Exercise to the Lecture Astroparticle Physics
KIT, Wintersemester 2023/24

Prof. G. Drexlin, J. Storek, S. Mohanty, N. Gutknecht

Lectures	Thur. 14:00 + Wed 14:00 (every 14 days), Phys-HS Nr. 3
Exercises	Wed 14:00 (alternating with lecture), Phys-HS Nr. 3
ILIAS	`https://ilias.studium.kit.edu/goto.php?target=crs_2238561&client_id=produktiv`

Sheet 5 – Due 24.1.2024

1) Development of extensive air showers

(2 + 1 + 1 = 4 points)

Electrons create extensive electromagnetic showers when going through the Earth's atmosphere. The dominating processes are bremsstrahlung and pair production. In a simplified and semi-empirical model[1] it is assumed, that an electron with energy E_0 radiates a photon with energy $E_\gamma = E_0/2$ after a *splitting length* $d = \ln(2)\,X_0$. After the length d the photon converts into an electron and a positron with energies $E_{e^-} = E_{e^+} = E_0/4$ and the primary electron radiates another photon with $E_\gamma = E_0/4$. The shower develops in the same manner until the energy of the primary particle reaches a critical energy $E_{c,em}$. From now on the ionization dominates and the electrons are absorbed in the atmosphere.

(a) What is the maximum number of particles N_{max} that is reached in the shower maximum? After what distance (in units of the radiation length X_0) will the shower maximum X_{max} be reached?

(b) For electromagnetic air showers $E_{c,em}$ is approximately 100 MeV. What shower length is possible for electrons with a primary energy of 10^{12} and 10^{20} eV assuming that the maximum is reached after 2/3 of the shower length?

(c) The atmospheric depth x (in g/cm^2) scales exponentially with the height h above the sea level (in km): $x = X\,e^{-h/H}$, where $H = 6,5\,\text{km}$ and $X = 1030\,\text{g/cm}^2$ is the total atmospheric depth. In which height do the electrons from b) develop their shower maximum for a given radiation length of $X_0 = 37\,\text{g/cm}^2$?

[1] The basic concept of this model was developed in 1936 by Walter Heinrich Heitler (*1904, †1981) in Karlsruhe.

2) Inverse Compton scattering

(3 + 2 + 1 = 6 points)

An indirect method of the dark matter searches is the dark matter annihilation process. In this process a high energetic γ of energies between GeV an TeV are produced. One of the background processes in these studies is the inverse Compton scattering. In the inverse Compton effect a high energetic electron transfers energy to the photon.

The calculation is done in two steps: first the energy of the outgoing photon is derived in the rest frame of the electron and then it is transformed into the laboratory frame of the observer.

(a) Derive energy of the outgoing photon $E_\gamma^{*\prime}$ in the rest frame of the electron and compare the result with the usual Compton scattering formula.

(b) Use the Lorentz transformation to express the energy of the outgoing photon in the laboratory frame. Assume that the scattering angle of γ is close to 180° and that $E_\gamma^* \ll m_e$, i.e. $E_\gamma^{*\prime} \approx E_\gamma^*$, and derive the outgoing photon energy in laboratory frame using the incoming photon energy in the laboratory frame. (Suggested notation: asterisk for the electron's rest frame and no special sign for the laboratory frame.)

(c) The result of the inverse Compton scattering can for relativistic velocities of the electron be approximated by formula $E_\gamma' = 4\gamma^2 E_\gamma$, where E_γ and E_γ' are the energies of the photon before and after the scattering, respectively, and γ is the Lorentz factor. Assume energy of the cosmic microwave background photons to be $E_\gamma = 7 \cdot 10^{-4}$ eV and calculate their energy after the inverse Compton scattering with a 1 TeV electron.

3) Direct DM searches

(5 points)

Write down a basic principle of the direct dark matter searches and think about a possible experimental realization. Focus on the spin independent interaction and design a suitable detector. Explain the detection technique, different background contributions and also background suppression methods.