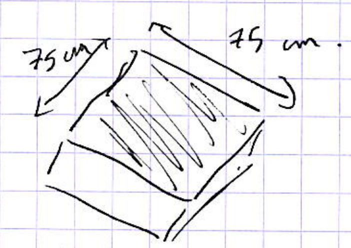
COSMIC RAYS - Exercises set 1

1. Cosmic rays at Earth



$\Rightarrow$ $A = 0.5625 \ m^2$

$\Omega = \pi \sin^2 \theta_{max}$ ×2 ?

$\theta_{max} = 80^\circ \Rightarrow \Omega = 1.7 \ sr$

$$\phi(E) = \frac{d^4 N}{dE\, dA\, d\Omega\, dt} = \phi_1 \left(\frac{E}{E_1}\right)^{-\gamma}$$

a) $N(\Delta t = 1 \text{ day}, \Omega = 1.7\,sr, A = A, E \geq 10^{14} eV) = \int_A \int_\Omega \int_{\Delta t} \int_{E_0 = 10^{14} eV}^{\infty} \phi_1 \left(\frac{E}{E_1}\right)^{-\gamma} dE$

$$N = \Omega A \Delta t \ \phi_1 \left[\left(\frac{\infty}{E_1}\right)^{-\gamma+1} - \left(\frac{E_0}{E_1}\right)^{-\gamma+1}\right] \times \frac{E_1}{-\gamma+1}$$

$$\gamma = 2.7 \Rightarrow \boxed{N = \Omega A \Delta t \ \phi_1 \frac{E_1}{\gamma - 1}\left(\frac{E_0}{E_1}\right)^{1-\gamma}}$$

$\Delta t = 1 \text{ day} = 86400 \ s$

$\phi_1 = 1.1 \times 10^{-18} \ /eV/m^2/sr/s$

$E_1 = 10^{14} \ eV$

$\boxed{N = 9.7}$

b) $E_0 = 10 \ GeV = 10^{10} \ eV$

$A_{\oplus} = 4\pi R_{\oplus}^2 = 5.11202 \ 10^{14} \ m^2$

$\Delta t = 1 \ s$

$$\Rightarrow \frac{dN}{dt}(E \geq 10 \ GeV) = 2.6 \ 10^{18} \ /s.$$

m_p $\Rightarrow t = 1 \ yr \Rightarrow 3.8 \cdot 10^{-4} \ g$

$t = 4.54 \ Gyr \Rightarrow 1.7 \ 10^{6} \ g \sim 1.7 \ Ton.$

[c] convert to GeV !!

$$\phi(E) = 1.1\ 10^{-18} \left(\frac{E}{10^{14}\,eV}\right) \ /eV\ m^2\ s\ sr$$

$$\Downarrow$$

$$\phi(E_{in\ GeV}) = 1.1\ 10^{-9} \left(\frac{E}{10^{5}\,GeV}\right) \ /GeV\ m^2\ s\ sr$$

[d]

$$E^2 \phi(E) = E^2 \frac{dN}{dE\, dA\, d\Omega\, dt}$$

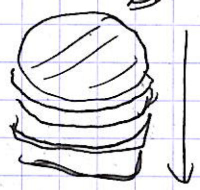
$$N(E_1) = \int_{E_1}^{\infty} \phi(E)\, dE = \Omega A \Delta t\, \phi_0 \underbrace{\frac{E_0}{\gamma - 1}\left(\frac{E_1}{E_0}\right)^{1-\gamma}}_{\frac{E_1}{\gamma-1}\phi_0\left(\frac{E_1}{E_0}\right)^{-\gamma}}$$

$$N(E_1) = \frac{\Omega A \Delta t}{(\gamma - 1) E_1} \underbrace{E_1^2 \phi_0 \left(\frac{E_1}{E_0}\right)^{-\gamma}}_{E^2 \phi(E)}$$

$$\Rightarrow \boxed{E^2 \phi(E) = \frac{N(\gamma - 1) E}{\Omega A \Delta t}}$$

2. Direct detection of CRs

$A = 6.7\ m^2$



6 layers

design (1)

superconducting magnet

length 1 m

$B_1 = 0.75$ T

$\Delta t_1 = 3$ yrs

design (2)

Permanent magnet

length L

$B_2 = 0.15$ T

$\Delta t_2 = 15$ yrs.

(a)

$$\frac{\sigma(p)}{p} = \frac{\sigma_{pos}}{\sqrt{N} L^2 B}\, p$$

$$\left(\frac{\sigma(p)}{p}\right)_{(1)} = \left(\frac{\sigma(p)}{p}\right)_2 \Rightarrow \frac{1}{L_1^2 B_1} = \frac{1}{L_2^2 B_2}$$

$$\Rightarrow L = L_1 \sqrt{\frac{B_1}{B_2}}$$

$$L = 1 \sqrt{\frac{0.75}{0.15}} = 2.24\ m$$

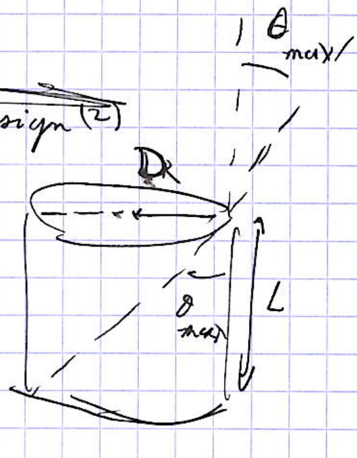
(b) ~~design (1)~~

$$A = S \pi \sin^2 \theta_{max}$$

$$\sin \theta_{max} = \frac{D}{\sqrt{D^2 + L^2}}$$

$$= \frac{D}{\sqrt{4\frac{S}{\pi} + L^2}}$$

design (2)



$$R = \sqrt{\frac{S}{\pi}} \qquad D = 2R = 2\sqrt{\frac{S}{\pi}}$$

aperture : $\mathcal{A} = \dfrac{S\pi}{\sqrt{\frac{4S}{\pi} + L^2}} \sqrt{\frac{S}{\pi}}$

design (1)

$\mathcal{A} = 18.84\ m^2\ sr$

design (2)

$\mathcal{A} = 13.3\ m^2\ sr$

from Sullivan.

$$G = \frac{1}{2}\pi^2\left[R_1^2 + R_2^2 + L^2 - \sqrt{(R_1^2 + R_2^2 + L^2)^2 - 4R_1^2R_2^2}\right]$$

In our case $R_1 = R_2 = R$

$$\Rightarrow \boxed{G = \frac{1}{2}\pi^2\left[2R^2 + L^2 - \sqrt{(2R^2 + L^2)^2 - 4R^4}\right]}$$

$R = \sqrt{\frac{S}{\pi}}$

design 1 :

$G = \cancel{[illegible]}\ m^2\ sr$

10.75

design 2 :

$G = \cancel{[illegible]}\ m^2\ sr$

5.13

Exposure : $\mathcal{E} = G\,\Delta t$

$\Delta t = 3$ yrs

$\Delta t = 15$ yrs.

$\mathcal{E}_1 = [illegible]\ 10^{[illegible]}\ m^2\ sr\ s$

$\mathcal{E}_2 = 2.[illegible]\ 10^9\ sr\ s\ m^2$