Astroteilchenphysik II – Cosmic Rays — Wintersemester 2022/23

Please submit via ILIAS by **Monday January 30th**. Solutions will be discussed Feb. 2nd 9:45 in room 10/1.

1. Hadronic Particle Production

In general, 'energy scaling' of particle production means that the probability of producing particle A in reactions of particle B depends only on the ratio $x = E_\mathrm{A}/E_\mathrm{B}$, $E\,\mathrm{d}\sigma/\mathrm{d}E_\mathrm{A} = f(x)$ (examples: A=photons from the bremsstrahlung of B=electrons or A=neutral pions from the collision of B=protons with protons). Please show that if the energy distribution of the projectiles B follows a power law $E^{-\alpha}$, then the energy distribution of produced particles A is a power law as well with same spectral index α. **(3 points)**

2. Charged Pion Decay

Consider the decay $X^\pm \to \mu^\pm + \overset{(-)}{\nu}_\mu$, where X is a meson of mass M_X moving with speed β_X.

(a) Derive the energy distributions of neutrinos and muons, $\mathrm{d}n/\mathrm{d}E_\nu$ and $\mathrm{d}n/\mathrm{d}E_\mu$, in the laboratory frame. Assume an isotropic distribution of particle emission in the rest frame of the meson. **(2 points)**

(b) What is the average fraction of energy with respect to the meson's laboratory energy, $z_{\nu/\mu} = E_{\nu/\mu}/E_X$, carried away by neutrinos and muons in the limit $\beta_X = 1$? Please give numerical values for $z_{\nu/\mu}$ for charged π, K and D mesons. **(1 point)**

3. Atmospheric depth

Consider a primary cosmic-ray particle which, upon entering the Earth's atmosphere, interacts with a nucleus of air and produces secondary particles. For the primary proton and primary iron nucleus, crossing the atmosphere with a zenith angle of 45°, solve the assignments given below. The interaction lengths of protons and iron nuclei in air are $\lambda_\mathrm{p} = 80\,\mathrm{g/cm^2}$ and $\lambda_\mathrm{Fe} = 12\,\mathrm{g/cm^2}$.

(a) On average, at which atmospheric depth (measured in $\mathrm{g/cm^2}$) is the first interaction taking place? **(1 point)**

(b) A detector is flown with a balloon at large heights so that the remainder of the atmosphere above the detector amounts only to $5.5\,\mathrm{g/cm^2}$ of vertical column density. Calculate the fraction of protons and iron nuclei which interact at least once before reaching the detector. **(1 point)**

(c) What is the mean height of the first interaction if you assume an isothermal atmosphere with the scale height of 8.4 km? **(2 points)**