Exercise to the Lecture Astroparticle Physics
KIT, Wintersemester 2022/23

Prof. G. Drexlin, A. Huber, N. Kovac, J. Lauer

Karlsruher Institut für Technologie

Lectures	Thur. 11:30 + Wed 14:00 (every 14 days), Phys-HS Nr. 3
Exercises	Wed 14:00 (alternating with lecture), Phys-HS Nr. 3
ILIAS	`https://ilias.studium.kit.edu/goto.php?target=crs_1902412&client_id=produktiv`

Sheet 4 – Due 14.12.2022

1) Imaging Atmospheric Cherenkov Technique

To illustrate the detection of high-energy gamma rays with *Imaging Atmospheric Cherenkov Telescopes* we consider electromagnetic air showers, which are supposed to propagate through the atmosphere vertically to the ground. For the emission of Cherenkov light by the charged particles in the shower ($e^{\pm}$) the refractive index n at the respective height above the ground is relevant, which can be parameterized as

$$n(h) = 1 + 0{,}000283 \cdot e^{-h/H_0}, \qquad \text{with } H_0 = 8\,\text{km}.$$

At an altitude of 4.5 km, Cherenkov light is now generated by $e^{\pm}$, whose energy distribution is at its maximum at about 50 MeV , i.e. $\beta \approx 1$.
What is the opening angle of the Cherenkov light cone when emitted at this altitude? Use this to calculate the radius of the light cone when it hits the ground.
Furthermore, derive an expression for the radius of the Cherenkov cone as a function of the height h and sketch the function. What is the radius at an emission height of 39 km?

2) Hadron acceleration in supernova remnants

In 2005, the H.E.S.S. telescopes succeeded in detecting TeV gamma radiation from the shell frmigenous supernova remnant (SNR) *Vela Junior*[1]. In a measurement time of 11400 s, 670 ± 60 photons (above the background) in the energy range from 0.5 to 10 TeV were observed. This became at 1 TeV a differential flux of

$$\begin{aligned}\Phi_0(E = 1\text{ TeV}) &= (2{,}1 \pm 0{,}2) \cdot 10^{-11}\text{TeV}^{-1}\text{cm}^{-2}\text{s}^{-1}\\ \frac{\mathrm{d}\Phi}{\mathrm{d}E} &= \Phi_0 \left(\frac{E}{E_0}\right)^{-\Gamma} \text{ with } E_0 = 1\,\text{TeV}\end{aligned}$$

determined with a spectral index of $\Gamma = 2.1 \pm 0.1$.

(a) Determine the detector area $\overline{A}$ averaged over the energies from the number of events N and the differential flow. Then determine the energy flow w_γ for the range 0.5 to 10 TeV.
Note: The number of detected events is given by $N = T \int_{E_1}^{E_2} \frac{\mathrm{d}\Phi}{\mathrm{d}E} A(E)$, $\mathrm{d}E$, with the observation time T and the energy-dependent detector area $A(E)$. The definition of the energy flow is $w_\gamma = \int \frac{\mathrm{d}\Phi}{\mathrm{d}E} E\,\mathrm{d}E$.

[1] `http://www.mpi-hd.mpg.de/hfm/HESS/pages/home/som/2005/03/`

In the following we try to calculate their share of the total energy of the supernova from the energy flow of the high-energy cosmic particles. We assume that the entire γ flow comes about through the „π^0 cooling“ of accelerated protons trapped in magnetic fields in the shock front of the SNR. In order to generate gamma rays via the π^0 decay, one needs protons with an energy 10 times higher than that of gamma rays, i.e. in our case the energy must be between 5 and 100 TeV.

(b) Calculate the mean cooling time ($\tau_p = E/\langle \mathrm{d}E/\mathrm{d}t \rangle$) at a proton density in the SNR of $n_p = 1\,\mathrm{cm}^{-3}$. The mean energy loss is given by: $\langle \mathrm{d}E/\mathrm{d}t \rangle = R \cdot f \cdot E$, with the interaction rate $R = n_p \cdot \sigma_{pp}$ *cdotc* and the inelasticity f (assume $f = 0.2$). σ_{pp} can be taken as 50 mbarn.
Which total energy must the protons have in the energy range between 5 and 100 TeV in the SNR in order to be able to explain the observed spectrum? Note the distance of the SNR with $d_{SNR} = 200\,\mathrm{pc}$.
Note: To calculate the total energy, the result of the energy flow w_γ from part (a), the average cooling time and the distance must be offset against each other.

(c) Now calculate the total energy of the protons in the energy range between 1 GeV and 100 TeV. Assume that the protons have the same spectral index as the photons. What fraction of the total SN energy ($W_{SNR} = 10^{44}$ J) is that?

3) Inverse Compton Effect

The inverse Compton effect is another process that can produce TeV gamma rays. In the inverse Compton effect, energy is transferred from a high-energy electron to a low-energy photon. This effect is examined in more detail below.

(a) An electron with energy E_e and momentum $\vec{p}_\mathrm{e}$ collides with a photon with energy E_γ and momentum $\vec{p}_\gamma$ (with $|\vec{p}_\gamma| = E_\gamma$). Transform the energy of the photon into the rest frame of the electron. Then find an expression for the energy of the scattered photon in the rest frame of the electron. *Hint:* Assume that the photon runs towards the electron in the laboratory system. A sketch in the rest frame of the electron is helpful.

(b) Now transform the result of the previous subtask back into the laboratory system. Assume that $\theta = 180^\circ$ and that $E_\gamma^* \ll m_\mathrm{e}c^2$ (E_γ^* here denotes the energy of the gamma in the rest system of the electron).

(c) The end result of the previous subtask is $E_\gamma' = 4\gamma^2 \cdot E_\gamma$, where E_γ' denotes the energy of the photon after scattering and γ is the Lorentz factor of the electron. Calculate the energy of a photon of the cosmic microwave background ($E_\gamma = 7 \cdot 10^{-4}\,\mathrm{eV}$) after a collision with an electron of energy 1 TeV . What energy must an electron have to accelerate a photon of cosmic microwave radiation to 1 TeV?

4) The Cherenkov Telescope Array (CTA)

The Cherenkov Telescope Array will be built from 2020. It will also use the imaging Cherenkov technique to detect high-energy gamma rays. Familiarize yourself with the project using the collaboration's website `https://www.cta-observatory.org/`. In particular, briefly discuss the following points: location of the observatory, number of telescopes, effective areas of the telescopes, detectable energy range, energy and angular resolution of the observatory (compared to MAGIC), current status of the project, scientific goals.