Written Examination in Physics **Spring 2020**
Modern Physics (KSOP)

Problem 1 (4 Points)

a) Give the definition of entropy.
b) What is the definition of temperature?
c) Write up the formula of Maxwell's velocity distribution function without the prefactor and sketch of the distribution function.
d) Explain the definition of the Kelvin temperature scale.

Problem 2 (4 Points)

The Lorentz-transformation between frame *S* and *S*' is
$t = \gamma(\mathrm{t}' + \frac{V}{c^2}\mathrm{x}')$, $x = \gamma(\mathrm{x}' + Vt')$, $y = y'$, $z = z'$ with $\gamma^{-1} = \sqrt{1 - V^2/c^2}$.

a) Write up the classical Galilei-transformation for small values of the relative velocity *V*. Sketch the relative motion of frame *S* and *S*'.
b) Write up the Lorentz-transformation of the relativistic wave vector.
c) Calculate the longitudinal Doppler effect starting with the Lorentz transformation.
d) Describe the velocity measurements of streaming gases by means of Laser beams.

Problem 3 (4 Points)

a) The nucleus is formed by protons and neutrons. Write up and explain the general notation of a nucleus.
b) Give the formula for the quantized energy of the hydrogen atom determined by Bohr's model of the atom.
c) Write up the formula for the energy of x-rays according to Moseley.
d) What denotes a K_α-line in x-ray spectroscopy?

Problem 4 (4 Points)

a) Make a sketch of the quantized angular momentum vector. Indicate the length and the z-component of the vector.
b) Write up the eigenvalues equations of the angular momentum operator.
c) Explain, why no values can be assigned to the x- and the y-component of the angular momentum operator.
d) Spherical harmonics are the eigenfunctions of the orbital angular momentum operator. Explain the general structure of the spherical harmonics and write up the spherical harmonics $y_{1,-1}$, $y_{1,0}$ and $y_{1,+1}$. Omit the normalization factors.

Problem 5 (4 Points)

a) Write up the magnetic moment due to the orbit and the spin of the electron.
b) Explain by means of Bohr's model of the atom why spin and orbital angular momentum align antiparallel for electrons in an atom.
c) Write up the Hamilton-operator of the spin-orbit coupling and calculate the energy eigenvalues.
d) For the ground state of Eu^{3+} are the quantum numbers of the orbit and the spin 3, i.e. $\mathrm{L} = 3$ and $\mathrm{S} = 3$. Write up the quantum states of the spin-orbit multiplet of the ground state starting with the state of lowest energy. Use the atomic notation of quantum states.

Please note reverse side.

Problem 6 (4 Points)

a) Write up the entangled quantum state of two fermions.
b) How many electrons can occupy a quantum state with the orbital angular momentum quantum number $\ell = 1$?
c) Give the entangled states of two p-electrons.
d) Which of these states has the smallest energy, when electrons have the same main quantum number n?

Problem 7 (4 Points)

a) Sketch the phonon dispersion $\nu(\vec{k})$. Consider one propagation direction and a crystal lattice with one atom in the unit cell.
b) Sketch the phonon dispersion, when there are two atoms within the unit cell and give the typical frequency range of phonon modes.
c) Explain why the phonon modes are confined to the 1st Brillouin zone.
d) There are how many phonon modes in a crystal lattice with *n* atoms per unit cell?

Problem 8 (4 Points)

a) Write up the Fermi-distribution function and make a sketch for $T = 0$ and $T \neq 0$.
b) Calculate the density of states

$$D(E) = \frac{1}{A}\frac{\mathrm{d}N}{\mathrm{d}E}$$

of a two-dimensional quasi free electron gas.
c) Compare the heat capacity of a gas formed by atoms or molecules with the heat capacity of a free electron gas. Ignore all prefactors.
d) Plot the energy spectrum of nearly free electrons for waves propagating along the edges of a simple cubic unit cell with lattice parameter *a*.