Exercise to the lecture Astroparticle Physics
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Lectures	Thur 14:00 + Wed 14:00 (every 14 days), Phys-HS Nr. 3
Exercises	Wed 14:00 (alternating with lecture), Phys-HS Nr. 3
ILIAS	`https://ilias.studium.kit.edu/ilias.php?ref_id=2238561&cmdClass=ilrepositorygui&cmdNode=x1&baseClass=ilrepositorygui`

Sheet 2 – Due 29.11.2023

1) **The IceCube Neutrino Obervatory**

The IceCube Neutrino Observatory was built at the South Pole to detect high-energy neutrinos with $E_\nu > 100\,\mathrm{GeV}$ For this purpose, a cubic kilometer of ice is instrumented with photomultipliers and neutrinos are detected via the Cherenkov radiation of charged leptons with the same flavor.

(a) For muon neutrinos, this charged-current (CC) interaction looks like this:

$$\nu_\mu + \mathrm{N} \rightarrow \mu^- + \mathrm{M}\,. \tag{1}$$

Here N denotes a resting nucleon before and M one after the interaction, where $m_\mathrm{N} \approx m_\mathrm{M} \approx 939\,\mathrm{MeV}$ should hold. Calculate the minimum energy $E_{\nu,\mathrm{min}}$ required by the muon neutrino for the CC interaction to occur.

(b) Calculate the mean free path of muon neutrinos with $E = 100\,\mathrm{TeV}$ in ice ($\rho_\mathrm{Eis} = 934\,\mathrm{kg\,m^{-3}}$) if the effective cross section σ for the CC interaction at this energy is $6{,}7 \cdot 10^{-38}\,\mathrm{m^2}$.

(c) The photomultiplier tubes (PMTs) are located at a depth of 1500 to 2500 m below the ice surface. The neutrino flux Φ of muon neutrinos is for $E_\nu \geq 100\,\mathrm{TeV}$ integrated $1 \cdot 10^{-12}\,\mathrm{cm^{-2}\,s^{-1}}$. Calculate the event rate of muon neutrinos in IceCube per year for the CC interaction in the instrumented volume crossing the detector from top to bottom. *Note:* The event rate is calculated using $R = \Phi \cdot W \cdot A$, where A is the cross-sectional area of the detector and W is the interaction probability. To calculate W, assume a mean free path length of $\lambda = 2{,}5 \cdot 10^7\,\mathrm{m}$ or your result from the previous subtask. You can assume the shape of the instrumented volume of IceCube to be cube-shaped.

(d) How long does a muon travel through ice before it has lost so much energy that the energy falls below the Cherenkov threshold? The muon's energy loss is described by

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = A + B \cdot E,$$

where $A = 2\,\mathrm{MeV\,cm^{-1}}$ and $B = 4{,}2 \cdot 10^{-6}\,\mathrm{cm^{-1}}$ are quantities already corrected for the density of the ice. Assume a muon with energy $E_\mu = 5\,\mathrm{TeV}$. Comment on the result from task part c) on the basis of this result.

(e) Muons above a minimum energy of 160 MeV produce Cherenkov radiation in ice. Why can IceCube only detect muons above an energy of 100 GeV? Briefly describe the planned expansion stages of IceCube, PINGU, and IceCube-Gen2, and address the scientific goals and energy ranges covered in each case.

2) The Pierre-Auger-Observatory

The Pierre Auger Observatory[1] in Argentina was built to study high-energy cosmic rays. On an area of 3000 km^2 1660 water Cherenkov detectors with a volume of 12000 ℓ each were set up for this purpose. The detectors have a distance of about 1,5 km from each other.

(a) Explain the physical principle of the Cherenkov effect used for particle detection in the Pierre Auger surface detector. Derive the formula for the opening angle θ of the resulting Cherenkov cone from Huygens' principle.

(b) What is the minimum energy required for a muon to produce Cherenkov radiation in the water detectors (refractive index $n_{H_2O} = 1{,}333$)? What would be the minimum energy required in air ($n_{air} = 1{,}0003$)? What is the opening angle of the Cherenkov cone of a typical muon with $E = 1$ GeV in a Pierre Auger water Cherenkov detector?

(c) The flux $\Phi(E)$ of cosmic rays describes how many particles dN per energy interval dE and time unit dt impinge on a surface dA from the solid angle interval dΩ:

$$\Phi(E) = \frac{\mathrm{d}^4 N}{\mathrm{d}A\,\mathrm{d}\Omega\,\mathrm{d}E\,\mathrm{d}t} \propto E^{-\alpha} \,. \tag{2}$$

Thus, the differential flux of cosmic rays follows a power law. The spectral index α has the value 2,7 for $E < 1 \cdot 10^{15}$ eV, 3,1 for $1 \cdot 10^{15}\,\text{eV} < E < 3 \cdot 10^{18}$ eV, 2,7 for $3 \cdot 10^{18}\,\text{eV} < E < 40 \cdot 10^{18}$ eV and finally 4,2 for $E > 40 \cdot 10^{18}$ eV. The integrated flux for $E > 1 \cdot 10^{15}$ eV is about one particle per square meter per year. Calculate the number of events for $E > 100$ EeV that can be observed in one year of operation from the Pierre Auger Observatory.
Note: Assume that the observatory has an efficiency of 100 % and can detect particles from the entire hemisphere.

(d) The Pierre Auger Observatory makes 10 % of the detected events available online. This dataset now includes nearly 50000 events. Go to `http://labdpr.cab.cnea.gov.ar/ED-en/index.php` and download the ascii file with all events. Plot the direction of origin of the particles in galactic coordinates for energies $E > 4$ EeV. Use a data processing program of your choice (Excel, OpenOffice, Python, Mathematica, ...). Can you detect sources of high-energy cosmic rays, e.g., the galactic center?
Note: You can find the data file also on Ilias. Bring your laptop for pre-calculating in the tutorial, so that you can explain your procedure to your fellow students with the help of the beamer.

3) Imaging Atmospheric Cherenkov Technique

To illustrate the detection of high-energy gamma radiation with *Imaging Atmospheric Cherenkov Telescopes*, we consider electromagnetic air showers propagating vertically through the atmosphere. For the emission of Cherenkov light by the charged particles in the shower ($e^{\pm}$), the refractive index n at the respective height h above the ground is relevant and can be parameterized as

$$n(h) = 1 + 0.000283 \cdot e^{-h/H_0}, \qquad \text{with } H_0 = 8\,\text{km}.$$

At 4.5 km of height, Cherenkov light is generated by $e^{\pm}$ with an energy distribution peaking at approximately 50 MeV, i.e., $\beta \approx 1$.

[1] see `www.auger.org`. There is strong participation of KIT scientist in the collaboration.

What is the opening angle of the Cherenkov light cone at emission at this height? Calculate the radius of the light cone upon reaching the Earth's surface.
Furthermore, derive an expression for the radius of the Cherenkov cone as a function of height h and sketch the function. What is the radius at an emission height of 39 km?