Introduction to Laser-Wakefield Acceleration

Matthias Fuchs

Outline

- Electron interacting with a strong electro-magnetic field
- Introduction to plasma physics
- Plasma waves
- Laser-plasma acceleration

Laser-Wakefield Acceleration

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- Invented by Tajima & Dawson, 1979

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Laser Electron Accelerator

T. Tajima and J. M. Dawson
Department of Physics, University of California, Los Angeles, California 90024
(Received 9 March 1979)

An intense electromagnetic pulse can create a weak of plasma oscillations through the action of the nonlinear ponderomotive force. Electrons trapped in the wake can be accelerated to high energy. Existing glass lasers of power density 10^{18}W/cm^2 shone on plasmas of densities 10^{18} cm^{-3} can yield gigaelectronvolts of electron energy per centimeter of acceleration distance. This acceleration mechanism is demonstrated through computer simulation. Applications to accelerators and pulsers are examined.

- Short-pulse high-power lasers no where near existence
- used computer simulations that today a cell phone could do in a fraction of the time

Electron in strong electromagnetic field I

- Plane wave, linearly polarized along y direction

$$\overrightarrow{E_l}(x,t) = \hat{y}E_0\cos(k_l x - \omega_l t)$$
$$\overrightarrow{B_l}(x,t) = \hat{z}B_0\cos(k_l x - \omega_l t)$$

$k_l = \frac{2\pi}{\lambda_l}$: wavenumber

ω_l : angular frequency

Electron in strong electromagnetic field II

- e in plane e-m wave (non-relativistic)

$$\frac{d\vec{p}_e}{dt} = \overrightarrow{F}_L$$ Lorentz force

$$\frac{d}{dt}(m\vec{v}_e) = -e(\overrightarrow{E}_l + \vec{v}_e \times \overrightarrow{B}_l)$$

$$\overrightarrow{E}_l(x,t) = \hat{y}E_0\cos(k_l x - \omega_l t)$$

$$\overrightarrow{B}_l(x,t) = \hat{z}B_0\cos(k_l x - \omega_l t)$$

$B_0 = \frac{E_0}{c}$ -> for v_e <<c : v x B term negligible

$$\vec{v}_e(t) = \hat{y}\frac{eE_0}{m\omega_l}\sin(k_l x - \omega_l t)$$

$$y(t) = \frac{eE_0}{m\omega_l^2}\cos(k_l x - \omega_l t)$$

Electron in strong electromagnetic field III

- Maximum velocity

$$\vec{v}_e(t) = \hat{y}\frac{eE_0}{m\omega_l}\sin(k_l x - \omega_l t)$$

$$v_{max} = \frac{eE_0}{m\omega_l}$$

- relativistic for $v_{max} \to c$:

$$\frac{v_{max}}{c} \to 1 \; : \; \underbrace{\frac{eE_0}{\omega_l mc}}_{\stackrel{!}{=}\, a_0} \to 1$$

a_0 : normalized laser field

$$a_0 = \frac{eE_0}{\omega_l mc}$$

$\gtrsim 1 \to$ relativistic interaction

$\to$ also need to consider $\vec{v} \times \vec{B}$ term

Interpretation of a_0

other way to look at it:

$$\omega = ck = c\,\frac{2\pi}{\lambda}$$

$$a_0 = \frac{e E_0}{\omega_\ell m c} = \frac{\overbrace{e E_0 \lambda_\ell}^{\text{work done by } E_0 \text{ over } \lambda_\ell}}{2\pi \underbrace{mc^2}_{\bar{e} \text{ rest mass energy (Einstein)}}}$$

-> for $a_0 \geq 1$: $\bar{e}$ gains energy $\approx mc^2$ over a distance λ_ℓ

Example: normalized laser field 100 TW

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engineering formula: $a_0 \approx \lambda[\mu m] \cdot \sqrt{\frac{I_0 [W/cm^2]}{1.4 \cdot 10^{18}}}$

100 TW laser @KIT: $P = \frac{2.5\ J}{25\ \underbrace{fs}_{10^{-15}\ s}} = 100 \cdot 10^{12}\ W$

· focused to 20 μm spot:

$I = \frac{P}{d^2} = \frac{100 \cdot 10^{12}\ W}{(20 \cdot 10^{-4})^2\ cm^2} = \frac{100}{\underbrace{400}_{1/4}} \cdot \underbrace{10^{12} \cdot 10^{8}}_{10^{20}} = 2.5 \cdot 10^{19}\ \frac{W}{cm^2}$

$\lambda = 1\ \mu m$:

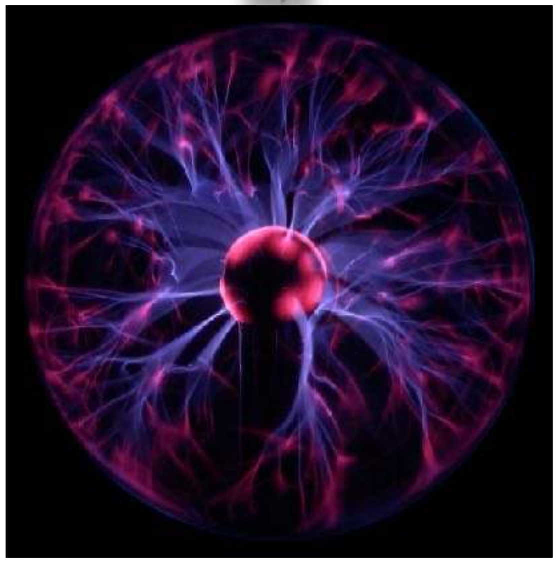
$a_0 \approx 1 \cdot \sqrt{\frac{2.5 \cdot 10^{19}}{\underbrace{1.4}_{\approx 2} \cdot \underbrace{10^{18}}_{10}}} \approx \sqrt{20} \approx \underline{\underline{4.5}}$

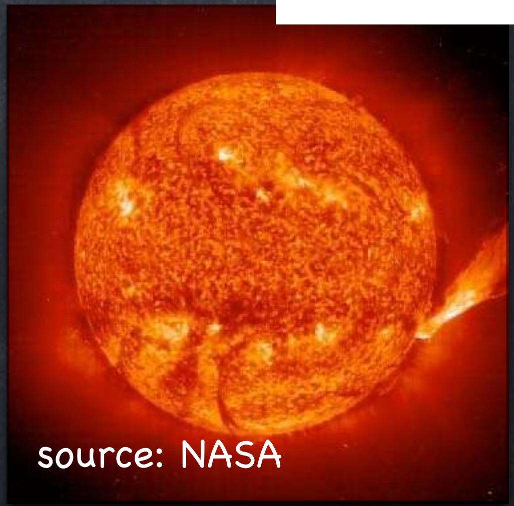
Outline

- Electron interacting with a strong electro-magnetic field
- Introduction to plasma physics
  - Plasma frequency
  - Light propagation in plasmas
- Plasma waves
- Laser-plasma acceleration

Plasma

- "fourth state of matter"
- consists of separated positive and negative charges (e.g. ionized gas)
- electrically neutral
- by separating electrons from the ions, enormous electric fields can be generated
- Unlike electrons, ions are static on the timescales of the interaction (ion movement on ~0.1 ns - scale) due to higher mass

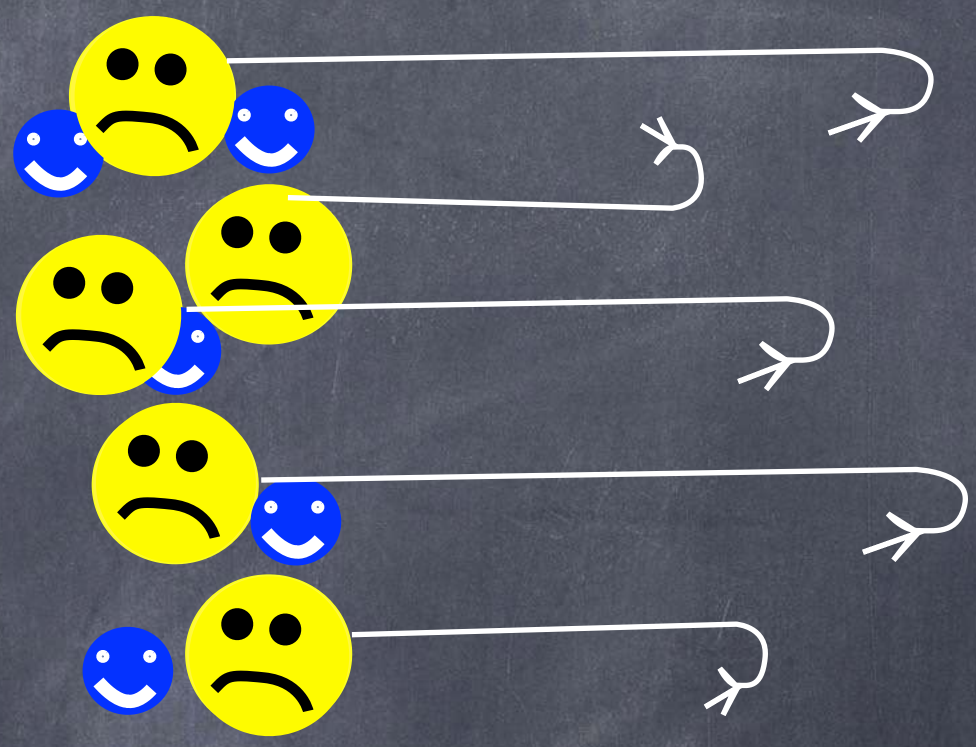




Plasma properties: Plasma oscillations

- displacement of electrons
- creates regions of positive and negative charges
- sets up restoring electrical field
- electrons are accelerated back, overshoot
- harmonic oscillation with "plasma frequency":

$$\omega_{p,e} = \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}}$$



Laser propagation in plasmas I

relevant Maxwell-eqns:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

plane waves: $\vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)}$ -> $\vec{\nabla} \times \vec{E} = i\vec{k} \times \vec{E}$:

$$i\vec{k} \times \vec{E} = i\omega \vec{B} \qquad (*)$$

$$i\vec{k} \times \vec{B} = \mu_0 \vec{j} + i\frac{\omega}{c^2}\vec{E} \qquad (**)$$

k x (*) & use $A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$

(**)

$$k^2 \vec{E} - \vec{k} \cdot (\vec{k} \cdot \vec{E}) = \frac{\omega^2}{c^2}\left(i\frac{\vec{j}}{\epsilon_0 \omega} + \vec{E} \right)$$

-> find expression for current density j

Laser propagation in plasmas II

current due to movement of electrons

(ions remain stationary for high frequencies)

-> current density: $\vec{j} = -n_e e \vec{v}_e$ n_e: electron density

dependence of j on E:

-> use Lorentz force: $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

-> to get first order eqn. of motion $m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -e\vec{E}$

=> $\vec{j} = -n_e e \vec{v} = \frac{n_e e^2}{m_e}\frac{1}{\mathrm{i}\omega}\vec{E}$ $\left(\vec{E} = \vec{E}_0 \mathrm{e}^{\mathrm{i}(\vec{k}\vec{r}-\omega t)}\right)$

remember: $k^2\vec{E} - \vec{k}\cdot(\vec{k}\cdot\vec{E}) = \frac{\omega^2}{c^2}\left(\frac{\vec{j}}{\mathrm{i}\epsilon_0\omega} + \vec{E}\right)$

for e-m waves: $\vec{k} \perp \vec{E}$ => $(c^2k^2 - \omega^2)\vec{E} = -\frac{n_e e^2}{m_e\epsilon_0}\vec{E}$

Laser propagation in plasmas III

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$$(c^2k^2 - \omega^2)\vec{E} = -\underbrace{\frac{n_e e^2}{m_e \epsilon_0}}_{\omega_p^2}\vec{E}$$

-> dispersion relation for an e-m in plasma:

$$\omega^2 = \omega_p^2 + c^2k^2$$

-> phase velocity of wave:

$$v_\phi = \frac{\omega}{|\vec{k}|}$$

-> refractive index of plasma:

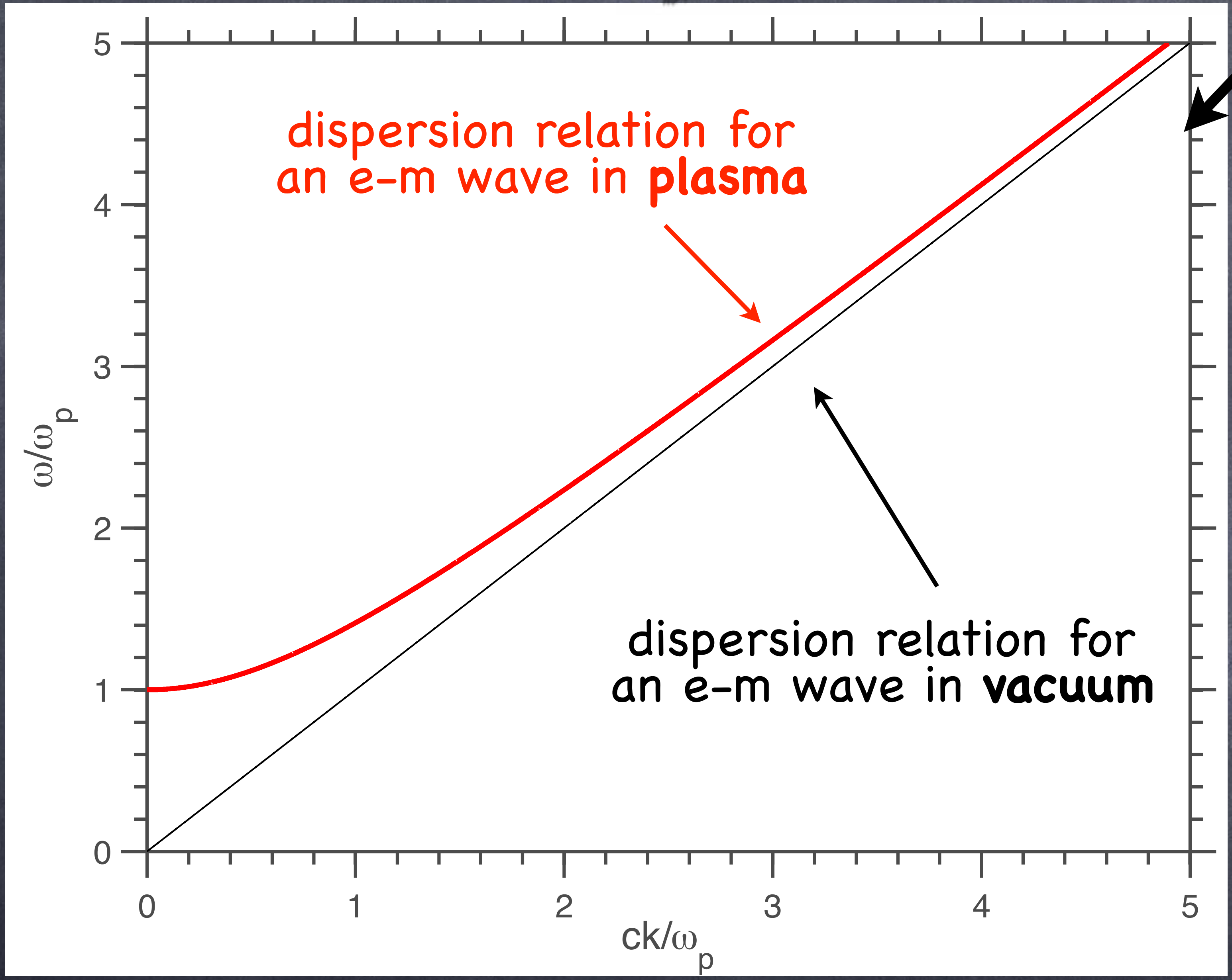
$$\eta = \frac{c}{v_\phi} = \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2}$$

-> group velocity of wave:

$$v_{\text{group}}^{\text{laser}} = \frac{d\omega_L}{dk_L} = c\sqrt{1 - \omega_p^2/\omega_L^2}$$

Dispersion relation I

-> dispersion relation: $\omega^2 = \omega_p^2 + c^2 k^2$



for $\omega \gg \omega_p$:

disp. rel. of plasma approaches that of vacuum
-> response of electrons (ω_p) too slow to respond to high frequencies

-> light field doesn't "feel" (couple to) electrons

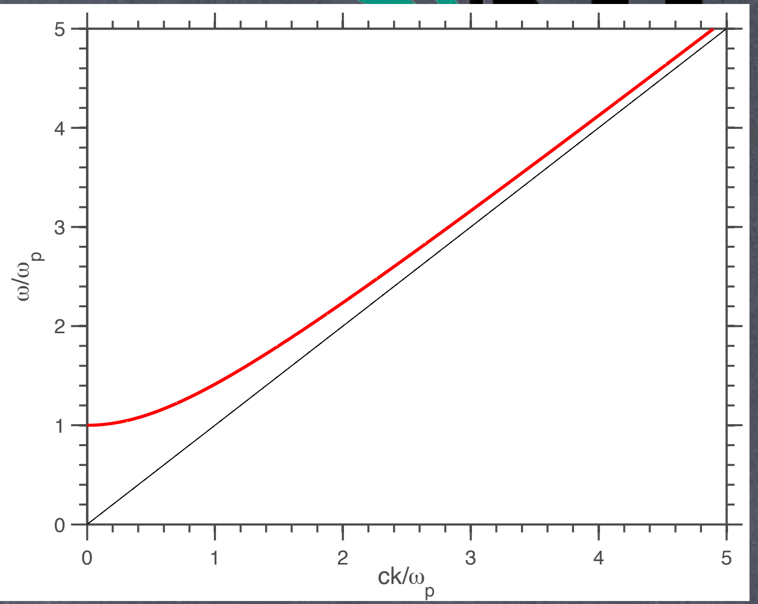
for $\omega < \omega_p$:

wave can't propagate in plasma

-> gets reflected or damped
-> plasma electrons shield fields that oscillate at a frequency < ω_p

Dispersion relation II

$$\left(\omega^2 = \omega_p^2 + c^2 k^2\right) \quad \rightarrow \quad k = \frac{\sqrt{\omega^2 - \omega_p^2}}{c}$$



for $\omega < \omega_p$: -> k is imaginary

=> wave decays as: $\exp\left(-x\sqrt{\frac{\omega_p^2 - \omega^2}{c}}\right)$

with skin depth: $\delta = |k|^{-1} = \frac{c}{(\omega_p^2 - \omega^2)^{1/2}}$

distance over which wave amplitude is decreased by factor 1/e

critical plasma density: ($\omega = \omega_p$)

remember: $\omega_{p,e} = \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}}$

critical plasma density: $n_c = \frac{m_e \epsilon_0 \omega^2}{e^2}$

overcritical plasma

Eqns of motion for an electron in vacuum interacting with a strong e-m plane field

Relativistic Electron Lagrangian

$$\frac{d}{dt}\frac{\partial L}{\partial v} - \frac{\partial L}{\partial r} = 0$$

$$L(v, x-ct) = -mc^2 \underbrace{\sqrt{1 - v^2/c^2}}_{1/\gamma} - (e/c)\, v \cdot A(x-ct)$$

$$\partial L/\partial v = \underbrace{m\gamma\, v}_{p} - (e/\mathrm{c})A_\perp$$

symmetry: $\partial L / \partial r_\perp = 0$, invariant: $\partial L / \partial v_\perp = p_\perp - (e/c)A_\perp =$ const

symmetry: $L(x-ct)$, $dH/dt = -\partial L/\partial t = c\ \partial L/\partial x = c\ dp_x/dt$

invariant: $\mathrm{E} - p_x c =$ const

For electron initially at rest:

$$\mathrm{E}_{\mathrm{kin}} = mc^2(\gamma - 1) = p_x c = p_\perp^2/2m = mc^2 a^2/2$$

(relativistically exact !)

Relativistic equations of motion

$$\mathbf{a} = \frac{e\mathbf{A}_\perp}{mc^2},$$
$$\hat{\mathbf{p}}_\perp = \frac{\mathbf{p}_\perp}{mc} = \mathbf{a} = (0, a_y, a_z),$$
$$\hat{E}_{kin} = \frac{E_{kin}}{mc^2} = \gamma - 1 = \hat{p}_x = \frac{\hat{p}_\perp^2}{2} = \frac{a^2}{2},$$

normalized vector potential:

$$a_0 = \frac{eA_0}{mc^2} = \sqrt{\frac{I_0[\mathrm{W/cm^2}]\lambda_0^2[\mu\mathrm{m}^2]}{1.37 \cdot 10^{18}}}$$

$a_0 > 1$: relativistic intensities
-> laser intensity ≈ electron rest mass
$a_0 = 1 \Rightarrow 10^{18}$ W/cm^2

light propagates in x, transv. polarized:

$$\gamma = 1 + \frac{a^2}{2} \qquad \text{for } a \gg 1$$

$$\hat{p}_y = \gamma\beta_y = \frac{\gamma}{c}\frac{dy}{dt} = a_y(\tau) \qquad \beta_y = \frac{a_y}{1 + a^2/2} \to 0$$

$$\hat{p}_z = \gamma\beta_z = \frac{\gamma}{c}\frac{dz}{dt} = a_z(\tau) \qquad \beta_z = \frac{a_z}{1 + a^2/2} \to 0$$

$$\hat{p}_x = \gamma\beta_x = \frac{\gamma}{c}\frac{dx}{dt} = a^2(\tau)/2 \qquad \beta_x = \frac{a^2/2}{1 + a^2/2} \to 1$$

e trajectories

for lin polarization in y:

$$a_y = a_0 \cos(\omega\tau)$$

drift w velocity

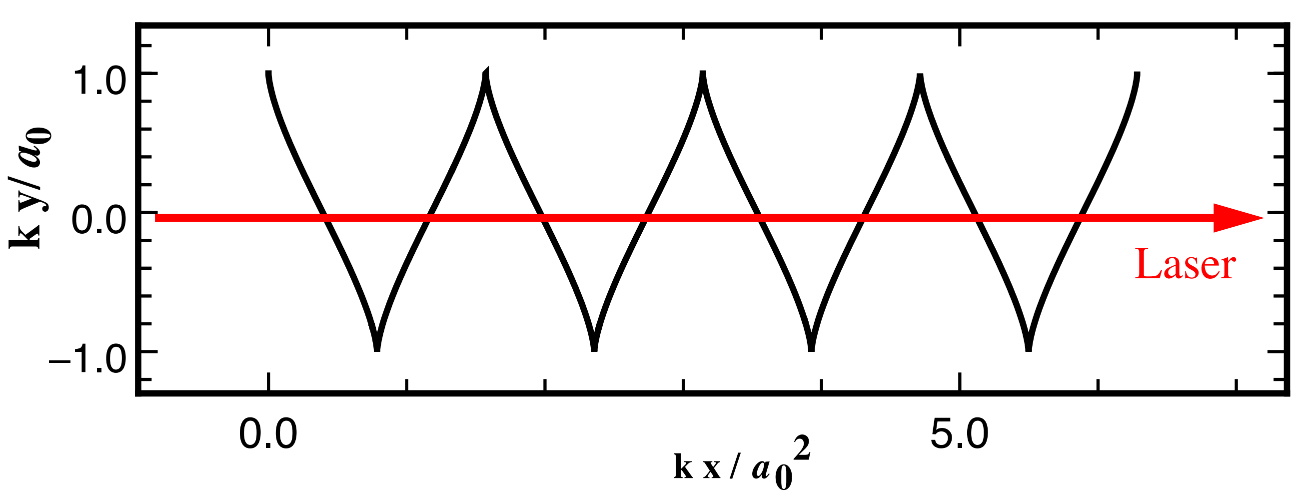
$$v_d = c a_0^2/(a_0^2 + 4)$$

e trajectories

$$x(\tau) = \frac{c a_0^2}{2}\int_0^\tau \cos^2(\omega\tilde{\tau})d\tilde{\tau} = \frac{c a_0^2}{4}\left[\tau + \frac{1}{2\omega}\sin(2\omega\tau)\right],$$

$$y(\tau) = c a_0 \int_0^\tau \cos(\omega\tilde{\tau})d\tilde{\tau} = \frac{c a_0}{\omega}\sin(\omega\tau),$$

$$\tau = t/\gamma \qquad \gamma = 1 + a_0^2/2$$



+ in e rest frame:

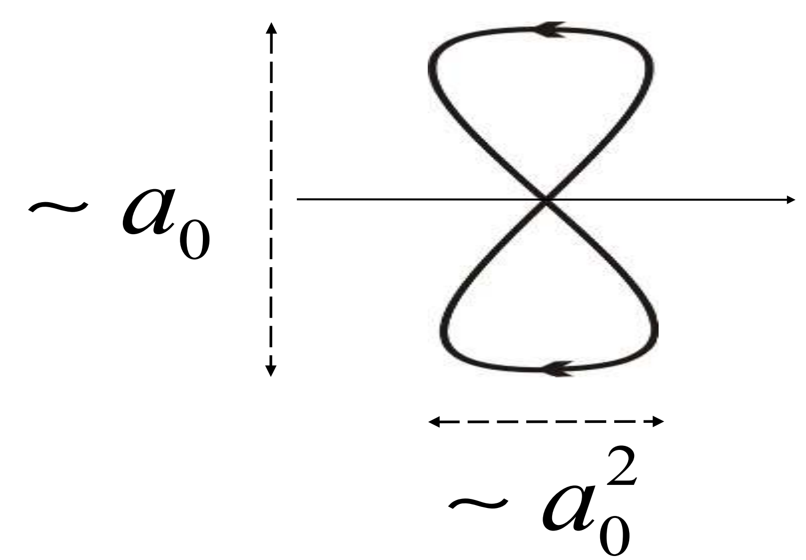


Figure-8 motion in drifting frame (ω=kc)

after laser passed:
e at rest
-> to acc e:
need to break symmetry, i.e. focus laser, ...

Outline

- Electron interacting with a strong electro-magnetic field
- Introduction to plasma physics
- Plasma waves
  - Ponderomotive potential
  - Ponderomotive plasma wave excitation
  - Properties of plasma wave
- Laser-plasma acceleration

Laser-matter interaction I: the ponderomotive force

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eqns of motion revisited:

from Lorentz force:

$$m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -e\vec{E}$$

for a wave with varying amplitude:

$$\vec{E} = \vec{E}_0(\vec{x},t)\cos(\vec{k}\vec{x} - \omega t + \Phi)$$

=>

$$\vec{v}_e(\vec{x},t) = -\frac{e}{m_e}\int \vec{E}_0(\vec{x},t)\cos\left(\vec{k}\vec{x} - \omega t + \Phi\right)\mathrm{d}t + \vec{v}_0$$

initial electron velocity == 0

-> quiver motion of electron

-> averaging over one oscillation period of the quiver energy ($E_\mathrm{q} = \frac{1}{2}m_\mathrm{e}|\vec{v}|^2$) :

=> ponderomotive potential:

$$U_\mathrm{P} = \langle E_\mathrm{q}\rangle = \frac{e^2}{4m_\mathrm{e}\omega^2}|\vec{E}_0|^2$$

$$\langle E_\mathrm{kin}\rangle = U_p = \frac{a_0^2}{2}m_e c^2$$

-> for a0>1: electron gains energy comparable with rest mass

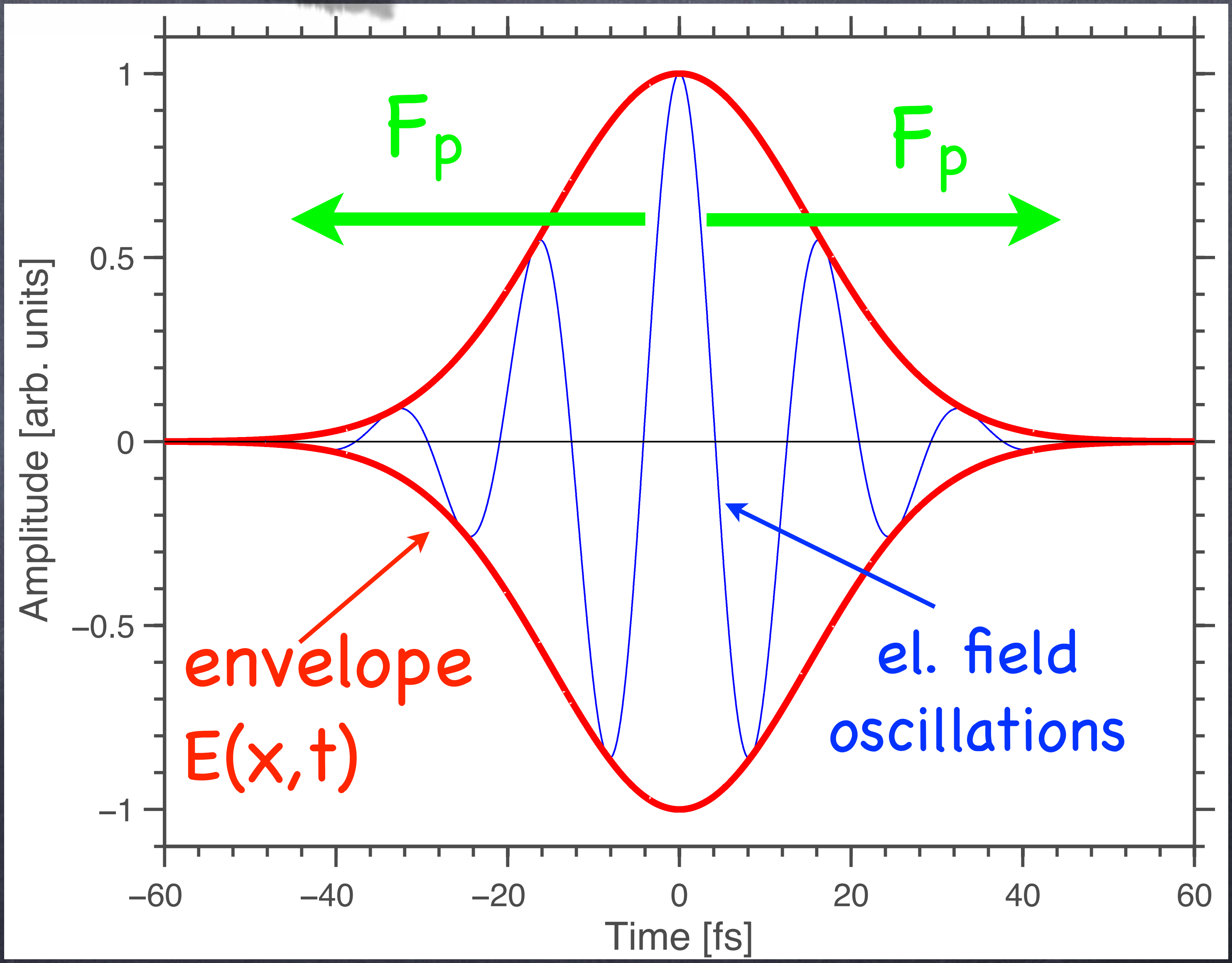
Laser-matter interaction II: the ponderomotive force

ponderomotive potential: $U_\mathrm{P} = \langle E_\mathrm{q} \rangle = \frac{e^2}{4 m_\mathrm{e} \omega^2} |\vec{E}_0|^2$

ponderomotive force: $\vec{F}_\mathrm{P} = -\vec{\nabla} U_\mathrm{P}$

-> is directed along the gradient of a laser-pulse envelope

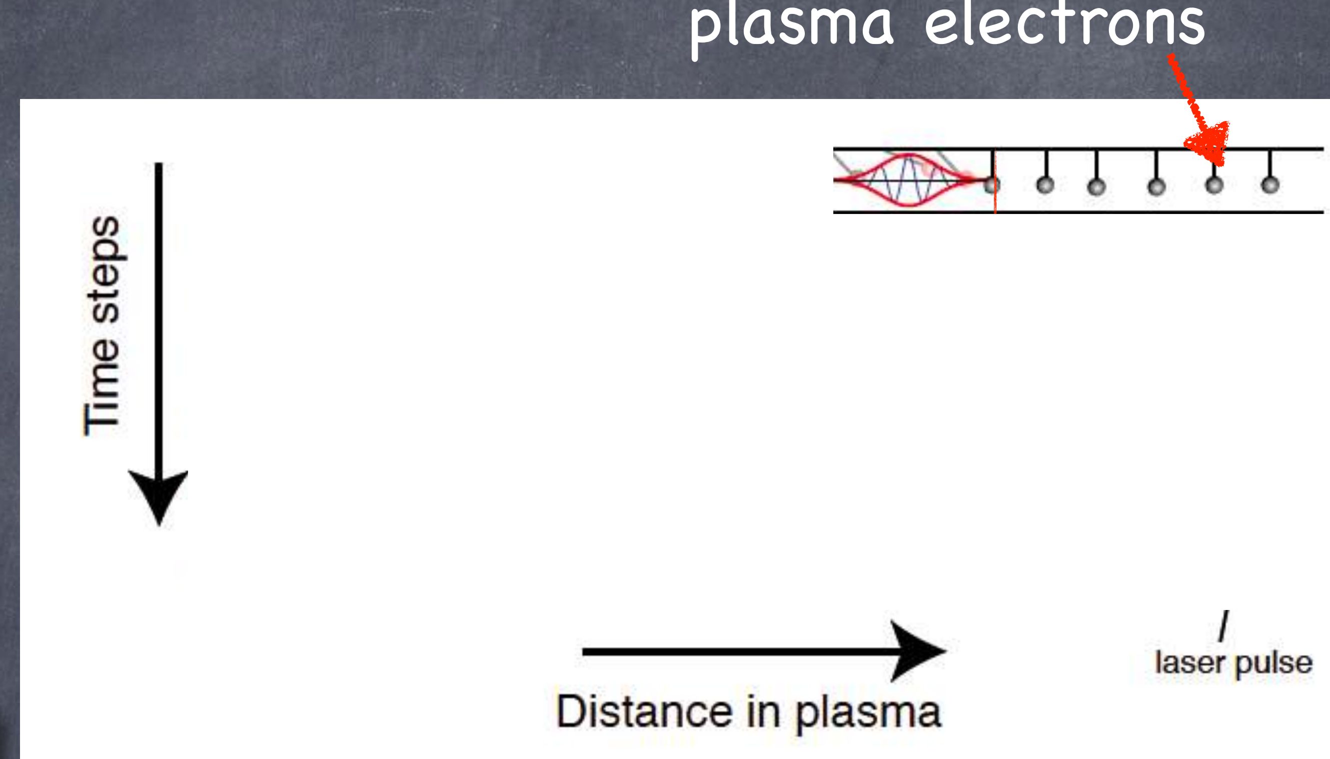
=> pushes electrons towards regions of lower laser intensity



Amplitude of a Gaussian laser pulse, 25 fs duration (FWHM)

Plasma Waves

- laser excites a plasma wave
- ultrashort laser pulse “kicks” electrons
- electrons are pulled back by stationary ions
- electrons oscillate with plasma frequency: $\omega_{p,0} = \sqrt{\frac{e^2 n_0}{m_e \epsilon_0}}$
- collective motion forms a plasma wave that is propagating at laser group velocity
- no charge transport: just oscillations

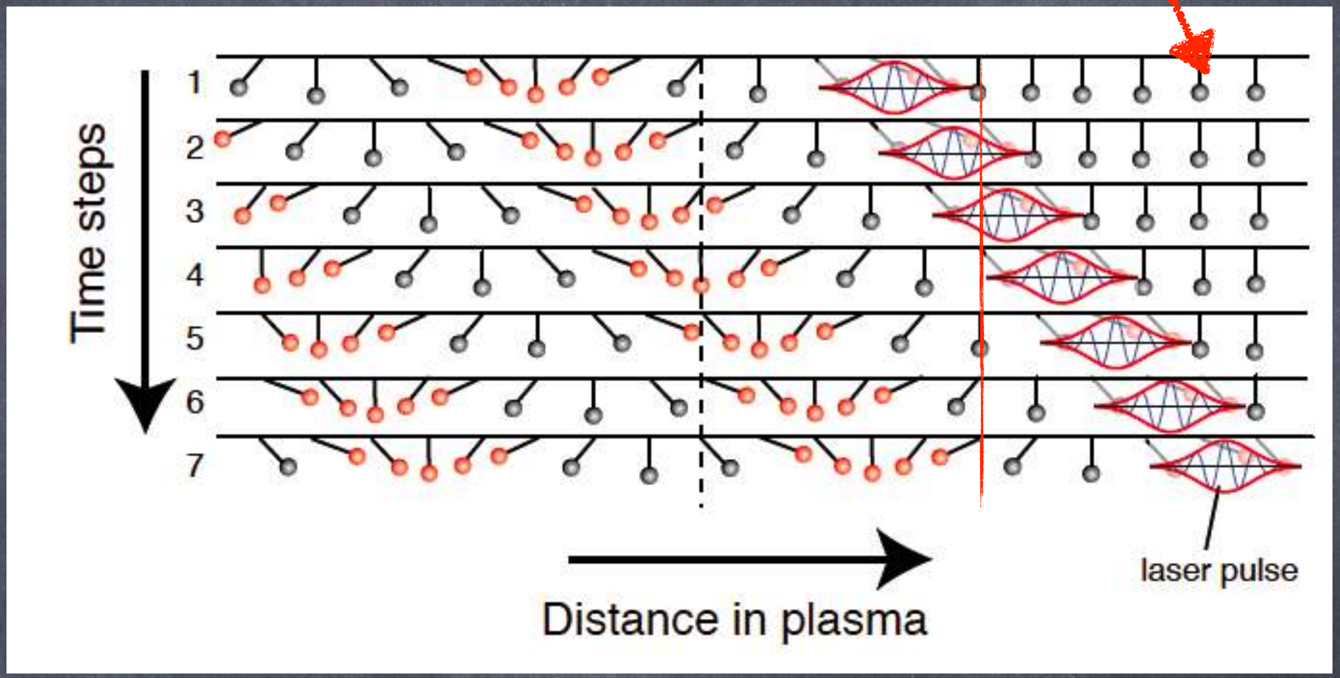


Dissertation M. F., after Dawson, Sci. American (1989)

Plasma Waves

- charge separation in the plasma wave
  - set up longitudinal electrical field: laser-wakefield
- particles injected into wakefield get accelerated!

plasma electrons



Dissertation M. F., after Dawson, Sci. American (1989)

Derivation of Plasma Wave 1

- "Delta" force moving with v_l :
  transfers momentum $m_e u_0$ at laser front

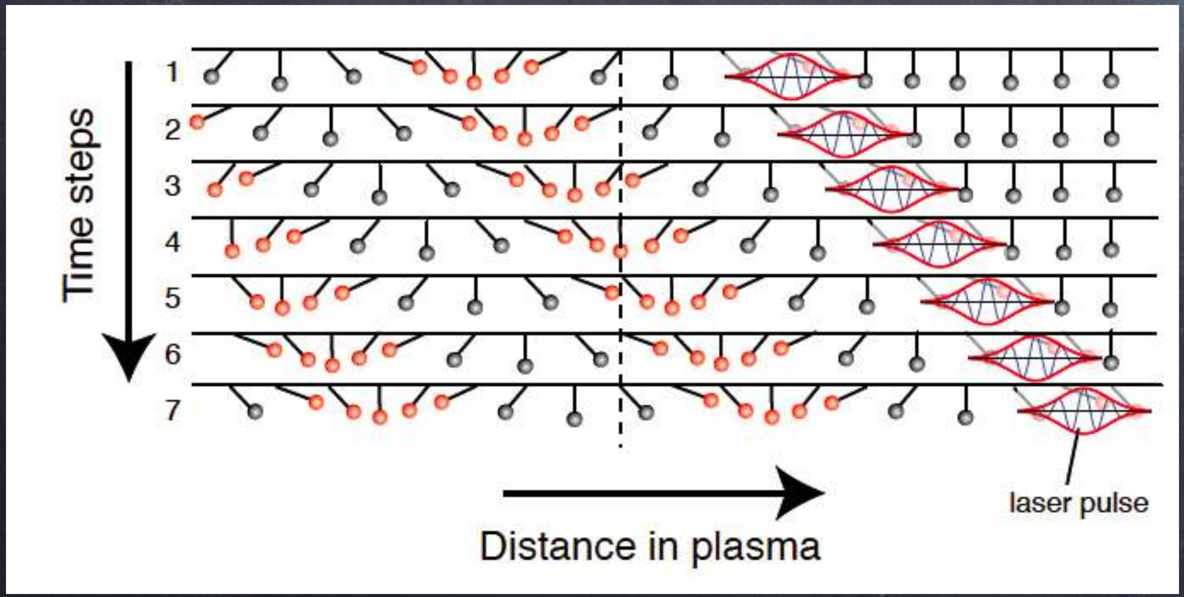
$$f \simeq m_e u_0 \delta(t - x/v_l)$$

- Electrons at laser front oscillate with plasma frequency.
  Electron Velocity:

$$u_x = u_0 \cos(\omega_p \tau)\ \Theta(\tau)$$

$$\tau = t - x/v_l$$

$$\Theta(\tau) = \begin{cases} 0 & \text{for } \tau \geq 0 \\ 1 & \text{for } \tau < 0 \end{cases}$$



Plasma Response

- linearized fluid equations: assume background with a small perturbation $\delta n_e = n_e - n_0$ $\quad n_e \ll n_0$

"cold fluid" equations:

- continuity equation: $\frac{\partial}{\partial t}\delta n + n_0 \vec{\nabla}\cdot\vec{u} \simeq 0$
- momentum equation: $\frac{\partial \vec{u}}{\partial t} \simeq \nabla\phi - \nabla a_0^2/2,$
- Poisson equation: $\nabla^2\phi \simeq k_p^2 \frac{\delta n}{n_0}$

n: electron density
u: fluid velocity
a_0: relativistic laser potential
(~ laser intensity)
k_p: plasma wavenumber
ϕ : electrostatic potential

$$\frac{\partial}{\partial t} n_e \simeq - n_0 \frac{\partial}{\partial x} u_x$$

(a_0 = 0)

$$\frac{\partial}{\partial t} u_x \simeq \frac{eE_x}{m_e}$$

Derivation of Plasma Wave 2

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linearized fluid equations:

$$\frac{\partial}{\partial t} n_e \simeq - n_0 \frac{\partial}{\partial x} u_x$$

$$\frac{\partial}{\partial t} u_x \simeq \frac{eE_x}{m_e}$$

Electron oscillation

$$u_x = u_0 \cos(\omega_p \tau)\ \Theta(\tau)$$

Electron density:

$$\delta n_e \simeq n_0 \frac{u_0}{v_l} \cos(\omega_p \tau)\ \Theta(\tau)$$

$$\delta n_e = n_e - n_0$$

$$\Theta(\tau) = \begin{cases} 0 & \text{for } \tau \geq 0 \\ 1 & \text{for } \tau < 0 \end{cases}$$

(longitudinal) electric field:

$$E_x \simeq \frac{m_e \omega_p u_0}{e} \sin(\omega_p \tau)\ \Theta(\tau)$$

Plasma Wave Properties

- describe plasma electrons as fluid (use continuity-, momentum- and Poisson equation)

solution for ultrashort Gaussian laser pulse with frequency >> ω_p

-> sinusoidal wave with period λ_p

- electron density:

$$\frac{\delta n}{n_0} \sim \frac{a_0^2}{2} \sin(k_p \xi)$$

a_0^2 ~ laser intensity

- electric field

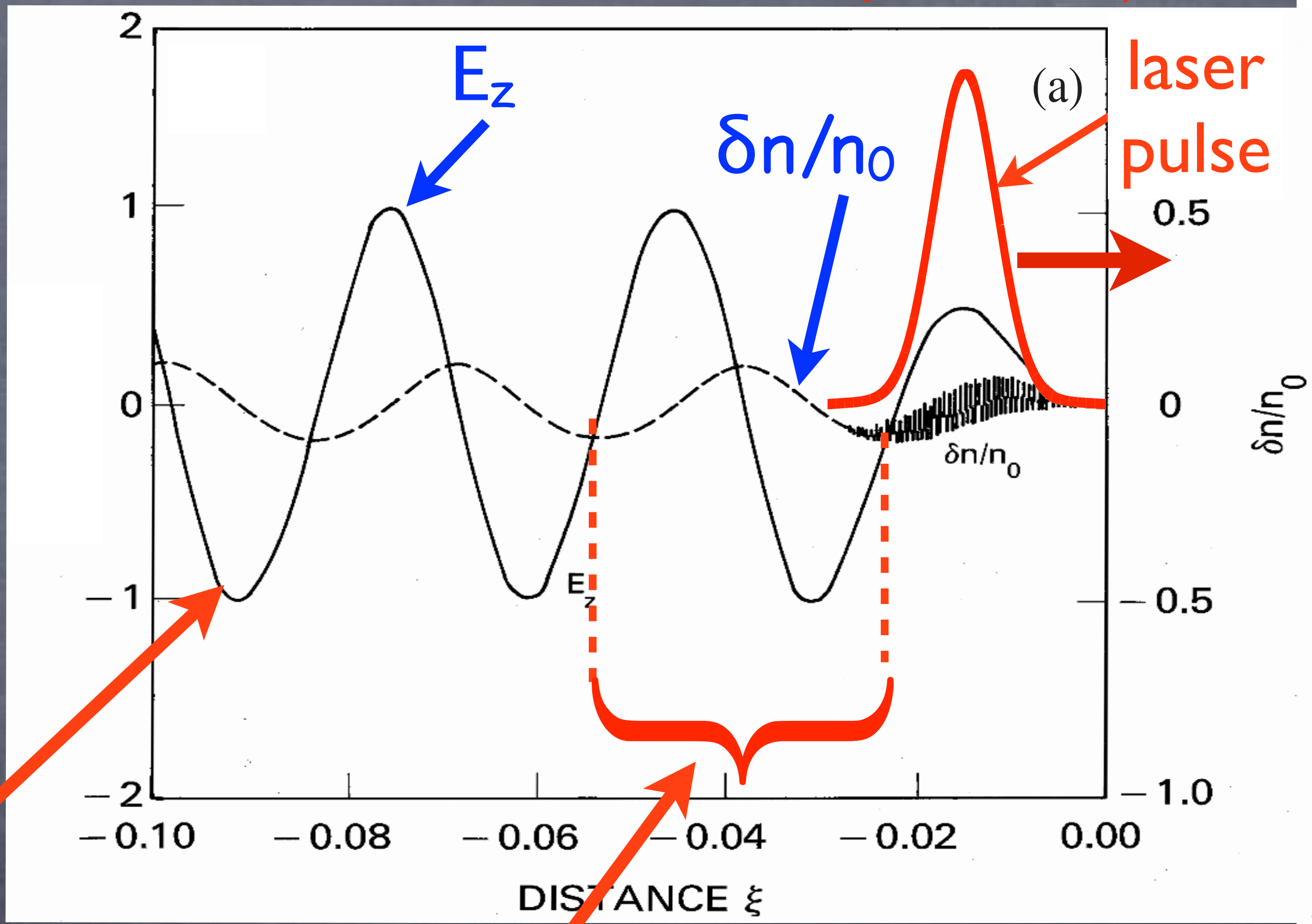
$$E_z \sim \frac{m_e c \omega_p}{e} \frac{a_0^2}{2} \cos(k_p \xi)$$

$$E_0[\mathrm{V/m}] \simeq 96\sqrt{n_0[\mathrm{cm}^{-3}]}$$

-> typical density: $n_0 = 10^{18}\ \mathrm{cm}^{-3}$

=> $E_0 \approx 100$ GV/m !!

(RF: 20 MV/m)



graph: E. Esarey, IEEE Tr... Plasma Sci. 24, 252-288

λ_p = 10 μm (30 fs)

Matthias Fuchs

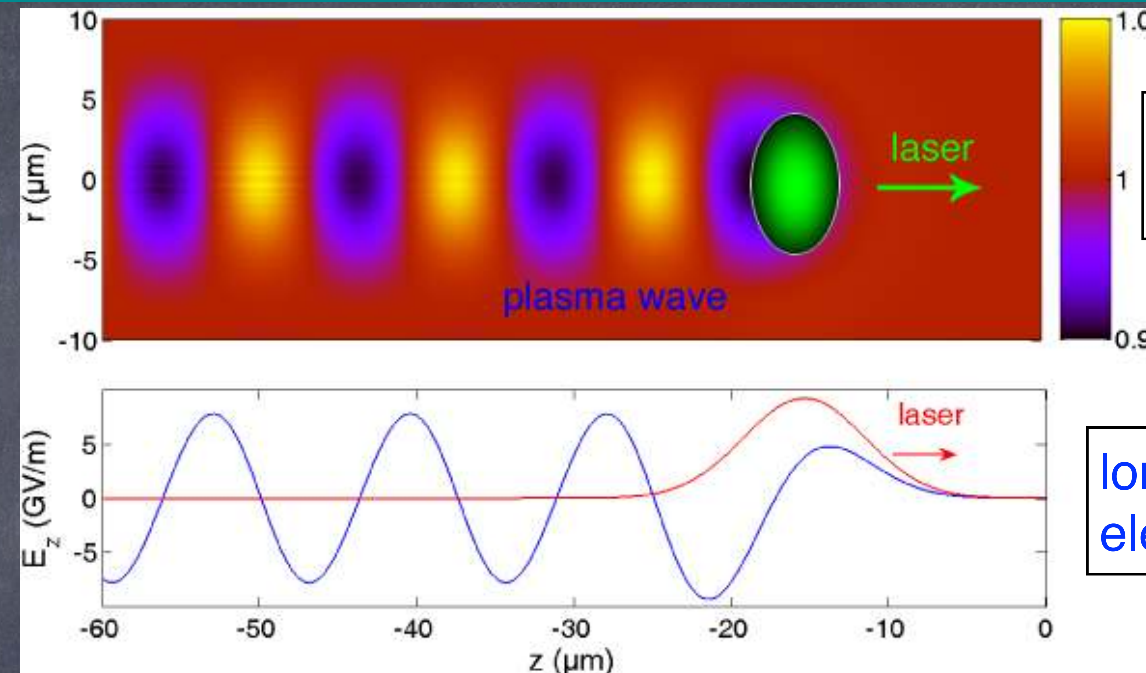
Outline

- Electron interacting with a strong electro-magnetic field
- Introduction to plasma physics
- Plasma waves
- Laser-plasma acceleration
    - Discussion: Laser-wakefield acceleration
    - Maximum energy gain
    - Limits of laser-wakefield acceleration

Wakefield

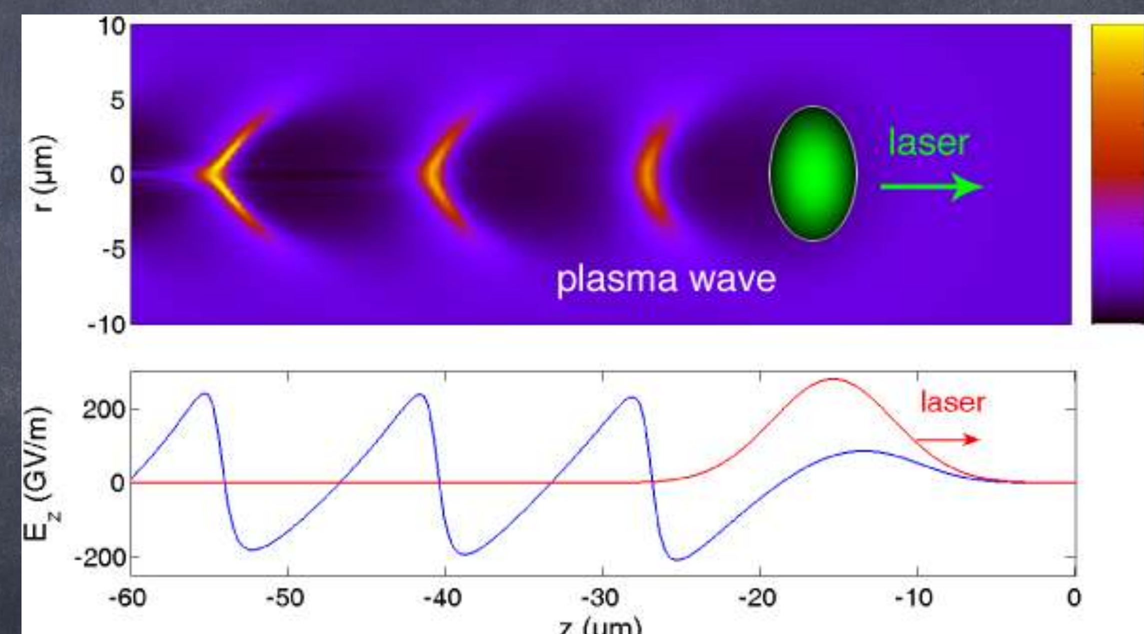
- <u>Linear Wakefield</u>
  laser: $a_0 = 0.35$
  -> sinusoidal density perturbation
  -> ~GV/m sinusoidal longitudinal el. field

- <u>Nonlinear Wakefield</u>
  laser: $a_0 \sim 2$
  -> can only be solved numerically
  -> density spikes
  -> sawtooth el. field
  (~100 GV/m amplitude)

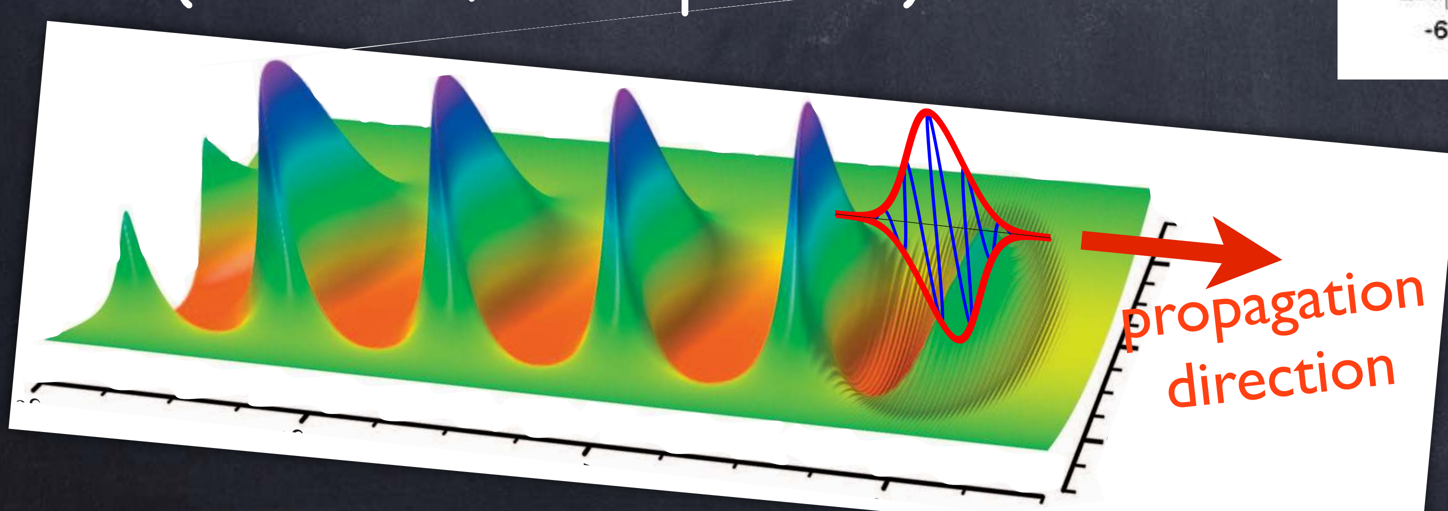


electron density

longitudinal electric field



V. Malka, in Proc. of the CERN Accelerator School (2016)



source: Shadwick, UNL

Wake Properties

For linear wakes:

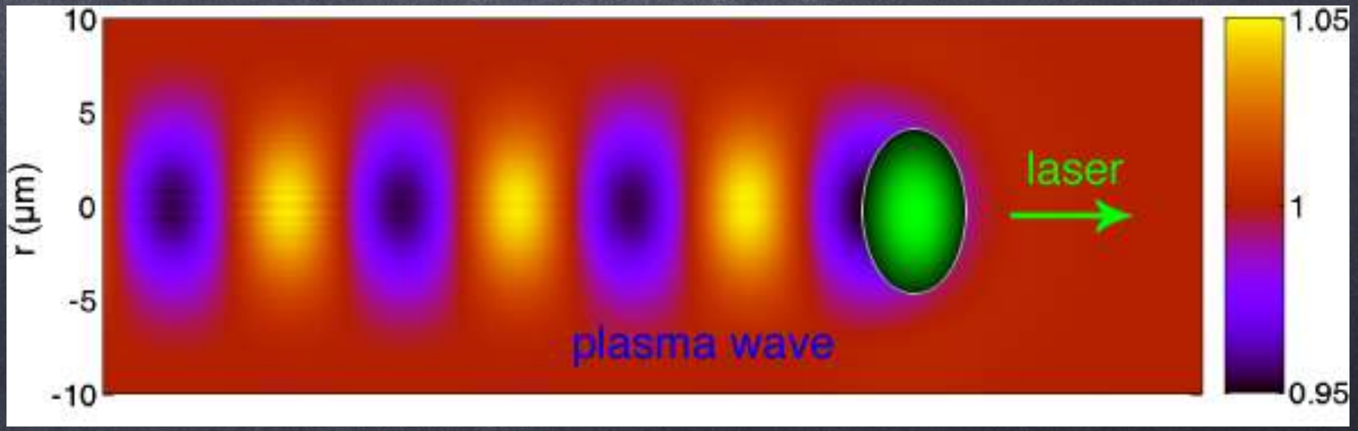
- estimated accelerating field (cold non-relativistic wavebreaking limit)

$$E_0[\mathrm{V/m}] \simeq 96\sqrt{n_0[\mathrm{cm}^{-3}]}$$

- for typical densities $n_0 = 1\times10^{18}$ cm^{-3}: E_0 = 100 GV/m !! (= 10 V/Å; close to atomic unit electric field !!)

- size of accelerating structure: (~plasma wavelength)

$$\lambda_{p,0}[\mu\mathrm{m}] = \frac{2\pi c}{\omega_{p,0}} = 3.33\times10^{10}\left(n_0[\mathrm{cm}^{-3}]\right)^{-1/2}$$



- for $n_0 = 1\times10^{18}$ cm^{-3}: λ_p = 30 μm !! (=> or 100 fs)

Experimental Observation

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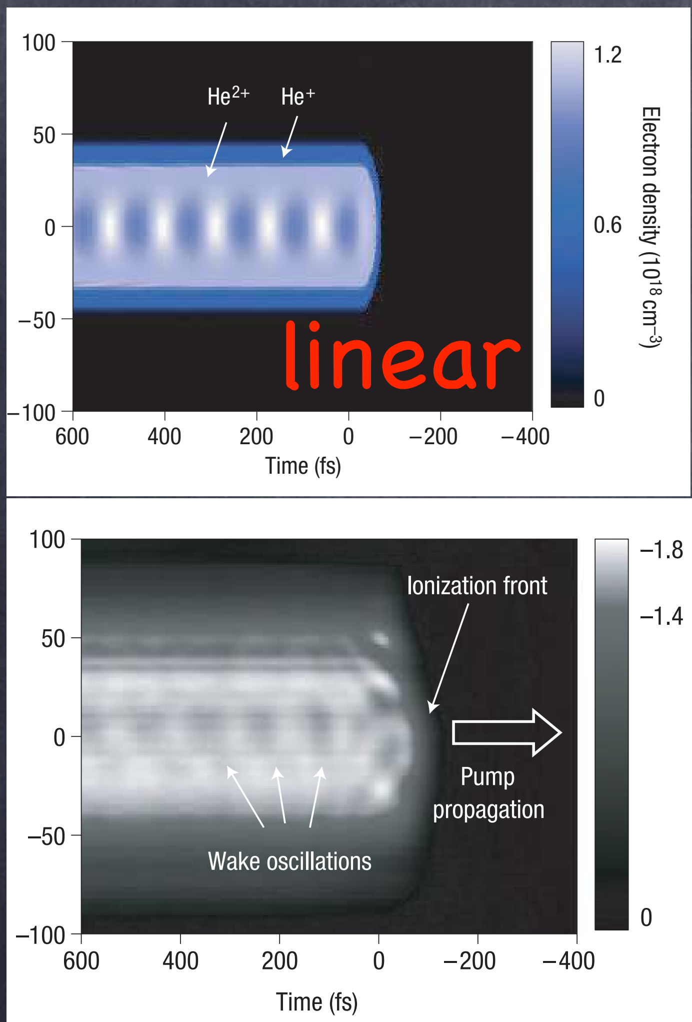
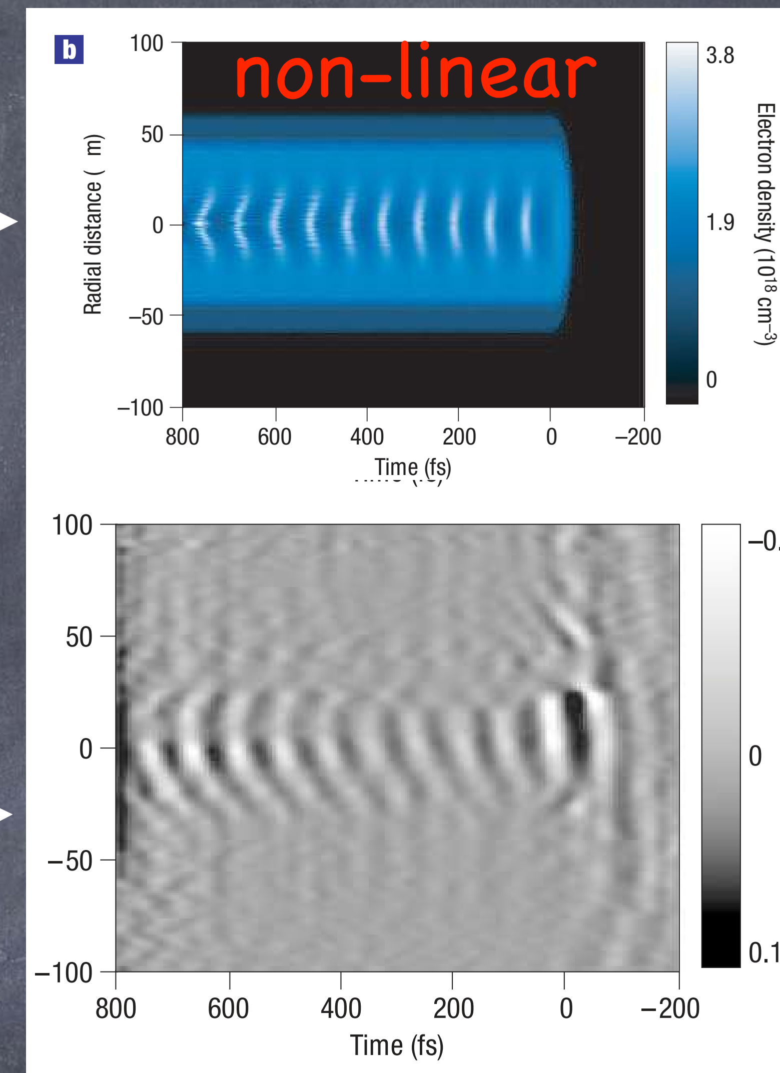


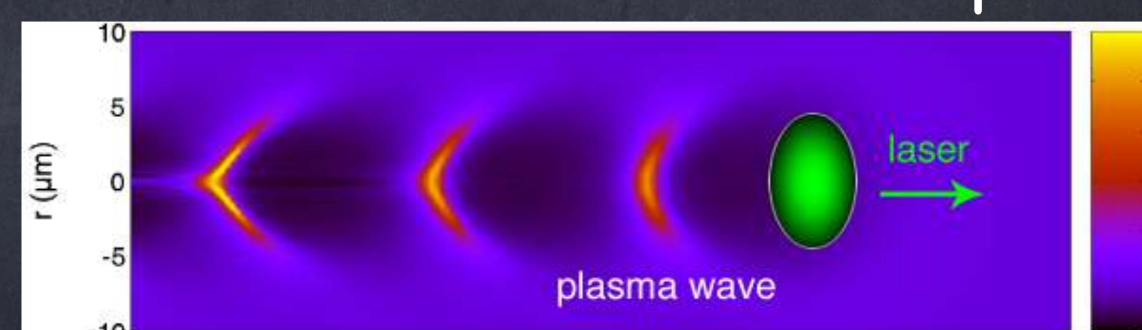
simulation

N. Matlis, et al.
Nature Phys., 2, 749 (2006)

experiment
using frequency-domain holography



non-linear: later buckets become "horseshoe" shaped



Electron Acceleration

- Hamiltonian of electron motion

laser intensity

electrostatic potential of wake

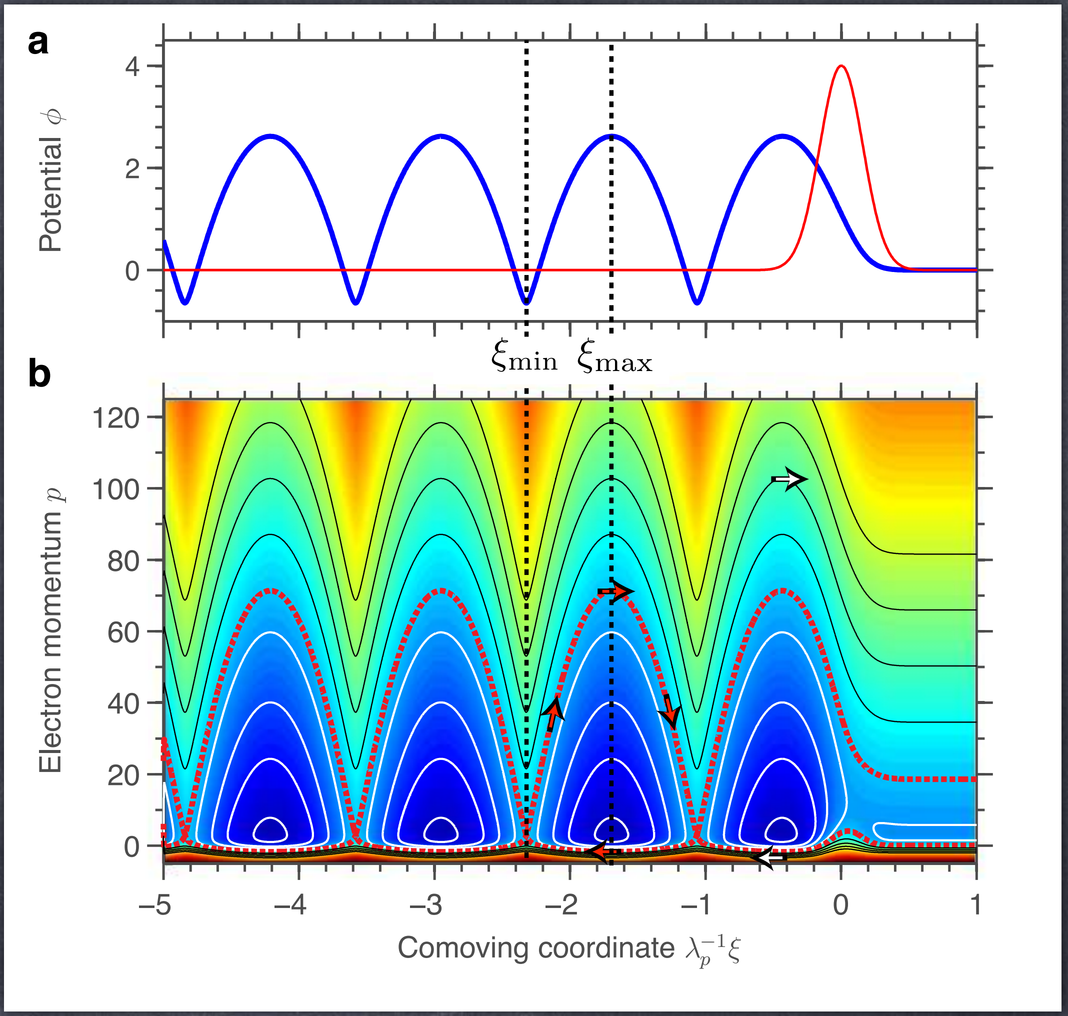
$$H(p_z,\xi) = \sqrt{p_z^2 + 1 + a^2} - \beta_p p_z - \phi(\xi)$$

long. é momentum

velocity of plasma wave

$\xi = z - v_g t$

co-moving frame -> distance from laser pulse



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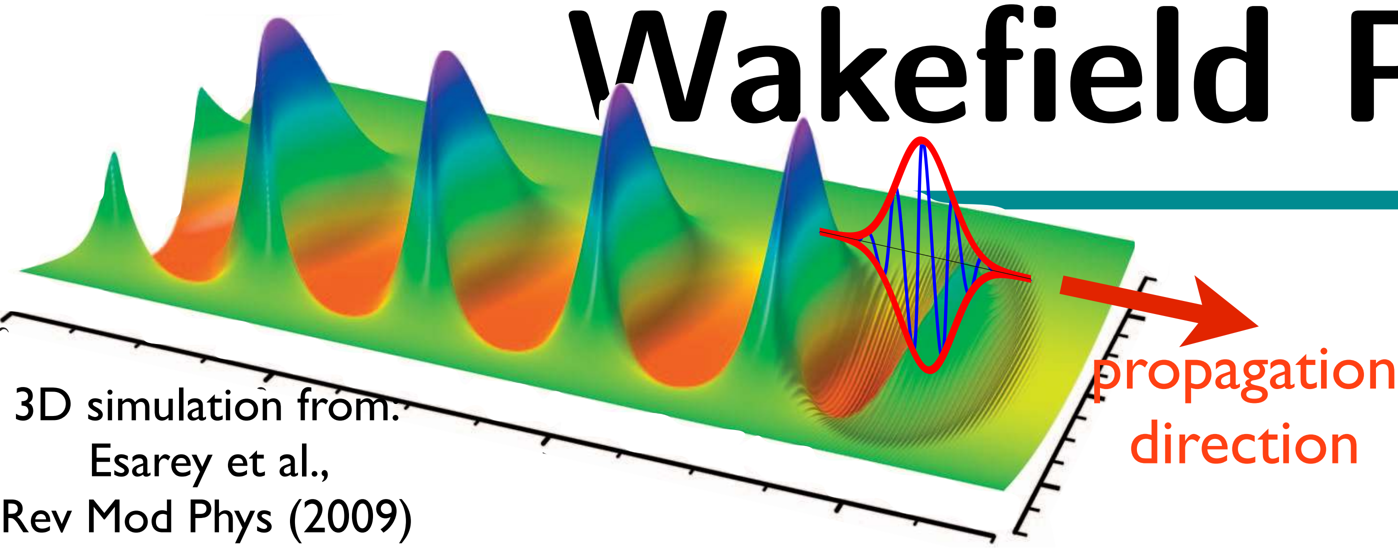
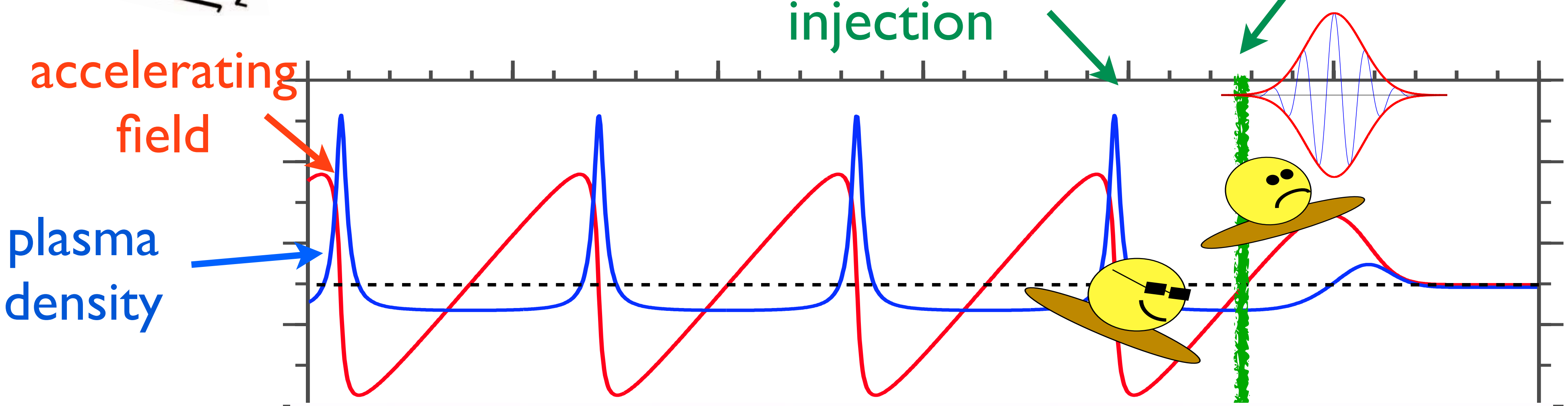
- electrons inside separatrix are accelerated (red)
- electrons outside (white): background plasma electrons
(flow backwards wrt laser or slowly overtake wake)

Wakefield Properties I

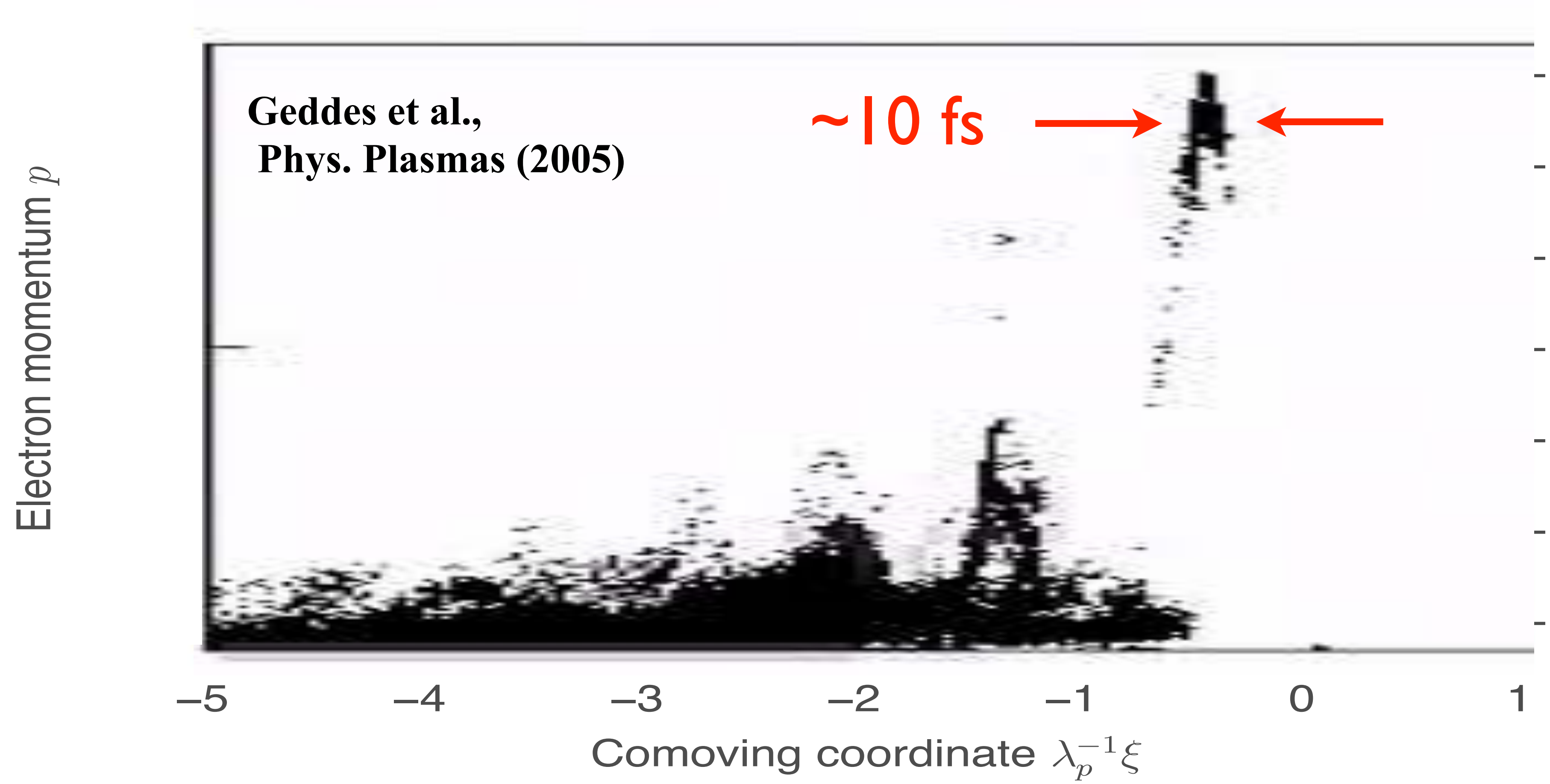


3D simulation from:
Esarey et al.,
Rev Mod Phys (2009)



- <u>electron bunch duration</u>

quasi-monoenergetic peak indicates: fraction of plasma wavelength:
λ_p = 15 μm (50 fs)



Wavebreaking

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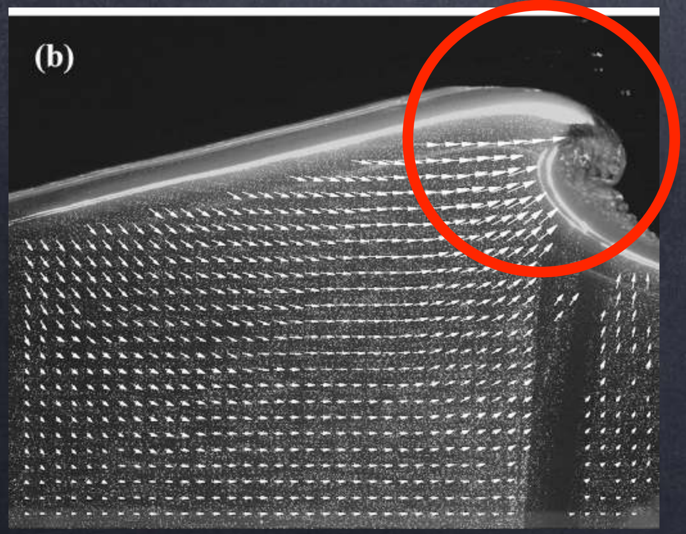
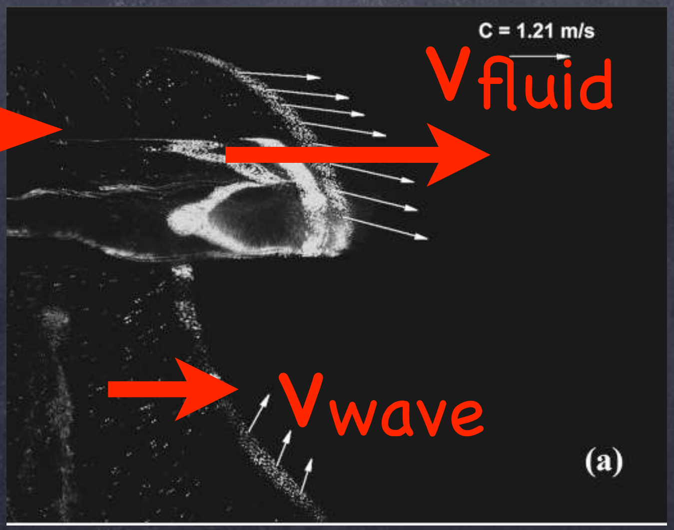
quick summary:

- oscillating plasma electrons form plasma wave
- plasma wave propagates ~with laser group velocity: $\gamma_p = \gamma_{\text{laser,g}} \simeq \frac{\omega}{\omega_p}$
  typically: $\gamma_p \approx$ 10-100 (including 3D effects)
- electron fluid velocity: $\gamma_e = \frac{(1+a^2)+(1+\phi)^2}{2(1+\phi)}$
  -> depends on laser intensity
- "self" injection through wavebreaking

-> at sufficiently high laser intensity, the electron fluid velocity is higher than plasma wave phase velocity $\gamma_e > \gamma_p$ => wave breaking





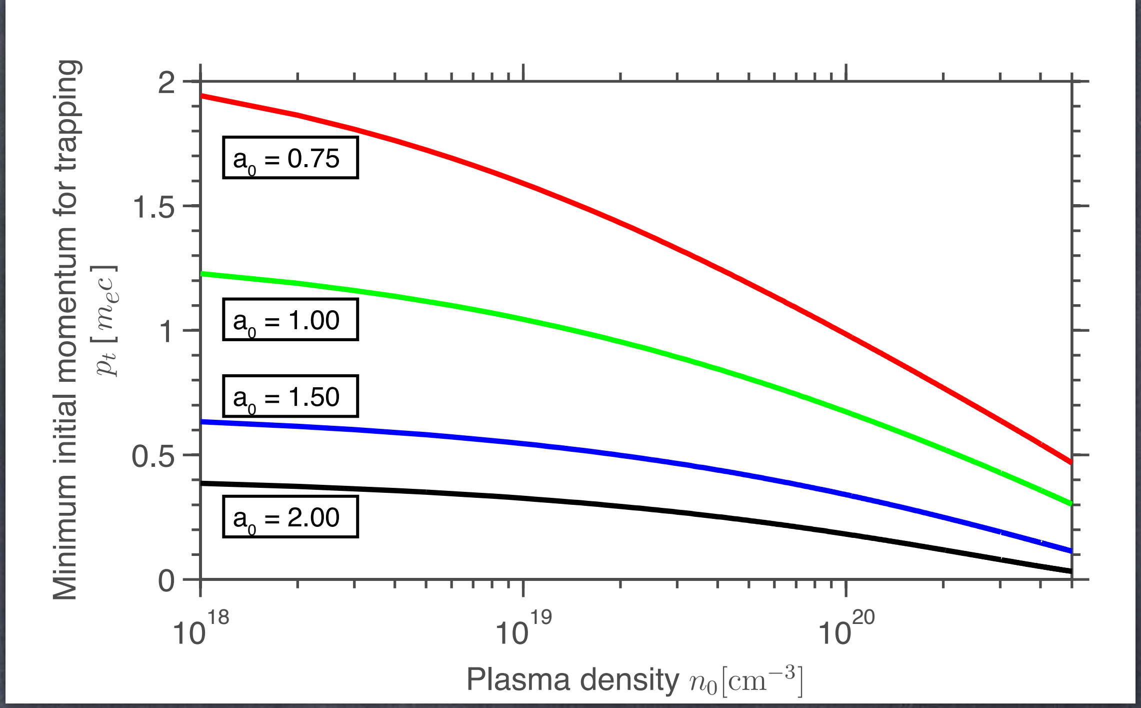
Chang & Liu, Phys. Fluids 10 (1) 1998

Electron Injection

Electron Trapping

- electrons get trapped (cross into separatrix) at the back of the bucket (at the potential minimum)
- minimum momentum required for trapping

$$p_t = \beta_p \gamma_p (1 - \gamma_p \phi_{\min}) - \gamma_p \sqrt{(1 - \gamma_p \phi_{\min})^2 - 1}$$



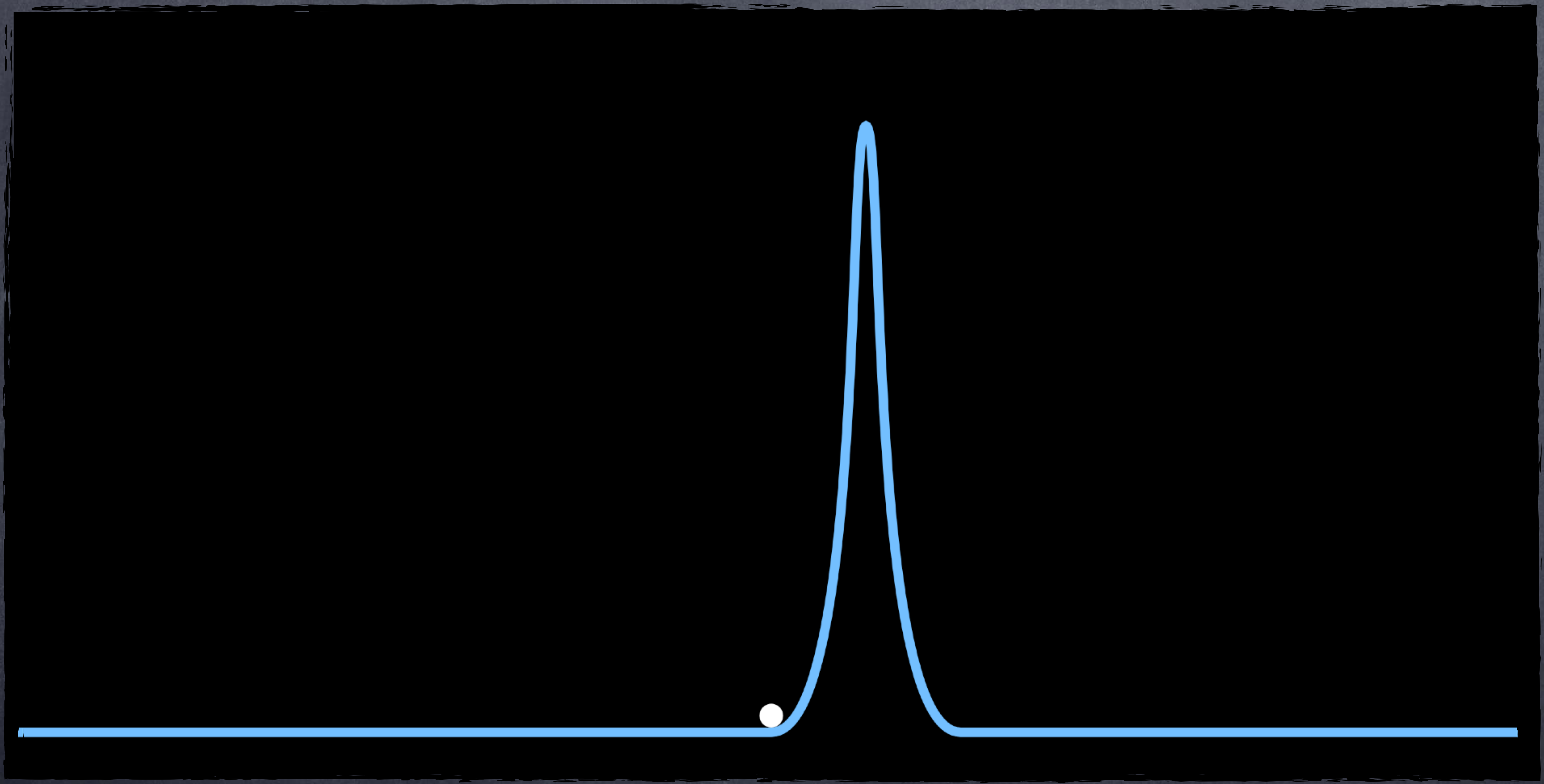
Dissertation M.F.

- implicitly depends on laser intensity a_0 and plasma density n_0
- lower momentum required for higher laser intensities and higher plasma densities

bigger separatrix amplitude

slower wake velocity

Untrapped electron



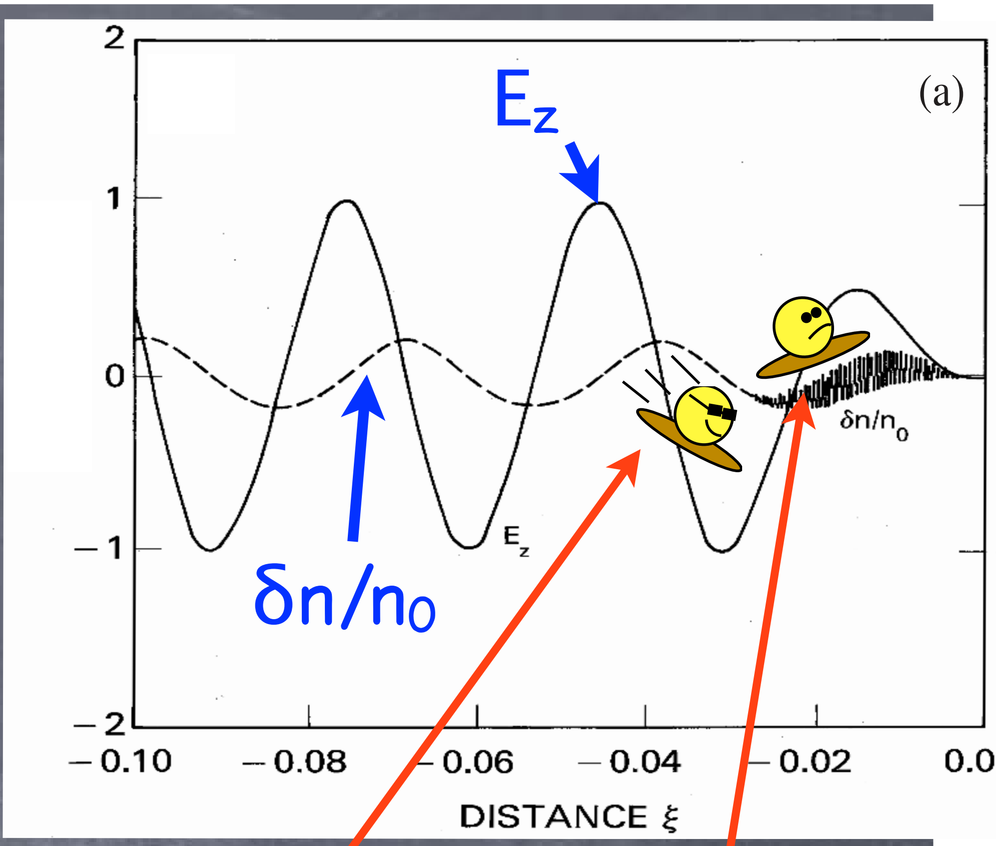
Electron velocity too small for trapping

Trapped & accelerated electron

Electron velocity sufficiently high

Accelerator Length, Maximum Energy

- dispersion relation : $\omega^2 = \omega_p^2 + c^2k^2$
  (for laser in plasma)
- plasma wave moves
  w laser group velocity: $v_g = \frac{d\omega}{dk} = \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2} \cdot c$
  slightly slower than c



- dephasing length L_d: $L_d \simeq \frac{\lambda_p^3}{\lambda^2} \sim n_0^{-3/2}$

distance until electrons (moving with c) outrun accelerating phase

$$t_{wave} = \frac{L_d}{V_g}$$

$$t_e = \frac{L_d + \frac{\lambda_p}{2}}{c}$$

$$t_{wave} = t_e$$

Accelerator Length, Maximum Energy

- for $\omega_p \ll \omega$: $\quad V_g = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}\, c \approx \left(1 - \frac{\omega_p^2/\omega^2}{2}\right) c$

$$\frac{\omega_p^2}{\omega^2} \stackrel{!}{=} x^2$$

$$t_{wave} = \frac{L_d}{V_g}$$

$$t_e = \frac{L_d + \frac{\lambda_p}{2}}{c}$$

$t_{wave} = t_e$:

$$\frac{L_d}{\left(1 - \frac{x^2}{2}\right) c} = \frac{L_d + \frac{\lambda_p}{2}}{c}$$

$$L_d \left(\frac{1}{1 - \frac{x^2}{2}} - 1 \right) = \frac{\lambda_p}{2}$$

$$\frac{1 - \left(1 - \frac{x^2}{2}\right)}{1 - \frac{x^2}{2}} = \frac{x^2}{2\left(1 - \frac{x^2}{2}\right)} = \frac{x^2}{2 - x^2}$$

$$L_d = \frac{2 - x^2}{x^2} \frac{\lambda_p}{2}$$

$\left(\frac{\omega_p}{\omega}\right)^2 \ll 1$

$$= \frac{2}{x^2} \frac{\lambda_p}{2}$$

$$= \frac{\omega_p^2}{\omega^2} \lambda_p = \frac{\lambda_p^3}{\lambda^2}$$

($\frac{\omega^2}{\omega_p^2} = \frac{\lambda_p^2}{\lambda^2}$)

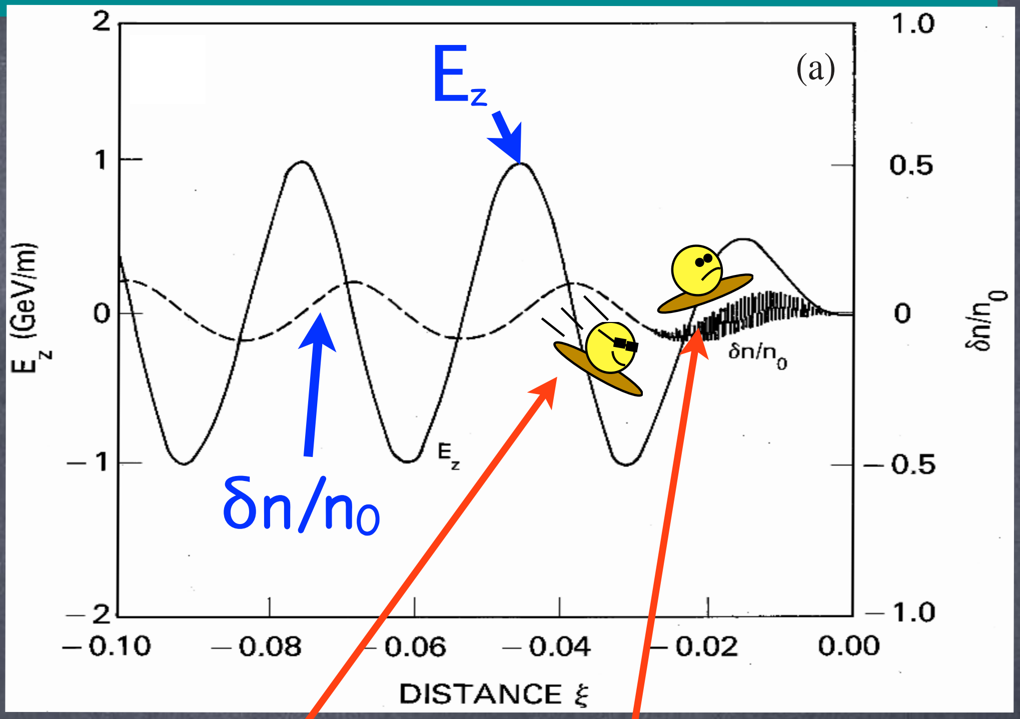
Accelerator Length, Maximum Energy

- dephasing length:

distance until electrons (moving with c) outrun accelerating phase

$$L_d \simeq \frac{\lambda_p^3}{\lambda^2} \sim n_0^{-3/2}$$

$\lambda_p \sim \frac{1}{\sqrt{n_0}}$



- max electron energy:

assume const. E_0

$$W_{max} = eE_0 L_d \sim \frac{1}{n_0}$$

$$E_0[\mathrm{V/m}] \simeq 96\sqrt{n_0[\mathrm{cm}^{-3}]}$$

⇒ overall higher energy gain for lower plasma densities

Accelerator Length, Maximum Energy

- dephasing length:

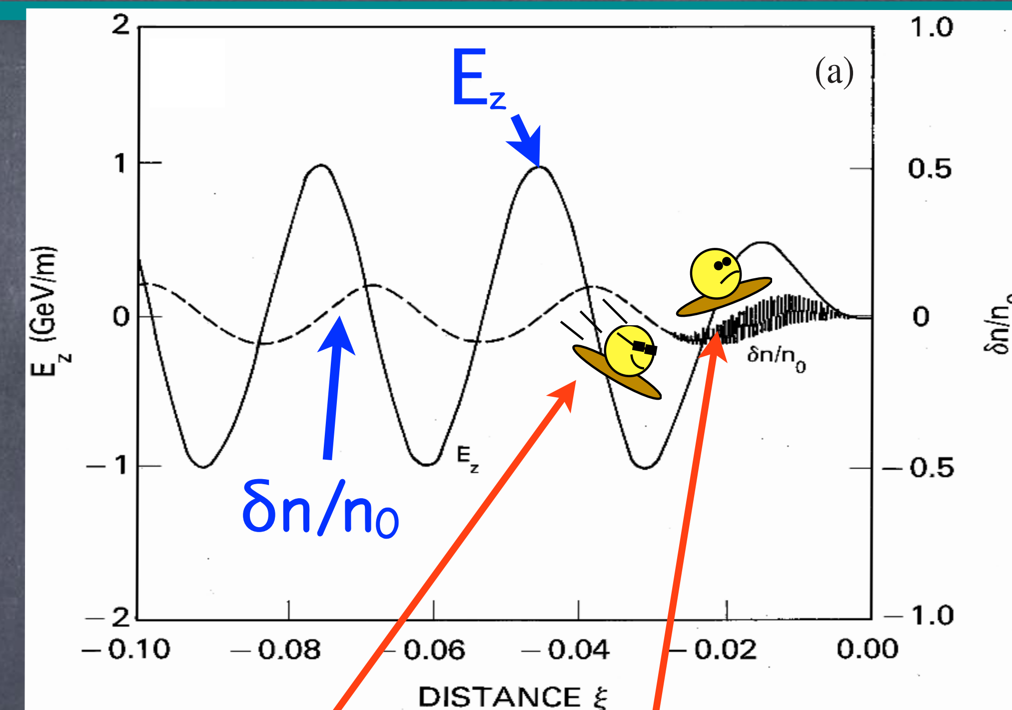
$$L_d \simeq \frac{\lambda_p^3}{\lambda^2} \sim n_0^{-3/2}$$

for $n_0 = 3 \cdot 10^{17}\,\mathrm{cm}^{-3}$:

$$\lambda_{p,0}[\mu\mathrm{m}] = \frac{2\pi c}{\omega_{p,0}} = 3.33 \times 10^{10}\,(n_0[\mathrm{cm}^{-3}])^{-1/2} = 60\,\mu\mathrm{m}$$

$$L_d = \frac{(60\,\mu\mathrm{m})^3}{(0.8\,\mu\mathrm{m})^2} = 0.3\ \mathrm{m}$$

laser → $(0.8\,\mu\mathrm{m})$



- max electron energy:

assume const. E_0

$$W_{max} = eE_0 L_d \sim \frac{1}{n_0}$$

$$E_0[\mathrm{V/m}] \simeq 96\sqrt{n_0[\mathrm{cm}^{-3}]} \approx 52\ \frac{\mathrm{GV}}{\mathrm{m}}$$

$$\Rightarrow W_{max} = 15.6\ \mathrm{GeV}$$

- Experimental results:

PHYSICAL REVIEW LETTERS **122**, 084801 (2019)

Editors' Suggestion | Featured in Physics

Petawatt Laser Guiding and Electron Beam Acceleration to 8 GeV in a Laser-Heated Capillary Discharge Waveguide

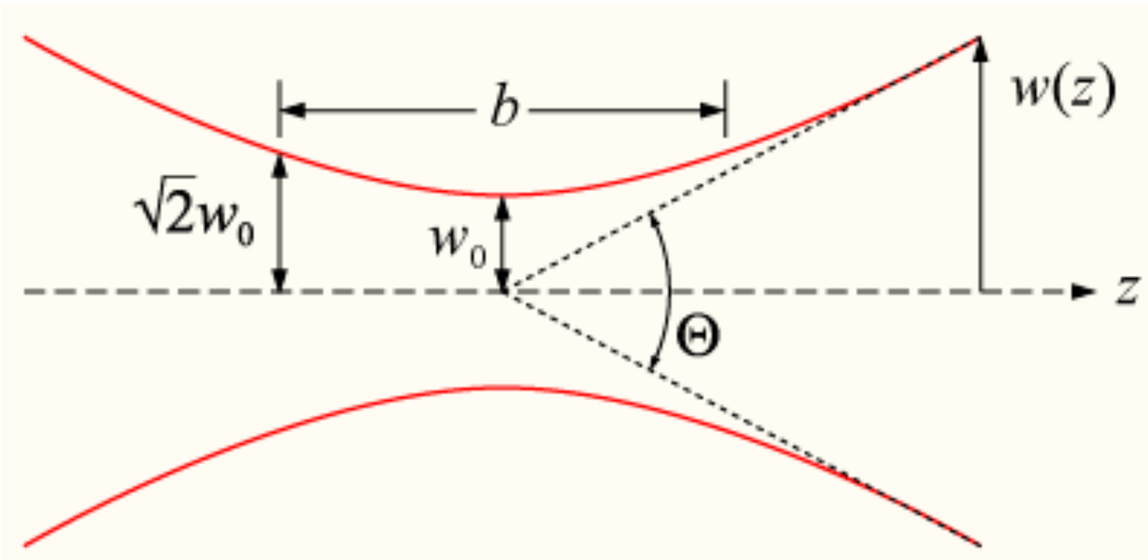
A. J. Gonsalves,[1,*] K. Nakamura,[1] J. Daniels,[1] C. Benedetti,[1] C. Pieronek,[1,2] T. C. H. de Raadt,[1] S. Steinke,[1] J. H. Bin,[1] S. S. Bulanov,[1] J. van Tilborg,[1] C. G. R. Geddes,[1] C. B. Schroeder,[1,2] Cs. Tóth,[1] E. Esarey,[1] K. Swanson,[1,2] L. Fan-Chiang,[1,2] G. Bagdasarov,[3,4] N. Bobrova,[3,5] V. Gasilov,[3,4] G. Korn,[6] P. Sasorov,[3,6] and W. P. Leemans[1,2,†]

along the capillary axis [20], and that this structure can extend the LPA length to 20 cm (15 diffraction lengths) at low ($\approx 3.0 \times 10^{17}\ \mathrm{cm}^{-3}$) density. This enabled the generation of electron beams with quasimonoenergetic peaks in energy up to 7.8 GeV using a peak laser power of 850 TW.

For L = 20 cm:

W_{max} = 52 GV/m *0.2m = 10.4 GeV

Other Acceleration Limits: Laser Diffraction



$$Z_R = \pi r_0^2/\lambda$$

for 25 μm focus: Zr = 0.5 mm

Guiding

plasma index of refraction:

$$\eta = \sqrt{1 - \left(\frac{\omega_p}{\omega_{las}}\right)^2} = \sqrt{1 - \frac{4\pi e^2 n_e}{m_e}}$$

phase velocity:

$$v_{ph} = \frac{c}{\eta}$$

Matthias Fuchs

How to guide a laser?

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$$\eta = \sqrt{1 - \left(\frac{\omega_p}{\omega_{las}}\right)^2} = \sqrt{1 - \frac{4\pi e^2 n_e}{m_e}}$$

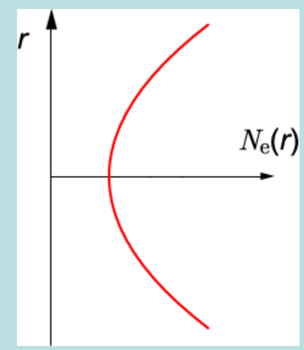
guiding using density gradient: "Plasma fiber"

$$v_{ph} = \frac{c}{\eta}$$

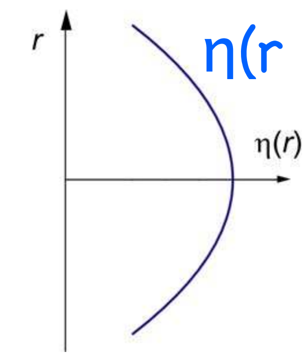
relativistic self focussing
-> self guiding

engineering transverse plasma profile (low density on axis)

Plasma channel
transverse variation of ***density*** yields refractive index profile

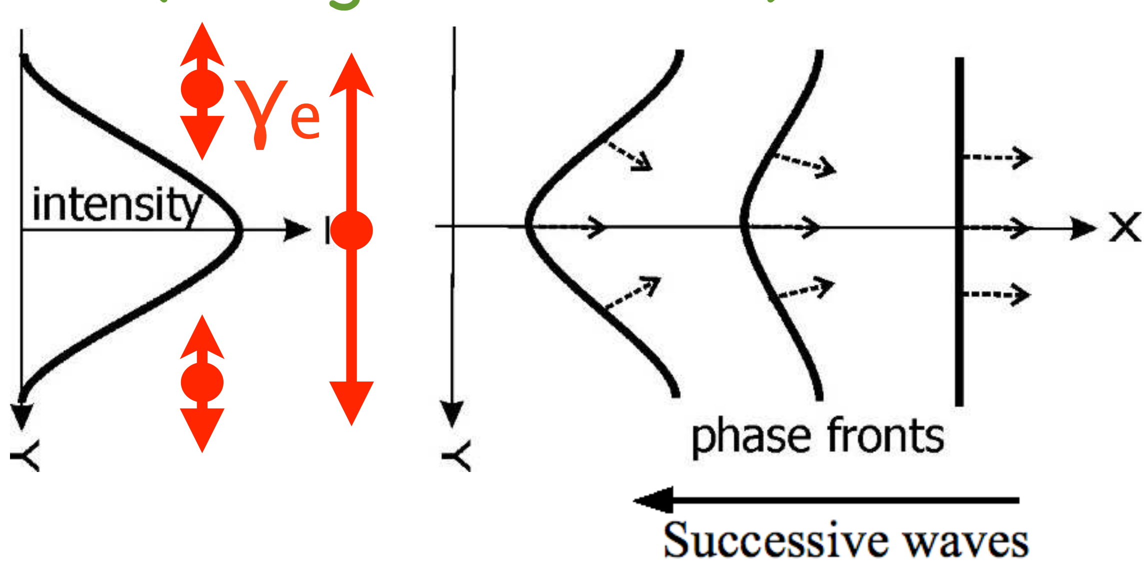


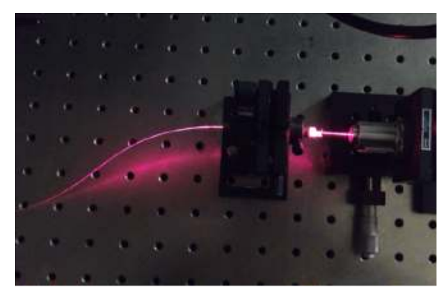
$$\eta \approx 1 - \frac{1}{2}\frac{n_e(r)e^2}{\gamma m_e \varepsilon_0 \omega^2}$$

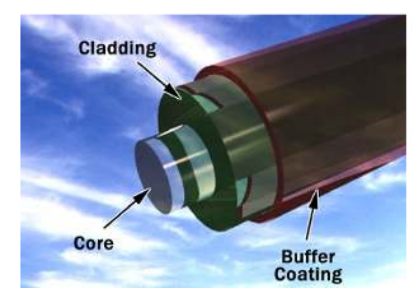


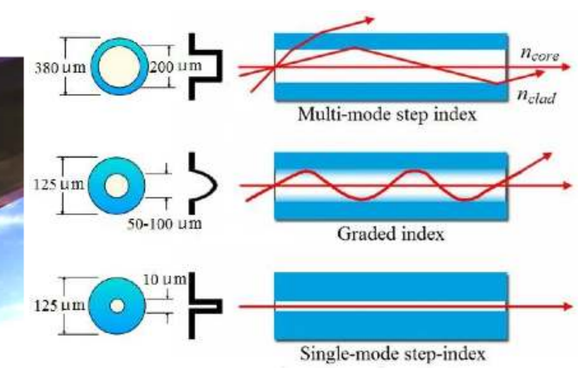
Self-focussing: $v_{ph} = c/n_R$

increase in electron mass on axis (at high intensities)

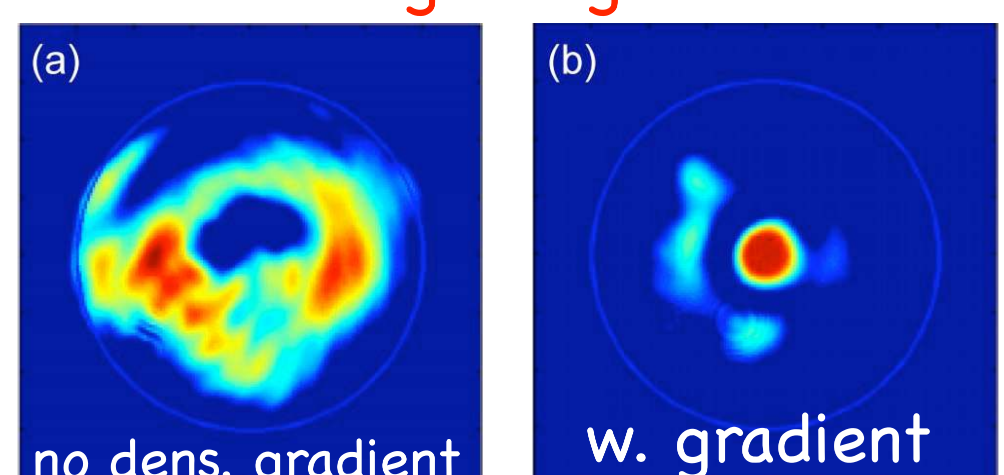








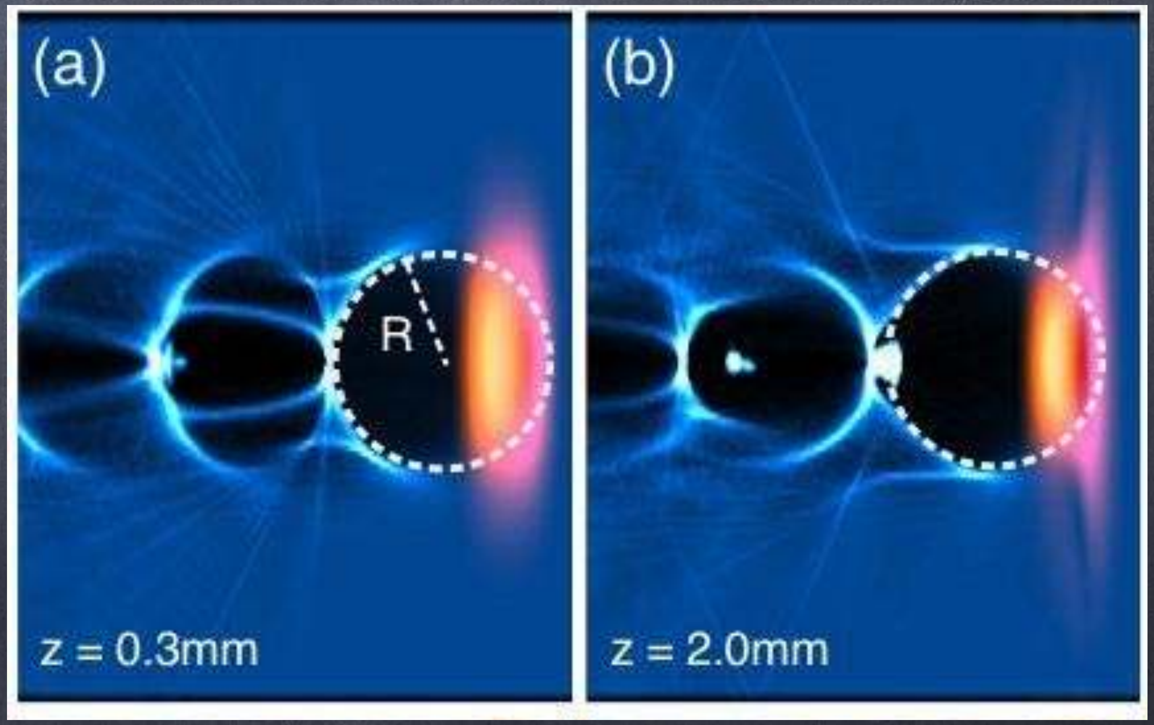
laser guiding



Matthias Fuchs

“Bubble”-Regime

- for $a_0 \gg 1$
- electrons also transversally expelled
- completely cavitated spherical ion “bubble” trailing laser pulse
- radius of bubble depends on laser and plasma properties



Lu et al., PRSTAB (2007)

The Bubble

- Assume spherical cavity: fields can be derived using Gauss' Law

$$E_z(\xi) \simeq \frac{\xi}{2} k_p E_0$$

E_0: nonrel wave breaking limit

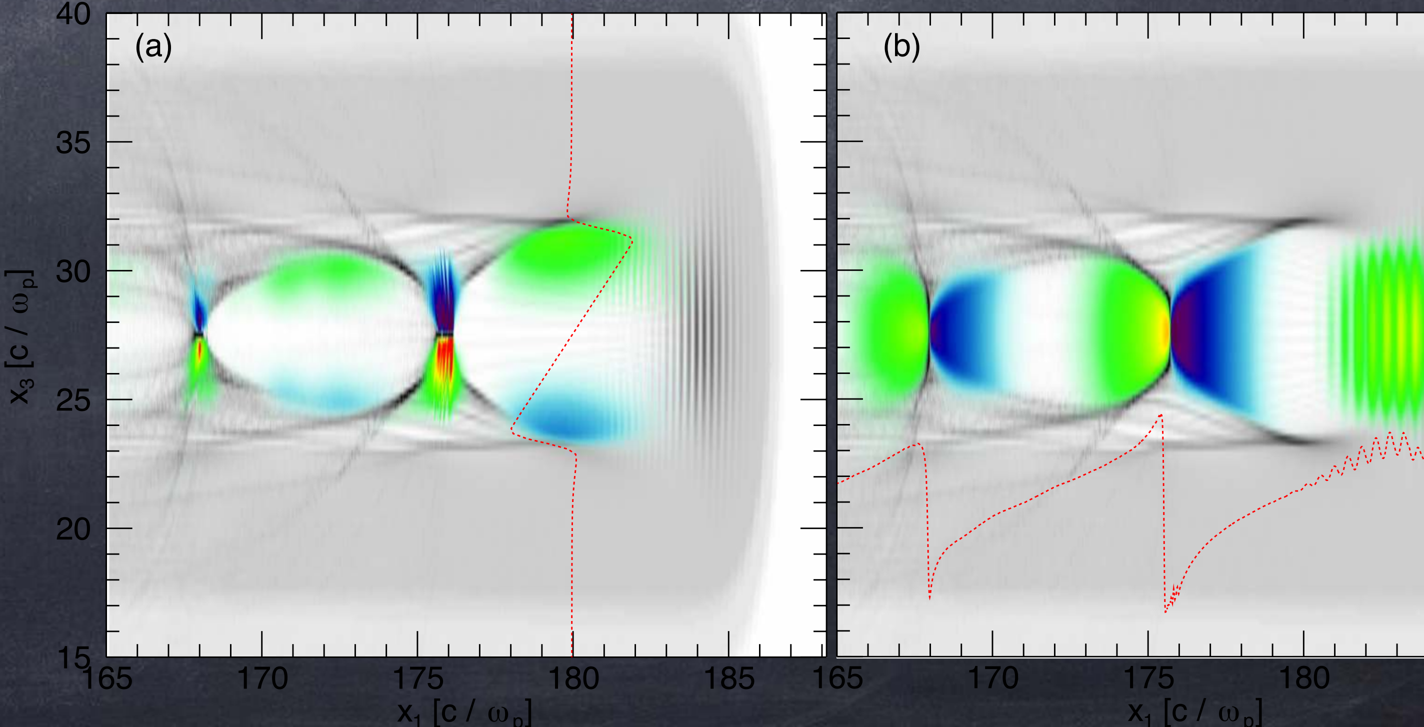
- Accelerating field: (linear with distance)
  - same field strength across different transv. positions
  - max field at ξ=R:

$$E_{\max} = \sqrt{a_0} E_0$$

- Transverse (restoring) field

$$E_r(r_\perp) - B_\Theta(r_\perp) = \frac{r_\perp}{2} k_p E_0$$

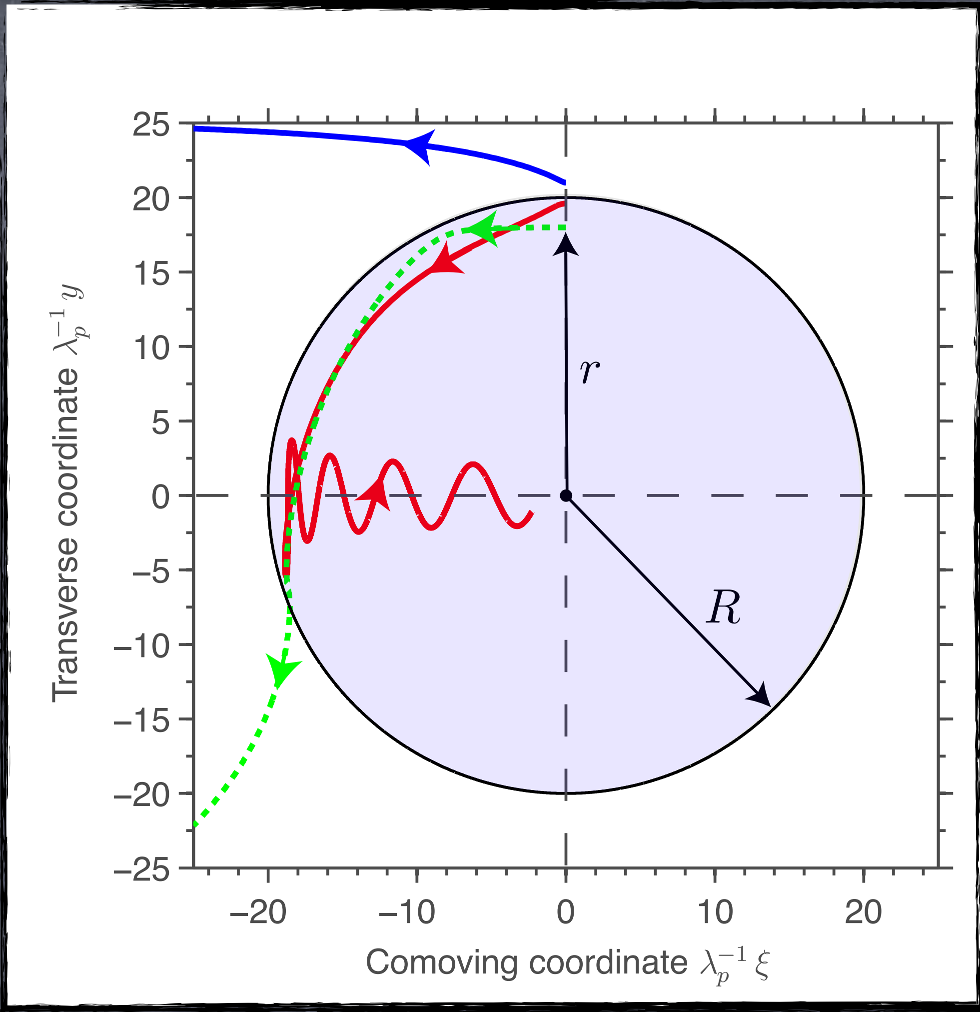
Vieira, in Proc. of the CERN Accelerator School (2016)



Injection into Bubble

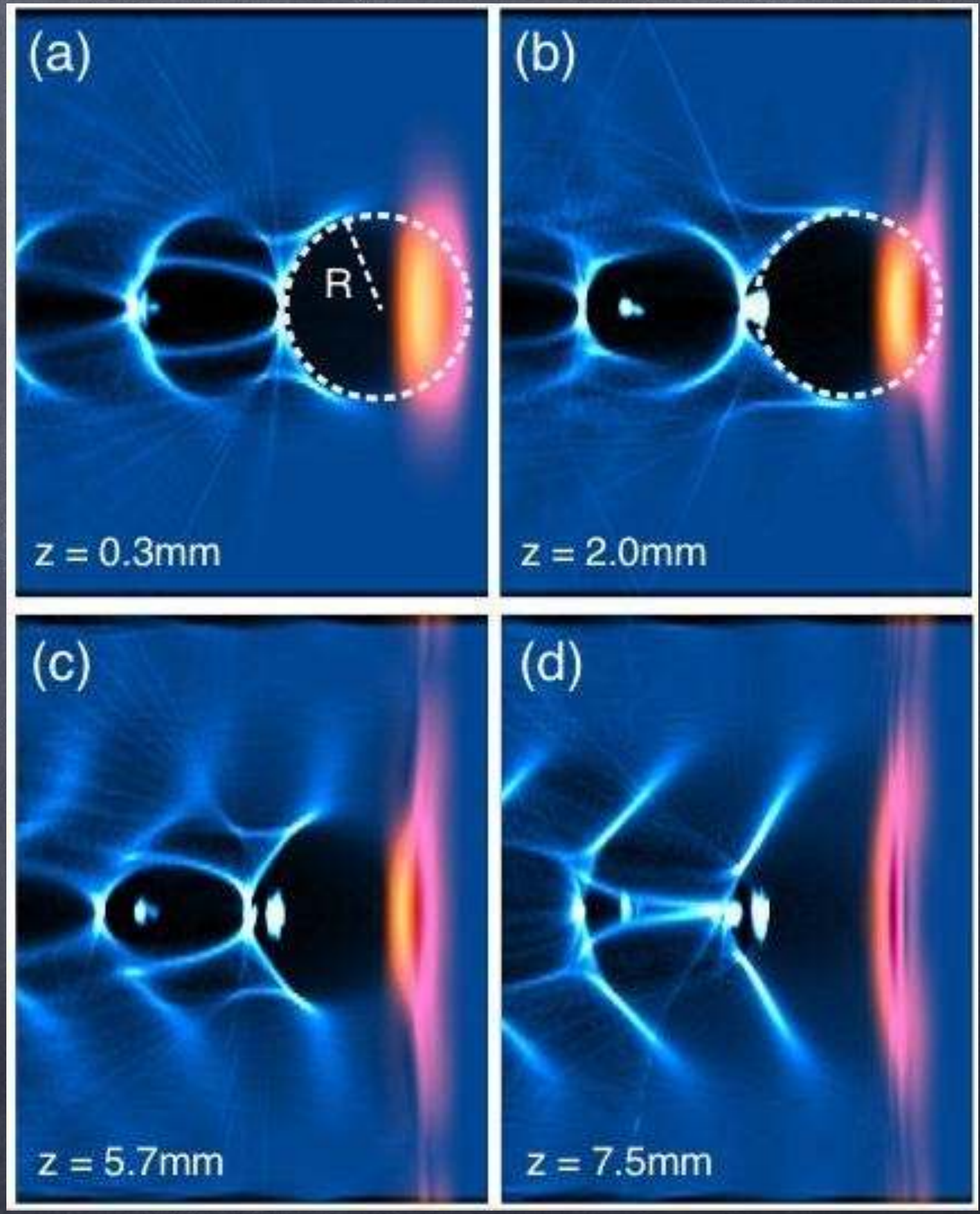
"bubble" or blowout regime:

- transversally expelled electrons -> pulled back by space charge field of ions
- electrons with suitable initial conditions (red) undergo sufficient longitudinal acceleration while bubble passes by
- injection at back of bubble
- electron have finite transverse momentum
- perform transv. (betatron) oscillations -> move on sinusoidal trajectory during acceleration
- amplitude of transv. motion decreases with increasing electron energy as: $(r_\beta \propto \gamma^{-1/4})$



Bubble dynamics

- dynamics and evolution highly nonlinear
- detailed description including laser evolution, space charge effects of injected electrons, ... and their feedback on bubble structure need large-scale 3D particle-in-cell (PIC) simulations



Lu et al., PRSTAB (2007)