Exercise to the Lecture Astroparticle Physics
KIT, Wintersemester 2023/24

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Lectures	Thur. 14:00 + Wed 14:00 (every 14 days), Phys-HS Nr. 3
Exercises	Wed 14:00 (alternating with lecture), Phys-HS Nr. 3
ILIAS	`https://ilias.studium.kit.edu/goto.php?target=fold_2238589&client_id=produktiv`

Sheet 6 – Due 07.02.2024

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1) **Direct Search for Dark Matter** ($3 + 2 + 3 + 2 = 10$ points)

Experiments for the direct search of dark matter attempt to detect collisions of WIMPs with target nuclei in shielded detectors. In the following, some typical quantities for the direct search of dark matter are calculated.

(a) Show that an initially stationary nucleus of mass m_{nuc} with a WIMP of mass m_χ and energy E_χ receives a kinetic energy E_{nuc} due to the collision, for which the following expression applies in the laboratory system:

$$E_{\text{nuc}} = 4\frac{m_\chi m_{\text{nuc}}}{(m_\chi + m_{\text{nuc}})^2} \cdot E_\chi \cdot \cos^2\theta\,,$$

where θ is the angle between the direction of motion of the nucleus after the interaction and the direction of incidence of the WIMP.
Under what conditions for θ does this recoil energy become maximal?

(b) Sketch the maximum recoil energy $E_{\text{nuc}}^{\text{max}}$ for a WIMP of mass $m_\chi = 100m_{\text{p}}$ as a function of the mass of the target nucleus in the range from $10m_{\text{p}}$ to $1000m_{\text{p}}$ (proton mass m_{p}). Assume that the average speed of the WIMPs relative to the Earth is $230\,\text{km/s}$.
What target core mass results in the largest recoil energy? How large is the recoil energy in that case?

(c) Calculate the expected event rate per day and kilogram of target material (^{131}Xe) under the following assumptions:

- WIMP mass $m_{\text{DM}} = 35\,\text{GeV}$,
- Density of the WIMP halo of the galaxy $\rho_{\text{DM}} = 0.3\,\text{GeV/cm}^3$,
- Mean velocity of the WIMPs relative to Earth $v_{\text{DM}} = 230\,\text{km/s}$,
- Spin-independent cross section for WIMP nucleon scattering $\sigma_{\text{DM}} = 10^{-44}\,\text{cm}^2$.

Consider how the spin-independent WIMP nucleon cross section scales with the nucleon number. You can assume that the coherence condition is fulfilled.

(d) In particle physics, a measurement is considered an indication if the measured value deviates from the expectation by 3 standard deviations (3σ). A deviation of 5σ is considered a discovery. For counting experiments, the significance z of the signal observation in units of the standard deviation can be approximated by

$$z = s/\sqrt{b},$$

where s is the number of signal events and b is the number of background events. Using the result from (c), calculate how many events you expect in the XENON1T experiment within one year (if you have not solved (c), assume the result is $R = 5 \cdot 10^{-3}\,\mathrm{kg}^{-1}\mathrm{y}^{-1}$). According to the expectation, what is the maximum background rate that you can achieve a significance of 3σ with a measurement within one year? How long would you have to measure at this background rate in order to achieve a significance of 5σ with the expectation from task part (c)?

2) **Direct Search for Dark Matter: The CRESST Experiment** (3 points)

In the CRESST experiment, nuclear perturbations after interaction with a WIMP are to be detected by both an ionization and a phonon signal. For this purpose, the temperature increase in the detector caused by phonon excitation is measured. The dependence of the specific heat capacity c of the detector at low temperatures ($T \ll \theta_D$) is given by

$$c = \frac{4\pi^4}{5}\frac{kT^3}{m\theta_D^3},$$

where m is the molecular mass, θ_D is the Debye temperature and k is the Boltzmann constant. The CRESST experiment, which is set up in the Gran Sasso underground laboratory (Italy), used sapphire crystals as detection material, which were operated at a temperature of 15 mK (now $CaWO_4$ crystals).
Why is the measurement performed at such low temperatures? Calculate the temperature rise in a sapphire crystal (Al_2O_3, θ_D = 1041 K) of mass 262 g, which results from a nuclear recoil with an energy deposition in the crystal of 1 keV.

3) **Background in Liquid Noble Gas Detectors** (2 points)

A xenon detector has an intrinsic contamination of krypton in the 1 ppt range (1 ppt = 10^{-12}). Natural krypton (average mass 83,8 u) contains radioactive isopts such as ^{85}Kr (released during above-ground nuclear bomb tests and during the processing of radioactive waste, $T_{1/2}$ = 11 years). The naturally occurring fraction of ^{85}Kr in krypton is 2×10^{-11}. What is the expected background rate in a xenon detector with a mass of 1 t due to ^{85}Kr impurity?
Compare this background rate with the background rate due to ^{39}Ar in a ^{40}Ar detector with a detector mass of 1 t. In the case of argon, the specific activity of argon due to ^{39}Ar corresponds to 1 Bq/kg.