Exercise to the lecture Astroparticle Physics
KIT, Wintersemester 2023/24

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Lectures	Thur 14:00 + Wed 14:00 (every 14 days), Phys-HS Nr. 3
Exercises	Wed 14:00 (alternating with lecture), Phys-HS Nr. 3
ILIAS	`https://ilias.studium.kit.edu/ilias.php?ref_id=2238561&cmdClass=ilrepositorygui&cmdNode=x1&baseClass=ilrepositorygui`

Sheet 3 – Due 13.12.2023

1) **Experiments on Neutrinoless Double Beta Decay** (Points: 5)

(a) A promising candidate for the search for neutrinoless double β-decay is ^{76}Ge. The experimental observable is the half-life $T_{1/2}$. The sensitivity targeted by the future experiment LEGEND is $T_{1/2} = 1 \times 10^{28}$ years. What mass of ^{76}Ge is required to measure an event rate of 3 events per year for the given half-life? Assume the experiment is free of background. How much naturally occurring germanium would your laboratory need to purchase to achieve this goal? What would be the cost?

(b) The CUORE experiment also searches for neutrinoless double beta decay. To shield the tellurium bolometers, 8910 kg of Roman lead is used, containing, on average, 0.832 μg of Pb-210 per kilogram (molecular weight $M_{\text{Pb-210}} = 210\,\text{g/mol}$). Pb-210 has a half-life of 22.3 years. Calculate how long the Roman lead must have been in the sea for CUORE, assuming no further activation of Pb-210, so that the entire lead shielding of CUORE contributes less than 1×10^{-7} Bq to the experiment's background.

(c) Experiments on neutrinoless double beta decay always take place in underground laboratories. Radon-222 is an important underground source with an air activity of approximately 150 Bq/m^3 when laboratory ventilation is established. Assuming that the concentration of Radon-222 in the surrounding rock is 200 times greater, estimate how many grams of Uranium-238 are contained in one cubic meter of the rock.

2) **Dark Matter: Evidence** (Points: 5)

(a) Galactic Rotation Curves: At $r = 10^5$ light-years, a calculated or measured rotation speed of $v_b = 15\,\text{km/s}$ or $v_g = 220\,\text{km/s}$ is obtained. Determine the required galaxy mass in both cases. What is the proportion of dark matter in the galaxy?
Hint: Use the formula for Newton's law of gravitation and centrifugal force.

(b) Derive from the Virial Theorem

$$T = -\frac{1}{2}U$$

the mass M of a galaxy cluster as a function of the average velocity $\langle v \rangle$ in the line of sight. T is the kinetic energy of the galaxies, U is the potential energy.
Hint: Assume a homogeneous spherical distribution of galaxies in the galaxy cluster with radius R. Also, assume that the velocities of the galaxies are equal in all three spatial directions.

(c) Fritz Zwicky observed the Coma Cluster in the 1930s and estimated that it consisted of approximately 1000 galaxies. For the luminosity of a galaxy, he assumed about 8.5×10^9 times the luminosity of the Sun, L. He determined the radius of the Coma Cluster to be $R = 3\,\mathrm{Mpc}$, and the averaged square velocities in the direction of the line of sight were determined to be $\langle v \rangle^2 = 10 \times 10^{11}\,\mathrm{m^2\,s^{-2}}$. Determine the mass-to-luminosity ratio of the Coma Cluster in units of M/L (M is the solar mass). How do you explain the discrepancy to $M/L \approx 3$ from the study of local solar systems?
Hint: The solution to the previous subtask is $M = \frac{5R}{G}\langle v \rangle^2$.

3) **Underground in the Search for Rare Events** (Points: 5)

(a) Alpha particles with an energy of 4 MeV have a range of 2.5 cm in air. Calculate the range of alpha particles in lead and water. Assume that the range is inversely proportional to the density of the absorbing material ($\rho_{\mathrm{air}} = 1.29\,\mathrm{mg/cm^3}$, $\rho_{\mathrm{water}} = 1\,\mathrm{g/cm^3}$, $\rho_{\mathrm{lead}} = 11.2\,\mathrm{g/cm^3}$). Subsequently, calculate the mean range of electrons with an initial energy $E_0 = 4\,\mathrm{MeV}$ and a constant energy loss of $\frac{dE}{dx} = 8\,\mathrm{MeV/cm}$. Note: For electrons, the mean range is given by $R = -\frac{1}{\frac{dE}{dx}} \int_{E_0}^{0} dE$.

(b) In an iron block, the intensity of a neutron beam decreases by a factor of 2 over a distance of 3 cm. How thick must the iron block be for the intensity to decrease by a factor of 8 or even 128?
Hint: The neutron beam behaves like a plane wave. The intensity I of gamma radiation with an energy of 15 MeV is reduced by a factor of 2 through a 1 cm thick lead plate.

- By what factor is the intensity reduced through a 5 cm thick plate?
- How thick must the plate be for the intensity to be attenuated by a factor of 1000?