Exercise to the Lecture Astroparticle Physics
KIT, Wintersemester 2022/23

Prof. G. Drexlin, A. Huber, N. Kovac, J. Lauer

Karlsruher Institut für Technologie

Lectures	Thur. 11:30 + Wed 14:00 (every 14 days), Phys-HS Nr. 3
Exercises	Wed 14:00 (alternating with lecture), Phys-HS Nr. 3
ILIAS	`https://ilias.studium.kit.edu/goto.php?target=crs_1902412&client_id=produktiv`

Sheet 2 – Due 16.11.2022

1) Development of extensive air showers

The electromagnetic component of an extensive air shower is dominated by the processes of bremsstrahlung and pair production.In a simplified and semi-empirical model[1] it is assumed, that an electron with energy E_0 radiates a photon with energy $E_\gamma = E_0/2$ after on *splitting length* $d = \ln(2)\, X_0$. After the length d the photon converts into an electron and a positron with energies $E_{e^-} = E_{e^+} = E_0/4$ and the primary electron radiates another photon with $E_\gamma = E_0/4$.
The shower develops in the same manner until the energy of the primary particle reaches a critical energy of $E_{c,em}$. From now on ionization dominates and the electrons are absorbed in the atmosphere.

(a) What is the maximum number of particles N_{max} that is reached in the shower maximum? After what distance (in units of the radiation length X_0) will the show maximum X_{max} be reached?

(b) For electromagnetic air showers $E_{c,em}$ is approximately 100 MeV. What shower length is possible for electrons with a primary energy of 10^{12} and 10^{20} eV assuming that the maximum is reached after 2/3 of the shower length.

(c) The atmospheric depth x (in g/cm^2) scales exponentially with the hight h above the sea level (in km): $x = X\, e^{-h/H}$, where is $H = 6,5\,\mathrm{km}$ and the total atmospheric depth $X = 1030\,\mathrm{g/cm^2}$. In which hight do the electrons from b) develop their shower maximum for a given radiation length of $X_0 = 37\,\mathrm{g/cm^2}$.

The Heitler-Model can be extended for hadronic showers to the Heitler-Matthews-Model (see for example here `http://particle.astro.ru.nl/ps/astropart1415-wk7a.pdf` J. Matthews, Astroparticle Physics 22 (2005), 387-397). In hadronic showers it is assumed that the primary particle (e.g. a proton) generates N_{ch} electrically charged and $\frac{1}{2}N_{ch}$ uncharged pions in their first interaction. The uncharged pions immediately decay into photons, triggering electromagnetic showers, while the charged pions, after a radiation length X_0, again produce charged and uncharged pions in a renewed interaction, and so on.

(d) Calculate the number of interactions $n_{c,had}$ until the pions reach the critical energy $E_{c,had}$ and the showering process of the hadronic component comes to a halt as the decay length becomes shorter than the interaction length. For this, assume that $E_0 = 10^{15}\,\mathrm{eV}$, $N_{ch} = 10$, and $E_{c,had} = 20\,\mathrm{GeV}$ (in Earth's atmosphere). What happens to the charged pions below the critical energy $E_{c,had}$?

[1] The basic concept of this model was developed in 1936 by Walter Heinrich Heitler (*1904, †1981) in Karlsruhe.

(e) Calculate the fraction of the primary energy that passes into the electromagnetic part of the shower. Use again $E_0 = 10^{15}$ eV, $N_{ch} = 10$ and $E_{c,had} = 20$ GeV.

(f) Express the muon number of the shower as a function of E_0 and $E_{c,had}$. Show how the muon and electron numbers of an air shower can be used to reconstruct the primary energy of the particle. For this, assume that at the maximum of the electromagnetic part, one tenth of the particles are electrons.

(g) The air shower of a heavy nucleus of mass number A can be modeled by the simultaneous evolution of A air showers of protons with the corresponding fraction of the total energy. How does the muon number in the hadronic shower maximum change as a function of mass number A? Briefly discuss how electrons and muons are distinguished, for example, in the Pierre Auger Observatory.

2) The Pierre-Auger-Observatory

The Pierre Auger Observatory[2] in Argentina was built to study high-energy cosmic rays. On an area of 3000 km^2 1660 water Cherenkov detectors with a volume of 12000 ℓ each were set up for this purpose. The detectors have a distance of about 1,5 km from each other.

(a) Explain the physical principle of the Cherenkov effect used for particle detection in the Pierre Auger surface detector. Derive the formula for the opening angle θ of the resulting Cherenkov cone from Huygens' principle.

(b) What is the minimum energy required for a muon to produce Cherenkov radiation in the water detectors (refractive index $n_{H_2O} = 1{,}333$)? What would be the minimum energy required in air ($n_{air} = 1{,}0003$)? What is the opening angle of the Cherenkov cone of a typical muon with $E = 1$ GeV in a Pierre Auger water Cherenkov detector?

(c) The flux $\Phi(E)$ of cosmic rays describes how many particles dN per energy interval dE and time unit dt impinge on a surface dA from the solid angle interval dΩ:

$$\Phi(E) = \frac{d^4N}{dA\,d\Omega\,dE\,dt} \propto E^{-\alpha} \; . \qquad (1)$$

Thus, the differential flux of cosmic rays follows a power law. The spectral index α has the value 2,7 for $E < 1 \cdot 10^{15}$ eV, 3,1 for $1 \cdot 10^{15}\,\text{eV} < E < 3 \cdot 10^{18}$ eV, 2,7 for $3 \cdot 10^{18}\,\text{eV} < E < 40 \cdot 10^{18}$ eV and finally 4,2 for $E > 40 \cdot 10^{18}$ eV. The integrated flux for $E > 1 \cdot 10^{15}$ eV is about one particle per square meter per year. Calculate the number of events for $E > 100$ EeV that can be observed in one year of operation from the Pierre Auger Observatory.
Note: Assume that the observatory has an efficiency of 100 % and can detect particles from the entire hemisphere.

(d) The Pierre Auger Observatory makes 10 % of the detected events available online. This dataset now includes nearly 50000 events. Go to `http://labdpr.cab.cnea.gov.ar/ED-en/index.php` and download the ascii file with all events. Plot the direction of origin of the particles in galactic coordinates for energies $E > 4$ EeV. Use a data processing program of your choice (Excel, OpenOffice, Python, Mathematica, ...). Can you detect sources of high-energy cosmic rays, e.g., the galactic center?
Note: Bring your laptop to the tutorial, so that you can explain your procedure to your fellow students with the help of the beamer.

[2]see `www.auger.org`. There is strong participation of KIT scientist in the collaboration.