Cosmic Rays - Exercises set 4

1. Hadronic particle production

$$\phi_B = \frac{dN_B}{dE_B} = N_B^0 \, E_B^{-\alpha} \rightsquigarrow \phi_A = \frac{dN_A}{dE_A} = ?$$

Feynman scaling : $E_A \dfrac{dN(E_B, E_A)}{dE_A} = f\left(\dfrac{E_A}{E_B}\right) \Rightarrow \dfrac{dN_{B\to A}}{dE_A} = \dfrac{1}{E_A} f\left(\dfrac{E_A}{E_B}\right)$

$$\phi_A(E_A) = \int_{E_A}^{\infty} \phi_B(E_B) \underbrace{\frac{dN_{B\to A}}{dE_A}}_{\text{production of } B \to A} dE_B$$

$$= \int_{E_A}^{\infty} N_B^0 \, E_B^{-\alpha} \, E_A^{-1} f\left(\frac{E_A}{E_B}\right) dE_B$$

let's $x = \dfrac{E_A}{E_B} \Rightarrow E_B = \dfrac{E_A}{x} \Rightarrow dE_B = -E_A \dfrac{dx}{x^2}$

Then $\phi_A(E_A) = \displaystyle\int_1^0 N_B^0 \left(\frac{E_A}{x}\right)^{-\alpha} f(x) E_A^{-1} \left(-E_A \frac{dx}{x^2}\right)$

$$\boxed{\phi_A(E_A) = N_B^0 \, E_A^{-\alpha} \int_0^1 x^{\alpha-2} f(x) dx}$$

2. Charge pion decay

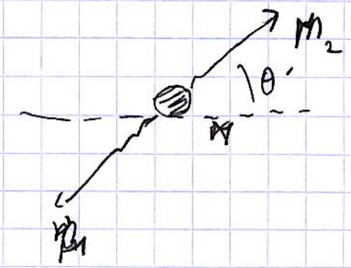
$$X^\pm \longrightarrow \mu^\pm + \overset{(-)}{\nu}_\mu$$

meson

mass M_X

speed $\beta_X \to 1$

Two-body decay (Rest frame)



quadri momentum

$$\hat{p}_0' = \hat{p}_1' + \hat{p}_2'$$

$$\hat{p}_2'^2 = \left(\hat{p}_0' - \hat{p}_1'\right)^2$$

$$\hat{p}_2'^2 = \hat{p}_0'^2 + \hat{p}_1'^2 - 2\,\hat{p}_0'\hat{p}_1'$$

$$m_2^2 = M^2 + m_1^2 - 2\begin{pmatrix} M \\ \vec{0} \end{pmatrix}\begin{pmatrix} E_1' \\ \vec{p}_1' \end{pmatrix}$$

$$\hat{p}_i = \begin{pmatrix} E_i \\ \vec{p}_i \end{pmatrix}$$

$$\hat{p}_i^2 = E_i^2 - \vec{p}_i^{\,2} = m_i^2 (c^4)$$

$$\hat{p}_0 = \begin{pmatrix} E_0 \\ \vec{0} \end{pmatrix} \Rightarrow$$

$$\Rightarrow \hat{p}_0^2 = E_0^2 = M^2$$

$$\hat{p}_0 = \begin{pmatrix} M \\ \vec{0} \end{pmatrix}$$

$$m_2^2 = M^2 + m_1^2 - 2ME_1'$$

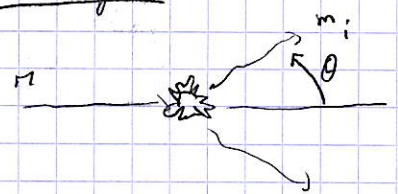
$$E_1' = \frac{m_2^2 - M^2 - m_1^2}{-2M} = \frac{M^2 + m_1^2 - m_2^2}{2M}$$

$$m_1^2 = E_1'^2 - \vec{p}_1^{\,2}$$

$$\Rightarrow \|\vec{p}_1'\| = \sqrt{E_1'^2 - m_1^2}$$

(1) $$\boxed{\|\vec{p}_1'\| = \sqrt{\frac{(M^2 + m_1^2 - m_2^2)^2}{4M^2} - m_1^2}}$$ same for $\|\vec{p}_2'\|$

Rest frame to lab frame



$$\begin{pmatrix} E_i \\ \vec{p}_i \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix}\begin{pmatrix} E_i' \\ \vec{p}_i' \end{pmatrix}$$

$$\Rightarrow \boxed{E_i = \gamma E_i' + \beta\gamma \|\vec{p}_i'\| \cos\theta'} \quad (2)$$

$$-\beta\gamma\|\vec{p}_i'\| + \gamma E_i' \leqslant E_i \leqslant \gamma E_i' + \beta\gamma\|\vec{p}_i'\|$$

In the present case: $M = M_X$, $m_1 = m_\nu = 0$, $m_2 = m_\mu$

for ν: $\hat{p}_i^2 = E_i^2 - \vec{p}_i^{\,2} = 0 \rightarrow$ $E_i = \|\vec{p}_i\|$

$$(1) \Rightarrow \|\vec{p}_\nu'\| = \frac{M_X^2 - m_\mu^2}{2M_X}$$

$$(2) \Rightarrow \gamma(1-\beta)\frac{M_X^2 - m_\mu^2}{2M_X} \leqslant E_\nu \leqslant \gamma(1+\beta)\frac{M_X^2 - m_\mu^2}{2M_X}$$

$$E_X = \gamma M_X c^2 \Rightarrow \gamma = \frac{E_X}{M_X}$$

$$(2) \Rightarrow (1-\beta)\frac{E_X}{M_X}\frac{M_X^2 - m_\mu^2}{2M_X} \leqslant E_\nu \leqslant (1+\beta)\frac{E_X}{M_X}\frac{M_X^2 - m_\mu^2}{2M_X}$$

$$\Rightarrow \boxed{(1-\beta)\frac{M_X^2 - m_\mu^2}{2M_X^2} \leqslant \frac{E_\nu}{E_X} \leqslant (1+\beta)\frac{M_X^2 - m_\mu^2}{2M_X^2}}$$

Isotropic distribution in θ' (uniform)

$\Rightarrow \frac{E_\nu}{E_\pi}$ uniformally distributed between

$$\Leftrightarrow \frac{dn}{d\Omega'} = \frac{dn}{2\pi d\cos\theta'} \propto \frac{dn}{dE_i} = cst \qquad \text{from (2)}$$

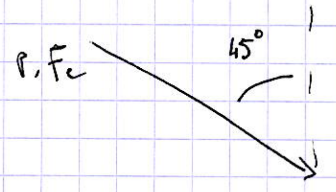
for $\beta_X = 1 \Rightarrow 0 \leqslant \frac{E_\nu}{E_X} \leqslant 1 - \frac{m_\mu^2}{M_X^2}$

$\left\langle \frac{E_\nu}{E_\pi} \right\rangle = 0,21 \qquad \left\langle \frac{E_\mu}{E_\pi} \right\rangle = 0.79$

$\left\langle \frac{E_\nu}{E_K} \right\rangle = 0.48 \qquad \left\langle \frac{E_\mu}{E_K} \right\rangle = 0.52$

$\left\langle \frac{E_\nu}{E_D} \right\rangle = 0.5 \qquad \left\langle \frac{E_\mu}{E_D} \right\rangle = 0.5$

3. Atmospheric depth



$\lambda_p = 80\ g/cm^2$

$\lambda_{Fe} = 12\ g/cm^2$

a) $X_\theta = \dfrac{X_V}{\cos\theta} \Rightarrow X_V = X_\theta \cos\theta \longrightarrow$ p: $56{,}57\ g/cm^2$, Fe: $8{,}49\ g/cm^2$

b) $\Rightarrow X_{balloon}^{balloon} = 5.5\ g/cm^2 \Rightarrow X_{45}^{balloon} = \dfrac{X_V^{balloon}}{\cos 45} \simeq 7.78\ g/cm^2$

$\Rightarrow$

$$\frac{dN_i}{dX} = -\frac{N_i}{\lambda_i} \Rightarrow N_i = N_0\, e^{-X/\lambda_i}$$

fract° of interaction: $\dfrac{N_i}{N_0}(x) = e^{-\frac{X}{\lambda_i}}$

$\Rightarrow$ fraction that interacts: $\dfrac{N_{int}}{N_0} = \left(1 - e^{-X/\lambda_i}\right)$

d) p: $\sim 9\%$

Fe $\sim 47.7\%$

c) $X_V = X_0\, e^{-h/h_0}$ $\qquad X_0 = 1030\ g/cm^2$

$h_0 = 8.4\ km$

$$\Rightarrow h = h_0 \ln\left(\frac{X_0}{X_V}\right)$$

$$= h_0 = \ln\left(\frac{X_0}{X_\theta \cos\theta}\right)$$

p $= 24{,}4$ km.

Fe $= 40$ km.