

```

In[ ]:= T[string_] := Style[string, FontFamily -> "TeX Gyre Pagella", FontSize -> 20]
customStyle = {
  PlotStyle -> Thickness[0.006],
  Frame -> True,
  FrameStyle -> Black,
  FrameTicksStyle -> {Directive[18, FontFamily -> "TeX Gyre Pagella"],
    Directive[18, FontFamily -> "TeX Gyre Pagella"]},
  ImageSize -> {550, 350}
};
σ[t_, D_] := Sqrt[2 D t]
(*n[t_, z_]:=1/Sqrt[4 π λ t]e^(-z^2/(4λ t)) *)
ndense[z_, t_, D_] := 1 / Sqrt[2 π σ[t, D]^2] e^(-(z)^2 / (2 σ[t, D]^2))

```

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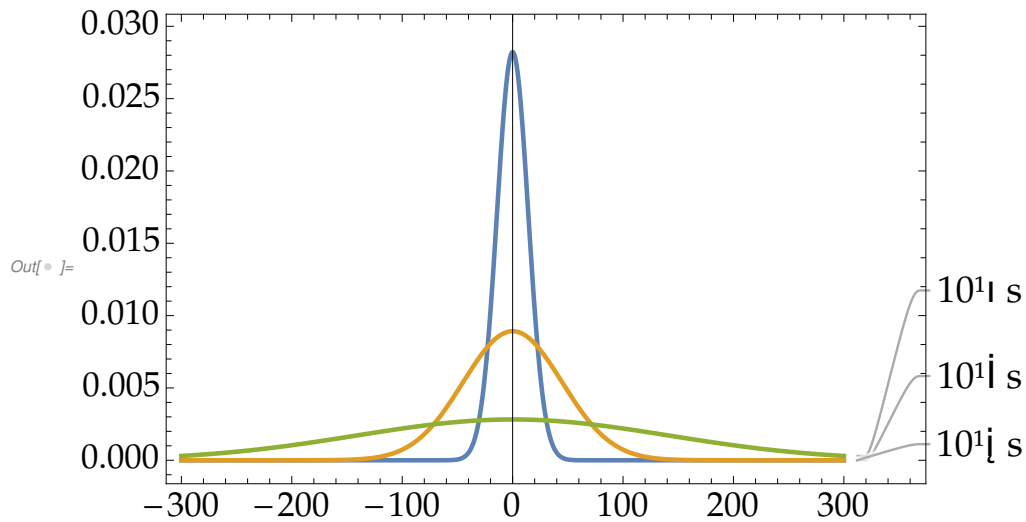
In[ ]:= Solve[dd D[1 / Sqrt[t c] e^(-(z)^2 / (2 c t)), z], z] - D[1 / Sqrt[t c] e^(-(z)^2 / (2 c t)), t] == 0, c]
(* age of galaxy, tG = 4.5 * 10^17 s*)
d := 10^(-15)
logt := 19
Plot[
  {ndense[z, 10^(logt - 2), d],
   ndense[z, 10^(logt - 1), d],
   ndense[z, 10^logt, d]},
  {z, -300, 300},
  PlotLabels -> {T["10^17 s"], T["10^18 s"], T["10^19 s"]},
  Evaluate@customStyle,
  PlotRange -> All]

```

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Out[ ]:= {{c -> 2 dd}, {c ->  $\frac{z^2}{t}$ }}

```



```
In[ ]:= a = D[D[ndense[z, t, 1], z], z]
      b = D[ndense[z, t, 1], t] // Expand
      a - b // FullSimplify
```

$$\text{Out[]} = -\frac{e^{-\frac{z^2}{4t}}}{4\sqrt{\pi}t^{3/2}} + \frac{e^{-\frac{z^2}{4t}}z^2}{8\sqrt{\pi}t^{5/2}}$$

$$\text{Out[]} = -\frac{e^{-\frac{z^2}{4t}}}{4\sqrt{\pi}t^{3/2}} + \frac{e^{-\frac{z^2}{4t}}z^2}{8\sqrt{\pi}t^{5/2}}$$

Out[] = 0

```
In[ ]:= Integrate[ndense[z, t, dd], {z, -H, H}] /. H -> 300 /. dd -> 10^(-14)
      Solve[% == 0.99, t]
      Integrate[ndense[z, t, dd], {z, -H, H}] /. H -> 300 /. dd -> 10^(-15)
      Solve[% == 0.99, t]
```

$$\text{Out[]} = \text{Erf}\left[\frac{1500000000}{\sqrt{t}}\right]$$

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$$\text{Out[]} = \{\{t \rightarrow 6.78232 \times 10^{17}\}\}$$

$$\text{Out[]} = \text{Erf}\left[\frac{1500000000\sqrt{10}}{\sqrt{t}}\right]$$

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$$\text{Out[]} = \{\{t \rightarrow 6.78232 \times 10^{18}\}\}$$

```
In[ ]:= Integrate[ndense[z, t, dd], {z, -H, H}] /. H -> 300 /. t -> 10^14 /. dd -> 10^(-15) γ
      gcrit = Solve[% == 0.5, γ]
      T["lg E crit:"]
      9 + Log10[γ /. gcrit]
```

$$\text{Out[]} = \text{Erf}\left[\frac{150\sqrt{10}}{\sqrt{\gamma}}\right]$$

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$$\text{Out[]} = \{\{\gamma \rightarrow 989149.\}\}$$

$$\text{Out[]} = \text{lg E crit:}$$

$$\text{Out[]} = \{14.9953\}$$

```
In[ ]:= FullSimplify[Integrate[ndense[z, t, dd], {z, -H, H}], Reals]
```

$$\text{Out[]} = \frac{\sqrt{dd} \sqrt{t} \operatorname{Erf}\left[\frac{H}{2\sqrt{dd} \sqrt{t}}\right]}{\sqrt{dd t}}$$

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In[ ]:=
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τ10 := 1.5 * 10^6
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nrat10 := 0.028
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```
nrat9 := 0.341
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```
nrat7 := 0.631
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((10 / (nrat10 / nrat9)) - 1) τ10
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% * 31536000
```

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Out[ ] = 1.81179 × 108
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```
Out[ ] = 5.71365 × 1015
```