Karlsruhe Institute of Technology

Institute for Beam Physics and Technology (IBPT)
Bastian Härer, Axel Bernhard, Bennet Krasch, Nathan Ray, Anke-Susanne Müller

Accelerator Physics WS24/25

Exercise sheet 2 (Submission: 19.11.2024)

Exercise 1: Symplectic Matrices (4 points)

In the lecture we've learned about symplectic matrices and why accelerator transport matrices are symplectic. Remember a real (2×2) matrix $\mathbf{A}$ is semplectic if

$$\mathbf{A}^T \mathbf{S} \mathbf{A} = \mathbf{S},$$

with

$$\mathbf{S} = \begin{pmatrix} \emptyset & \mathbb{I} \\ -\mathbb{I} & \emptyset \end{pmatrix}$$

(a) Show that the identity matrix $\mathbf{I}$ is symplectic. (1)

(b) Show that if $\mathbf{A}$ is symplectic, $\mathbf{A}^{-1}$ is too. (1)

(c) Show that if $\mathbf{A}$ and $\mathbf{B}$ are symplectic, $\mathbf{AB}$ is too. (1)

(d) Explain briefly in your own words why the transport matrices need to be symplectic and why the results in a) to c) matter. (1)

Exercise 2: Beam Optics (7 points)

(a) Explain the interpretation of the optical functions $\alpha(s)$, $\beta(s)$, $\gamma(s)$ and the emittance ϵ. (1)

(b) Give the transfer matrix $\mathbf{M}$ of a FODO lattice.
Hint: You have to multiply several matrices. Start and end with „half" a focusing quadrupole and use thin lens approximations for the magnets. (3)

(c) Why does the FODO lattice $\mathbf{M}$ exert an overall focusing effect on the beam?
Use the stability criterion ($|\mathrm{Tr}\{\mathbf{M}\}| \leq 2$) in your explanation. (2)

(d) Determine the focal length f of a quadrupole in LEP.
(100 GeV/c, magnetic field gradient of 11 T m^{-1}, length 2 m) (0.5)

(e) Comment if the thin lens approximation for the quadrupole in d) is suitable if the quadruple is 2 m long. (0.5)