



Particle Accelerator Physics

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V - Measurement and control of beam parameters

Beta functions and dispersion
 Gradient modulation / response matrix

All around the betatron tune
 FFT & LNP
 Results from multi-turn measurements
 Phasen space reconstruction

Beam energy
 Transverse polarisation
 Resonant spin depolarisation
 LEP and external effects

References to beam diagnostics



D. Brandt, Editor, CERN Accelerator School on Beam Diagnostics, CERN-2009-005, 2009.

http://cdsweb.cern.ch/record/1071486/files/cern-2009-005.pdf

M.G. Minty, F. Zimmermann, Measurement and Control of Charged Particle Beams, Springer, 2003.

A.W. Chao, M. Tigner, Handbook of Accelerator Physics and Engineering, World Scientific, 1999.

Examples for beam diagnostics



Device	Physical effect	Parameter	Destructive
Faraday cup	Charge collection	Beam intensity	yes
Wall current monitor	Mirror current	Beam intensity, long. profile	no
Current transformer	Magnetic field	Beam intensity	no
Wire scanner	Secondary particles	Transverse profile	interfering
Beam position monitor	Electric/magnetic field	Beam position	no
Fluorescent screen	Fluorescence	Transv. profile, position	yes
Residual gas monitor	Ionisation	Transverse profile	no
SEM	Secondary emission	Transv. profile, emittance	interfering

Faraday cup

- beam intensity -





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Wall current monitor

- beam current -

Measure the voltage created by the image current of the beam on resistors in the vacuum chamber



Integrated current transformer (ICT)



- beam charge, pulse duration -



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- Measure the charge of very short pulses via induced magnetic field
- Integrated signal, linear integrator
 - since eddy currents are negligible
 - but original shape of signal is lost



Wire scanner

- transverse beam profile -



Scattered particles are measured outside the vacuum chamber







Beam position monitor ("BPM")

- beam position, tunes, ... -
- Measurement of transverse beam position from the differential signal of the "buttons"

$$U_{\Delta x} = A \frac{(TR + BR) - (TL + BL)}{(TR + TL + BR + BL)}$$





TR

BR

BR

Lead shielding Protective cover

beam position monitor at KARA



Orbit measurement and correction







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Beam through one sector (1/8 ring), correct trajectory, open collimator and move on.

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Orbit response matrix

- Idea: A small dipole perturbation θ_k , caused by a corrector dipole, results in an orbit change of
 - $u_k = \frac{1}{2} \beta_k \theta_k \cot \pi Q$
- The resulting disturbed orbit therefore contains information about the beta function and the tune.
- Measurements of the beam positions along the ring for a large number of such "kicks" are summarized in the "Orbit Response Matrix".



Turn-by-turn measurements



Orbit BPM kick position Amplitude turn

Transverse signal: Tune, beta functions, ...

- Sum signal of all buttons gives spectrum of longitudinal components.
- In the spectrum there are orbital harmonics, sidebands of the betatron and synchrotron frequencies and synchrotron sidebands to the betatron sidebands.



Lomb normalized periodograms (LNP)



 $d_j = h_j - \frac{1}{N} \sum_{i=1}^N h_i$

 $\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} d_i^2$

 $\tan(2\omega\tau) = \frac{\sum_{j}\sin(2\omega t_{j})}{\sum_{i}\cos(2\omega t_{i})}$

Harmonic analysis without restriction to 2^N points or time interval of individual measurements
 Data is weighted "per point" (not "per time interval" as with Fourier transforms)

For each frequency, the significance with which it occurs is examined.

The LNP indicates how many points occur at a certain frequency.

The LNP is for N data points h_i at times t_i :

$$P_{N}(\omega) = \frac{1}{2\sigma^{2}} \left(\frac{\left[\sum_{j} d_{j} \cos\left(\omega(t_{j} - \tau)\right)\right]^{2}}{\sum_{j} \cos^{2}\left(\omega(t_{j} - \tau)\right)} + \frac{\left[\sum_{j} d_{j} \sin\left(\omega(t_{j} - \tau)\right)\right]^{2}}{\sum_{j} \sin^{2}\left(\omega(t_{j} - \tau)\right)} \right) \text{ with }$$

Only frequency information, no phase information

Faster convergence than FFTs (all points are used), higher resolution, independent of amplitude

Harmonic analysis

Gives tunes

Fast Fourier Transform (FFT) or Lomb Normalised Periodograms (LNP)



Multi-turn measurements

- Measurement of the transverse beam position with beam position monitors ("BPMs") for many consecutive transitions
- Only the center of gravity of the charge distribution is visible
- Many analyses possible, e.g.
 - tune as a function of e.g. amplitude from harmonic analyses,
 - phase space reconstruction,
 - energy loss measurement





Amplitude detuning

Beam position after single kick

Tune from LNPs for small sections of 50 envelopes
 It applies

$$\begin{aligned} x(t) &= x_0 + A_0 \ e^{-t/\tau} \ \sin\left(2\pi \ \Phi(t) + \varphi\right) \\ \Phi(t) &= \int_0^t dt' \left(Q_0 + \alpha_q \ A_0^2 \ e^{-2t'/\tau}\right) \\ &= tQ_0 + \frac{\tau}{2} \ \alpha_q \ A_0^2 \left(1 - e^{-2t/\tau}\right) \end{aligned}$$

Stability criterion
 Adjustment of sextupoles





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De- and recoherence

 Decoherence due to energy dependent single particle tunes
 Dependent on synchrotron frequency



Phase space and BPM position



Composite distribution: "ring + island" $f(x) = \alpha f_A(x) + \beta f_I(x)$



Measured beam position reflects asymmetry of the distribution.



normalised X



Filamentation



Injection into resonant island

Only mean values are actually observed!

Filamentation and emittance



Injection errors (mismatched position or angle) dilute are a source of filamentation.

- Non-linear effects (e.g. magnetic field multipoles) distort the harmonic oscillation and lead to amplitude dependent effects into particle motion.
- Over many turns, a phase-space oscillation is transformed into an emittance increase.





Measurement of beta functions: "k modulation"

The beta function can be determined in several ways:

- from a measurement of the tune change as a function of a gradient change of a single quadrupole ("local β ") 25
- from a measurement of the tune change as a function of a gradient change of a quadrupole family ("global β ")
- Relation for a gradient error:

$$\Delta Q = \frac{1}{4\pi} \oint \beta(s) \Delta k(s) ds$$
$$\beta_{x,y} \approx \pm \frac{4 \pi \Delta Q_{x,y}}{\Delta k}$$





s/m

Optical functions and emittance



The beam size at a specific location is given by $\sigma_{i,11}^2 = C_i^2 c_{11}^b + 2S_i C_i c_{12}^b + S_i^2 c_{22}^b$

• C_i and S_i are elements of the transport matrix between the location of the measurement and the location at which the parameters are to be determined.

• c_{ij}^{b} are the elements of the beam matrix at the location where the parameters are determined and $\sigma_{i,11}$ are the measured beam sizes.

Several measurements are carried out as a function of a quadrupole gradient. Fitting calculations then provide both the optical functions and the emittance.



Chromaticity

- Measurement of betatron tunes as a function of momentum offset
- Variation of momentum offset with

 $\frac{\Delta p}{p_0} = -\frac{1}{\alpha_c} \frac{f_{\rm RF} - f_{\rm RF}^c}{f_{\rm RF}^c}$

Slope gives chromaticity
 Curvature provides information about higher multipole orders





Bunch length, longitudinal profile



- Longitudinal profile can determined from image current or the electrical field of the bunch
- Length of SR light pulse: Interferometer or Streak Camera



Principle split-ring resonator (SRR)





Split-ring resonator (SRR)







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Split-ring resonator (SRR)



First streaking results (November 2024)

Electro-optical bunch profile monitor







e-field (t=0..end(0.005);x=0) [pb] ^ Component Abs Sample 92/535 Time 0.455 ns Maximum (Sample) 1.67994e+06 V/m Maximum (Global) 4.65824e+06 V/m V/m 1.65e+06

100 -

Electro-optical phase space tomography



electro-optic spectral decoding
 bunch profile measurement
 single-bunch @ 2.7 MHz





Complete phase space image reconstructed from time interval of 61 µm

External influences

The RF frequency keeps the orbit length constant

$$\frac{\Delta f_{\rm RF}}{f_{\rm RF}} \propto \frac{\Delta C}{C} \propto -\frac{\Delta E}{E}$$

- Daily drifts through injection and ramp
 Significant seasonal dependence
 - Hall floor expands with the temperature
- Long-term drifts:
 - Drying of the concrete leads to shrinkage of the hall floor



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Optics and energy offset



- Measurements of the quadrupole gradients show systematic shift relative to the model when the beam energy is incorrect.
- This applies to both lepton and hadron accelerators.
- ⇒ It is very important to know the beam energy precisely!



Energy measurement



There are several methods to measure the beam energy

- classical way: spectrometer
- exotic way: from particle/nuclear physics processes
- photon based measurements
- energy measurements from the central frequency
- from energy loss
- highest precision: resonant spin depolarisation
Spectrometer





Spectrometers measure the particle momentum by precisely determining the deflection angle in a dipole magnet

$$\theta \propto \frac{1}{E_0} \int B ds$$

Requirements:

- Beam position O(1 µm) at entrance and exit of the dipole magnet
- Magnetic field $\mathcal{O}(10^{-5})$ or better

Single Pass Systems

position measurement with position sensitive detector at the beam stop (possible attenuation)

Storage rings

circulating beam, measurement of beam position with BPMs after deflection



FLUTE spectrometer



Magnet parameters L = 157 mm **B** = 0.08 T ρ = 300 mm θ = 30°

Drift length to screen station: L = 0.609 m



LEP spectrometer



Position measurement: 1 µm
Magnetic field: $\Delta B/B = \mathcal{O}(10^{-5})$

Energy resolution:
$$\Delta E/E = \mathcal{O}(10^{-4})$$



LEP spectrometer II



- Pickup positions monitored with a stretched wire system (beware of thermal effects due to synchrotron radiation etc.)
- Take into account the local magnetic field of the earth and fields generated by close-by power lines







Determine energy from the Z boson

- Cross-check $E_{\rm cm}$ using events of the type $e^+e^- \rightarrow Z\gamma$, $Z \rightarrow f\bar{f}$ where the fermion f is a quark, electron, muon or τ -lepton
- $m_Z = 91 \,\text{GeV/c}^2$ is the well-known Z mass from LEP I
 - ⇒ invert the problem and conclude collision energy of the event

$$\Delta E/E = \mathcal{O}\left(3 \times 10^{-4}\right)$$



G. Abbiendi et al., Phys. Lett. B (604) 2004.

Central frequency and momentum

The particle velocity relative to the speed of light is

 $\beta = \frac{C f_{\text{rev}}}{c} = \frac{C f_{\text{RF}}}{hc} = \frac{pc}{E}$

- To simultaneously determine p and C, measure f_{rev} for two particle types with different Z/m under the same machine conditions (e⁺/p, p/Pb⁵³⁺)
- The momentum is given by

$$p \approx m_p c \sqrt{\frac{f_{\rm RF,p}}{2\Delta f_{\rm RF}} \left[\left(\frac{m_i}{Zm_p} \right)^2 - 1 \right]}$$

■ difficult for high energies since $\Delta f \propto (m_i/Zm_p)^2/p^2$ ■ maximize m_i/Zm_p (Pb⁵³⁺) ■ $\Delta p/p = \mathcal{O}(10^{-4})$



For details see: CERN note "AB-Note-2003-014 OP"



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next time

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Transverse polarisation

- Resonant spin depolarisation
- LEP and external effects





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© BERGOZ Instrumentation, Integrating Current Transformer, user's manual

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beam position monitor at KARA

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Beam energy

Transverse polarisation

- Resonant spin depolarisation
- LEP and external effects

Transverse spin polarization

- Spin polarization results from the emission of synchrotron radiation:
 - Asymmetry in the spin flip probability during photon emission leads to transverse polarization
 - maximum polarization from the size of the asymmetry term:

 $8/5\sqrt{3} \approx 92.4\%$

Polarization level increases exponentially with time

 $P(t) = P_0 \left(1 - e^{-t/\tau_p} \right)$ $\tau_p = \frac{8}{5\sqrt{3}} \frac{m_0^6 c^{10} \rho^3}{\hbar r_e E_0^5}$

- Typical polarization time for electrons: some minutes till hours
- LEP: 340', KARA: 10', FCC-ee: > 10 h







Spin precession

The movement of the spin vector \vec{s} of a relativistic electron in the presence of electric and magnetic fields \vec{E} and \vec{B} is described by the Thomas-BMT (Bargmann, Michel, Telegdi) equation:

$$\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} = \vec{\Omega}_{\mathrm{BMT}} \times \vec{s}$$

The spin precession frequency $\overrightarrow{\Omega}_{\rm BMT}$ is

$$\vec{\Omega}_{\rm BMT} = -\frac{e}{\gamma m_0} \left[(1+a\gamma) \vec{B}_{\perp} + (1+a) \vec{B}_{\parallel} - \left(a\gamma + \frac{\gamma}{1+\gamma}\right) \vec{\beta} \times \frac{\vec{E}}{c} \right]$$
$$a = (g_e - 2)/2 = 0.001159652193(10)$$

The mean value of the number of spin revolutions per turn of all particles is defined as the "spin tune"

$$\nu = \frac{f_{\rm spin}}{f_{\rm rev}} = \frac{(g_e - 2)/2}{m_0 c^2} E_0$$

 \overrightarrow{B} \overrightarrow{s}



Resonant spin depolarization



Spins precede in the vertical magnetic bending field with the frequency $f_{\rm spin}$:

$$\nu = \frac{f_{\rm spin}}{f_{\rm rev}} = \frac{(g_{\rm e} - 2)/2}{m_0 c^2} E_0$$

 \rightarrow Spin tune ν depends on beam energy!

- Apply a horizontal magnetic field B_x , which is modulated with frequency f_B Resonant depolarisation at $f_{spin} = f_B$ $\Rightarrow \nu \Rightarrow E_0$
 - Precision: $\Delta E/E \approx 10^{-5}$



Resonant spin/polarization vector rotation for $\nu = n + 1/2$, $n \in \mathbb{N}$

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Polarization and loss rate

- The Touschek effect describes the transformation of small transverse momenta into large longitudinal momenta by Coulomb scattering leading the loss of particles.
- The Touschek cross-section depends on polarization
 - \rightarrow Touschek loss rate is a measure of the polarization level
- A loss monitor in a region with high dispersion and small betafunctions serves as a polarimeter.





Proof of polarization

- For example, scintillators in lead shielding as protection against synchrotron radiation
- Or, lead-glass block with photo multiplier
 Photo multiplier pulses are converted into NIM signals and counted







scinfilla

Pb

Recent RDP results





Recent RDP results II





Obtain momentum compaction factor α_c from scans at different RF frequencies f_{RF}

 $\frac{\Delta L}{L} = \frac{\Delta f_{\rm RF}}{f_{\rm RF}} = \alpha_{\rm c1} \delta \delta + \alpha_{\rm c2} \ \delta^2 + \dots$

 α_{c2} is highly significant, but small

Automated scans





Recent RDP results II





- Depolarisation is also possible on the synchrotron side bands
- Due to the single-particle nature of the depolarization process, this happens at

 $f_{\rm dep}/f_{\rm rev} = [\nu] \pm Q_s^{\rm inc}$

Energy reproducibility









LEP energy calibration

Emission of synchrotron radiation

Asymmetry in the spin flip probability leads to spontaneous transverse polarization
 Maximum achieved transverse polarization: 92.4 %

Measurement with a polarimeter based on Compton scattering

- Circular polarized laser pulse collides with beam
- Measurement of the vertical photon profile in Si-W calorimeter
- $P_{\perp} \propto$ vertical displacement of the profile for the two polarization states



The Compton polarimeter of LEP






LEP and the moon

Tidal range affects oceans and earth's crust
Local change in radius Δ*R* due to a mass *M* in distance *d* and with the zenith angle θ:

$$\Delta R \propto \frac{M}{d^3} (3\cos^2\theta - 1)$$

Some facts:

Solar tides are 50% weaker than lunar tides

- Full moon tides at the equator $\Delta R \approx \pm 50 \,\mathrm{cm}$
- In Geneva: vertical motion $\approx \pm 12.5 \, \mathrm{cm}$
- Change of LEP circumference by $\,pprox\pm0.5\,\mathrm{mm}$



LEP and the moon II





Relation between energy and circumference:

$$\frac{\Delta E}{E} = -\frac{1}{\alpha_{\rm c}} \frac{(f_{\rm RF} - f_{\rm RF}^c)}{f_{\rm RF}}$$
$$= -\frac{1}{\alpha_{\rm c}} \frac{\Delta C}{C}$$

Human activity





Changes in the beam energy can be extrapolated from measurements of the magnetic field because

$$E_0 \propto \odot B \,\mathrm{d}s$$

In this case: B field measurements with NMR probes in some dipole magnets

NMR: nuclear magnetic resonance







 \Rightarrow Lower voltage, higher current

Historical anecdote from 1895



Nachtheile physikalischer Institute durch elektrische Bahnen

"Der bekannte, am 27. Mai in Charlottenburg angestellte Versuch hat eine beträchtliche Störung der feinen elektrischen Messinstrumente der physikalisch-technischen Reichsanstalt hervorgebracht. Die Abweichungen der Galvanometer betrugen bis 1,2 Bogenminuten, und die Unregelmässigkeiten bestanden in so fatalen, kurzen Stössen von so tückischem Charakter, dass sie jede Sicherheit der Beobachtung ausschliessen. Wenn diese von der Rückleitung des Stromes durch das Erdreich ausgehenden Störungen nicht zu beseitigen sind, so könne die Reichsanstalt nicht umhin, ihren Einspruch gegen den geplanten elektrischen Betrieb der Berlin-Charlottenburger Pferdebahn aufrecht zu erhalten. Die höchste zulässige Abweichung der Galvanometer sei 0,1 Bogenminute."

(Central-Zeitung für Optik und Mechanik, XVI. Jg., No. 13, Seite 151)





Historical anecdote from 1895 — translation

Disadvantages of physical institutes through electrical operation of railroads

"The well-known experiment carried out in Charlottenburg on May 27th produced a considerable disturbance in the fine electrical measuring instruments of the Physikalisch-Technische Reichsanstalt. The deviations of the galvanometers amounted to up to 1.2 arcminutes, and the irregularities consisted of such fatal, short shocks of such treacherous character that they precluded any certainty of observation. If these disturbances caused by the return of the current through the ground could not be eliminated, the Reichsanstalt could not avoid maintaining its objection to the planned electrical operation of the Berlin-Charlottenburg horse-drawn railroad. The maximum permissible deviation of the galvanometers is 0.1 arcminute."

(Central-Zeitung für Optik und Mechanik, XVI. Jg., No. 13, page 151)



