Please submit via ILIAS by **Monday, Nov. 21st**. Solutions will be discussed Nov. 24th 9:45 in room 10/1.

## 1. Cosmic Rays at Earth

Consider a simple cosmic-ray detector in space consisting of an ideal square scintillator with 75 cm sides.

(a) How many particles with energies above $10^{14}$ eV are detected in a day with such a detector? Assume the flux of cosmic rays is in a good approximation given as

$$\Phi(E) = \frac{\mathrm{d}N}{\mathrm{d}E\,\mathrm{d}A\,\mathrm{d}\Omega\,\mathrm{d}t} = \Phi_1 \left(\frac{E}{E_1}\right)^{-\gamma},$$

where $\Phi_1 = 1.1 \times 10^{-18}/\mathrm{eV\,m^2\,sr\,s}$, $E_1 = 10^{14}$ eV, and $\gamma = 2.7$. **(2 points)**

(b) Using the flux given above, calculate how many cosmic rays with energies $E$ above 10 GeV are hitting the Earth every second. What is the total corresponding mass of incoming cosmic rays with $E > 10$ GeV in a year or in the whole lifetime of Earth if you assume cosmic rays are protons? **(1 point)**

(c) Download and execute the code to plot the spectrum of different cosmic-ray particles at Earth from [1]. Change the axis title ("intensity") to something more appropriate and superimpose the approximate flux function used in (a). **(1 point)**

(d) Please derive the rate per area for a given point in the $(E, E^2\Phi)$ plane using a power law as given in (a) for arbitrary values of $\Phi_1$. Superimpose lines for $1/\mathrm{m}^2/\mathrm{s}$, $1/\mathrm{m}^2/\mathrm{yr}$ and $1/\mathrm{km}^2/\mathrm{yr}$ on the plot obtained in (c) and discuss the agreement of your result with the gray shaded areas of the figure. **(2 points)**

## 2. Direct Detection of Cosmic Rays

Assume a cosmic-ray spectrometer consisting of $N = 6$ identical layers of circular position detectors with an area of 6.7 $\mathrm{m}^2$ (i.e. similar to the Alpha Magnetic Spectrometer). Please compare two design options: (1) A superconducting magnet with a length of 1 m, a magnetic field strength of 0.75 T and a measurement time of 3 years. (2) A permanent magnet with a length $L$, a magnetic field strength of 0.15 T and a measurement time of 15 years.

(a) which length $L$ is needed to obtain the same momentum resolution in the two scenarios?

(b) calculate the apertures and exposures of the two designs (assuming $L$ obtained in (a)). Use the results of [2] for the aperture of a detector limited by two circular planes (i.e. the first and last tracking plane of the spectrometer).

Which design results in a larger exposure? **(2 points)**

## 3. Units and constants

A frequent source of errors in numerical calculations are incompatible units (see e.g. [3]). Write a program that does some simple calculations of your choice using unit packages (C ++ [4] or Python [5, 6]). For instance, convert the "spectrometer equation" ($r = 3.3\,\mathrm{m}(R/\mathrm{GV})/(B/\mathrm{T})$) to units appropriate for Galactic (EV, kpc, $\mu$G) and extragalactic ($10^{20}$ V, Mpc, nG) calculations. Be creative and try a few other conversions! Please send code as solution. **(2 points)**

[1] https://github.com/carmeloevoli/The_CR_Spectrum

[2] J.D. Sullivan "Geometrical factor and directional response of single and multi-element particle telescopes" Nucl.Instrum.Meth. 95 (1971) 5

[3] https://en.wikipedia.org/wiki/Mars_Climate_Orbiter#Cause_of_failure

[4] https://github.com/martinmoene/PhysUnits-CT-Cpp11

[5] http://docs.astropy.org/en/stable/units/

[6] http://docs.astropy.org/en/stable/constants/