Astroteilchenphysik II – Cosmic Rays **Wintersemester 2022/23**

Please submit via ILIAS by **Monday, Dec. 5th**. Solutions will be discussed Dec. 8th 9:45 in room 10/1.

1. Milky Way

The average orbital speed of stars and gas in the Galaxy is approximately constant ($v_\phi \sim 220$ km/s) over a wide range of galactocentric radii. Measurements of the pattern speed of the spiral arms of our Galaxy are compatible with a constant angular velocity of the spiral arms of $\Omega_{\mathrm{sp}} \approx 20 - 30\,\mathrm{km\,s^{-1}\,kpc^{-1}}$. Calculate the Galactic co-rotation radius, i.e. the radius at which the average rotation speed of stars equals the spiral pattern speed, and comment on the relative motion of the Sun with respect to the spiral arms. **(1 point)**

2. Diffusion Equation

Show that the distribution of particle positions x following a Gaussian distribution with mean $\mu = 0$ and variance $\sigma^2 = 2Dt$ is a solution to the one-dimensional diffusion equation. **(1 point)**

3. Cosmic-Ray Diffusion

Consider a simple model of cosmic rays in the Galaxy (height $H \ll$ radius R) in which the net diffusion of cosmic rays is mainly perpendicular to the Galactic disk. In this case the density of cosmic rays depends only on the vertical coordinate z and follows the diffusion equation

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial z^2} + Q(z,t)$$

where D is the diffusion coefficient and the source term is given by $Q(z,t)$. Use the approximation $Q(z,t) = Q_0\,\delta(z)$ to describe a time-independent concentration of sources close to $z = 0$.

(a) Find the steady-state solution to the diffusion equation given a vanishing cosmic ray density at the edges of the Galaxy, $n(z = +H) = n(z = -H) = 0$. **(2 points)**

(b) Calculate the cosmic-ray column density $N = \int_{-H}^{+H} n(z)\,dz$ and determine the average residence time τ_{res} from $N = Q_0\tau_{\mathrm{res}}$. What is the residence time for $H = 500$ pc and $D =0.05\ \mathrm{pc^2/yr}$? **(2 points)**

4. Particle Propagation

Assume a source of cosmic-ray carbon nuclei. Calculate the time t_{90} after which 90% of the nuclei will have been absorbed by interactions with the interstellar medium (proton density $1/\mathrm{cm}^3$, $\sigma_{\mathrm{abs}}(C + p) = 250$ mb). Derive the typical maximum distance from which particles can reach Earth from a source given by the square root of the average squared distance after diffusing for a time $t = t_{90}$ for a diffusion coefficient of $0.05\ \mathrm{pc^2/yr}$. **(2 points)**

5. Production of Anti-Protons

Cosmic-ray protons can produce antiparticles in collisions with the protons of the interstellar medium.

(a) What are the minimal final state particles for the production of one anti-proton, anti-deuteron or anti-helium? (hint: consider baryon-number conservation!) **(1 point)**

(b) What is therefore the minimal cosmic-ray proton energy to produce an anti-proton, anti-deuteron or anti-helium? **(1 point)**