{E_\gamma} = \frac{4.14 \cdot 10^{-15}\,\text{eVs} \cdot 3 \cdot 10^8 \text{ m/s}}{1173 \cdot 10^3 \text{ eV}} = 1.06 \cdot 10^{-12} \text{ m}$$

$$E_l = \frac{h \cdot c}{\lambda'} = \frac{h \cdot c}{(1.06 + 2 \cdot 2.24) \cdot 10^{-12} \text{ m}} = 224 \text{ keV}$$

c) The highest energies of the bremsstrahlung is
1333 keV – 229 keV = 1104 keV
and
1173 keV – 224 keV = 949 keV

d) An electron is removed from the K-shell either by the electron of the beta decay or by a photon of the gamma decays. The hole is filled with an electron of the L-shell and the photons of the K_α line are emitted. The energy is according to Moseley's law is

$$E_{K_\alpha} = 13.6 \text{ eV} (27\text{-}1)^2 \left(1 - \frac{1}{2^2}\right) = 6.9 \text{ keV}\,.$$

5.

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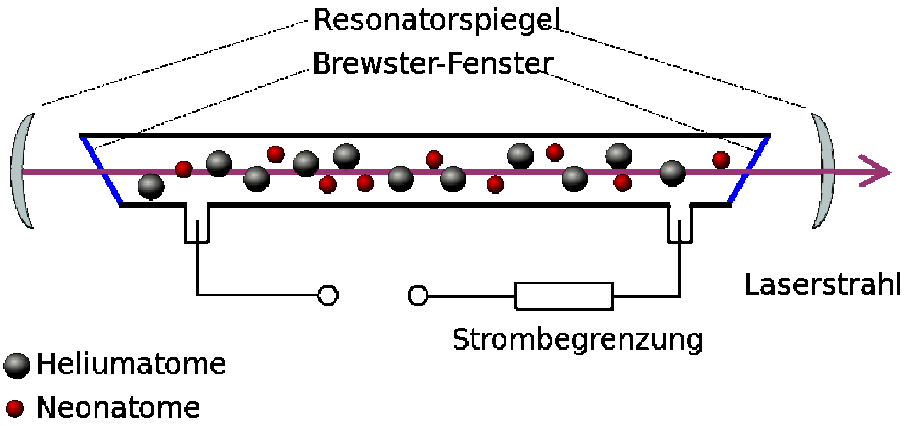
d) The light of the He-Ne laser is polarised. Why?

a) Light Amplification by Stimulated Emission of Radiation

b) Normal light: random $\vec{k}$ and random phase of the wave trains.
Laser light: all photons have the same $\vec{k}$, i.e. same direction and same wave length. Also there is phase coherence.

c)



The atoms are excited by collisions. The excitation energy is transferred from the He-atom to the Ne-atom by collision creating in the dense energy level spectrum of the Ne-atom population inversion for a number of energy levels. The wave length of the laser is selected by the length of the resonator.

d) Due to the Brewster windows the laser light is polarised. Light polarised in the drawing plane can pass the Brewster windows without reflection. The orthogonal polarisation is damped by reflection and not amplified.

6.

a) What is the definition of the angular momentum in classical physics?
b) Write up the equations defining the angular momentum in quantum mechanics.
c) What are the quantum numbers of the orbital angular momentum of a *d*-orbital?
d) Write down the quantum numbers of the angular momentum in the ground state of the hydrogen atom.

a) The angular momentum is $\vec{L} = \vec{r} \times \vec{p}$. $\vec{r}$: position of the particle, $\vec{p} = m\vec{v}$: momentum of the particle.

b) The eigenvalue equation for the square of the angular momentum

$$\vec{L}^2 \left| \ell, m \right\rangle = \hbar^2 \ell \left(\ell + 1 \right) \left| \ell, m \right\rangle$$

and the eigenvalue equation for the z-component of the angular momentum

$$L_z \left| \ell, m \right\rangle = \hbar m \left| \ell, m \right\rangle$$

c) *d*-orbital: $\ell = 2$ and $m = -2, -1, 0, +1, +2$

d) No orbital angular momentum (s-orbital) and the quantum numbers of the spin are $s = \frac{1}{2},\ m_s = \pm \frac{1}{2}$

7.

a) Sketch the potential experienced by an electron occupying
   i) a *s*- and
   ii) a *p*-orbital
   in the hydrogen atom as a function of the electron-proton distance and explain the difference between the potentials.

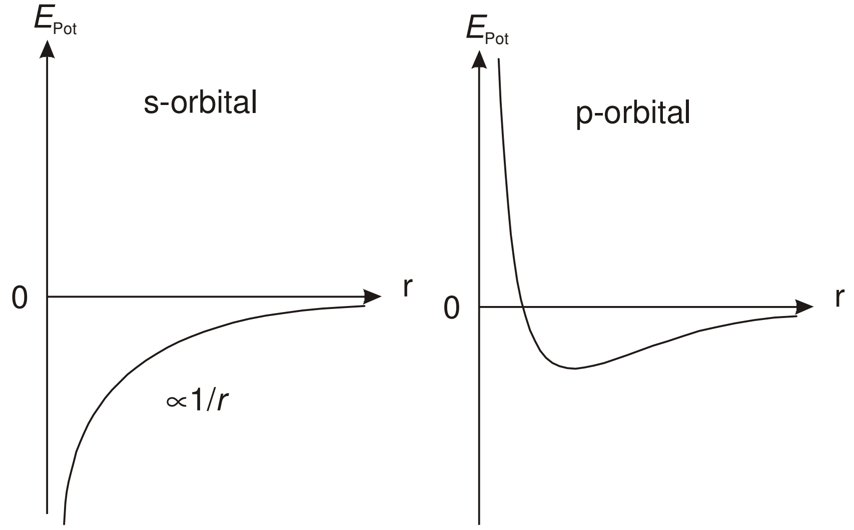
b) The difference between the potentials is of crucial importance for the periodic table of elements. Explain the consequences of this difference for the periodic table of elements.

c) A state of the hydrogen atom is denoted by $2^2D_{5/2}$. What is the meaning of $2^2D_{5/2}$?

d) $2^2D_{5/2}$ is one component of a so-called spin-orbit doublet. Write up the second component of the spin-orbit doublet and explain the reason for spin-orbit coupling.

e) Is the energy of the second component of the spin-orbit doublet larger or smaller than the energy of $2^2D_{5/2}$?

a) s-orbital $E_{Pot} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$ (Coulomb-potential)



p-orbital $E_{Pot} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} + \frac{\hbar^2\ell(\ell+1)}{2m_0 r^2}$ and $\ell = 1$ for a p-orbital

(Coulomb-potential + centrifugal potential)

b) The electron density of a s-orbital is not zero at the nucleus, whereas the electron density of a p, d etc. orbital is zero in the range of the nucleus due to the centrifugal potential. Therefore the binding energy of s-electrons is higher than the binding energy of p,d etc. electrons. In the periodic table of elements first the s-orbitals are occupied by electrons and then follow the other angular momentum states.

c) The meaning of $2^2D_{5/2}$: n $^{(2s+1)}\ell_j$. n is the main quantum number, $(2s+1)$ is the multiplicity with s=½ , D means $\ell = 2$ and *j*=5/2 is the total angular momentum.

d) The second component of the spin orbit doublet is $2^2D_{3/2}$. The magnetic moment of the electrons align in the magnetic field caused by the orbital motion of the electron.

e) The magnetic moments of spin and orbit prefer the antiparallel orientation $E(^2D_{3/2}) < E(^2D_{5/2})$.

8.

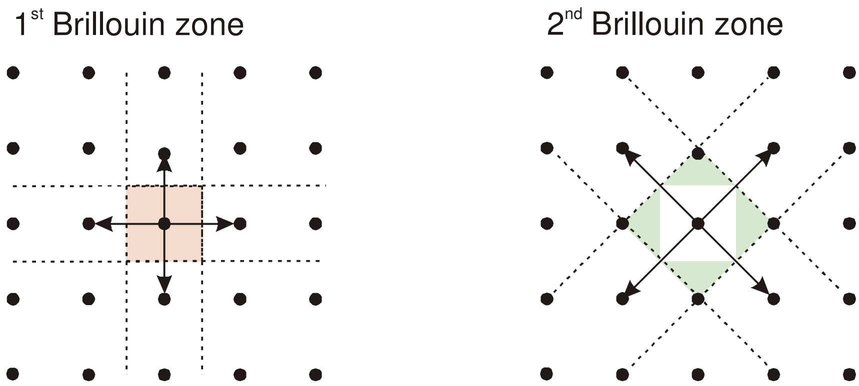
a) Sketch the construction of the first and second Brillouin zone of a square lattice.

b) What is a Bloch wave?

c) What is an energy band?

d) What is the Fermi surface?

a) 1^{st} /2^{nd} Brillouin zone: is confined by the planes perpendicular to the reciprocal lattice vectors connecting first/second neighbour points and passing through the midpoint.



b) A Bloch wave is a wave whose amplitude has the periodicity of the lattice:
$\varphi_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r})\exp(i\vec{k}\vec{r})$ and $u_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r}+\vec{R})$, $\vec{R}$ is a lattice vector

c) When the electron is not localized, the energy is not only a function of quantum numbers (main quantum number, angular momentum etc.) but also a quasi continuous function of the wave number.

d) Due to the Pauli principle each $\vec{k}$-state can be occupied by two electrons. The surface confining the occupied $\vec{k}$-states at zero temperature is called Fermi-surface.