Karlsruhe Institute of Technology

Institute for Beam Physics and Technology (IBPT)
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Accelerator Physics WS24/25

Exercise sheet 5 (Submission: 10.12.2024)

Exercise 1: Synchrotron and Betatron Tune (5 points)

Given is a synchrotron with a circumference of $U = 150.8\,\mathrm{m}$ in which electrons are stored at $E_0 = 3\,\mathrm{GeV}$. To compensate for losses, an energy of $U_0 = 500\,\mathrm{keV}$ should be supplied in the acceleration structures, and the desired phase for the bunch is $\Psi_\mathrm{S} = \frac{26\pi}{50}$. The momentum compaction factor is $\alpha_\mathrm{c} = 10^{-2}$, and the harmonic number is $h = 200$.

(a) Calculate the synchrotron frequency Ω and the synchrotron tune Q_S. (2)

(b) Calculate the betatron tune, split it in the integer and fractional part and mark it in the tune diagram using

$$\oint \frac{1}{\beta_x(s)}\mathrm{d}s = 57.9$$
$$\oint \frac{1}{\beta_y(s)}\mathrm{d}s = 24.19$$

(Hint: In the given template use the macros \integertunex,\integertuney, \fractionaltunex, \fractionaltuney to label the plot and mark the working point in the figure.) (1)

(c) Is the working point in (b) stable, if we only consider resonances up to the third order? Why? (1)

(d) Considering synchro-betatron resonances, is the working point in (b) still stable? Why? (1)

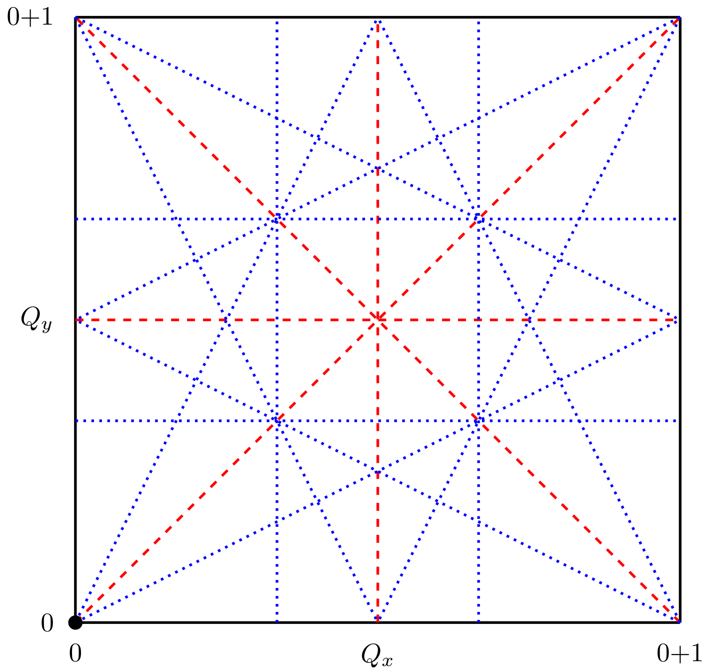

Figure 1: Betatron tune diagram.

Exercise 2: Beta function (2 points)

The diameter of the beam tube decreases when entering an insertion device, such as an undulator, since the distance of the magnets needs to be smaller than the normal beam tube diameter to achieve the desired magnetic field. Imagine a drift distance after a focussing structure with an effective aperture of 5 mm (realistic for undulators). How long may the drift distance for a beam with the emittance 20 nm rad be in total if the properties of the structure are chosen so that the waist of the beam lies in the centre of the drift distance and has a β_T of 1.8 m at this point?
Note: In a storage ring, the aperture of the vacuum chamber should be at least 10 times of the beam size sigma to prevent particle loss.

Exercise 3: Space charge (3 points)

Space charge effects can generally be neglected for relativistic particle beams, but must be considered for non-relativistic particle beams. This can be visualised with a simple model: Imagine a uniformly charged particle beam with a round profile (radius R, current I). You can assume that the beam is infinitely long (i.e. practically an infinitely long cylinder L with radius R). Furthermore, it may be assumed that all particles move along the cylinder axis z with velocity v_s.

(a) First, use Gauss's law to calculate the force acting on a particle located on the surface of the cylinder (radial position $r = R$) from the electric field E. (1)

(b) Second, use Ampere's law to calculate the force acting on this particle from the magnetic field B. (1)

(c) Which of the forces is focussing and which is defocussing? (0.5)

(d) Show that the two forces lift away from each other for $v_\mathrm{s} \to c$. (0.5)
Hint: $\frac{1}{\epsilon_0 \mu_0} = c^2$