Problem 1 (3 Points)

An observer at rest see a ruler moving with the velocity v. The length of the ruler in the direction of the motion is at rest ℓ_0.

a) How long is the ruler as measured by the observer?
b) What has to be considered when the measurement is carried out?
c) What is the velocity of the ruler when the observed relativistic effect is 0.5 %?
d) Show that the result of the measurement is compatible with the Lorentz transformation.

Problem 2 (5 Points)

The lifetime of radioactive particles at rest is $\tau = 5 \cdot 10^{-7}$ s. The particles move with the velocity $v = 0.95 \cdot c$ relative to an observer at rest.

a) How long is the time necessary to pass the distance of one kilometer as measured by the observer?
b) How long is the time as measured with a clock moving with the particles?
c) How many particles arrive at the end of the kilometer when there are 10^{10} particles at the beginning?
d) How many particles would be expected without relativistic effects?
e) Calculate the lifetime of the moving particles.

Problem 3 (4 Points)

a) Albert Einstein explained Planck's law by three fundamental processes. Explain these processes.
b) Make a sketch of Planck's law, i.e. plot the spectral radiance as a function of the wavelength λ.
c) Write up and explain Stefan-Boltzmann's law.
d) What is a black body?
e) A sphere (radius 10 cm) approximating a black body is exposed to the sun ($150\mathrm{W/m^2}$) and absorbs the whole arriving power of the sun. Calculate the temperature of the sphere in thermal equilibrium when the temperature of the surrounding is 300 K.

Problem 4 (4 Points)

A hydrogen atom ($m_\mathrm{H} = 1\,\mathrm{g/mol}$) moves in the one-dimensional harmonic oscillator potential $\Phi(x) = Dx^2/2$.

a) Give the time-independent Schrödinger equation for the motion in this potential.
b) Give the formula for the energy of the ground state.
c) The oscillation of the hydrogen atom can be excited by light of the wavelength $\lambda = 4.7 \cdot 10^{-6}$ m. Calculate the energy of the ground state.
d) Determine the value of the constant D.

Problem 5 (4 Points)

For the description of atomic wave functions the spherical coordinates r, θ and φ are used.

a) Which quantum numbers determine the radial and angular part of the wave function, $R(r)$ and $Y(\theta, \varphi)$, respectively.
b) Give the allowed range for each of these quantum numbers.

Please note reverse side.

c) Give the quantum numbers of the *d*-orbital.
d) Sketch the radial wave function of the *d*-orbital with the smallest energy. Give reasons for the plot.
e) How varies in this case $R(r)$ as a function of r in the limit of small and large values of r, respectively?

Problem 6 (4 Points)

a) Give the equations which determine the spin of the electron.
b) Explain Pauli's principle and its consequences for the wave function of electrons.
c) What is the exchange interaction and why is it important for the properties of atoms?
d) Give the Coulomb and the exchange energy of two electrons occupying the quantum states $|a\rangle$ and $|b\rangle$.

Problem 7 (4 Points)

For Pr^{3+} there are two electrons in the 4f shell.

a) Give the quantum numbers of these electrons.
b) Give the quantum numbers of the angular momenta and the spectroscopic notation of the ground state of Pr^{3+}.
c) Explain Landé's rule and sketch the spin-orbit splitting of the ground multiplet of Pr^{3+} according to Landé's rule.
d) Sketch the spin-orbit splitting of the ground multiplet according to Landé's rule for Tm^{3+} (hint: $4f^{12}$ configuration).

Problem 8 (4 Points)

The free electron gas trapped in a three dimensional box potential with infinitely high walls can be used as an approximation to describe the physical properties of a metal.

a) Give the time-independent Schrödinger equation for an electron trapped in a cubic box potential (length of the edge $L = 1\,\text{mm}$).
b) Give the wave function of an electron enclosed in this potential.
c) Give the quantum numbers characterizing the state of the electron.
d) Calculate the energy for the quantum states of the electron.
e) How large is the highest energy of an electron in the ground state when there are 10^{10} electrons in the box.

Required physical constants:

Velocity of light:	$c = 3 \cdot 10^8\,\text{m/s}$
Stefan-Boltzmann's constant:	$\sigma = 5.67 \cdot 10^{-8}\,\text{Wm}^{-2}\text{K}^{-4}$
Avogadro's number:	$N_A = 6 \cdot 10^{23}\,\text{mol}^{-1}$
Plancks's constant:	$h = 4,14 \cdot 10^{-15}\,\text{eVs}$
electron mass:	$m_e = 500\,\text{keV}/c^2$