Please submit via ILIAS by **Monday February 13th**. Solutions will be discussed Feb. 16th 9:45 in room 10/1.

1. Particle Detection with Cherenkov radiation

The Cherenkov light is produced by relativistic charged particles traversing matter with an index of refraction $n > 1$.

(a) For electrons and muons derive the energy threshold for Cherenkov emission, first in air at the sea level and then in water. **(1 point)**

(b) How large is the opening angle of the Cherenkov-light cone for an ultra-relativistic particle in air (at the sea level) and in water? How large is the radius of the Cherenkov-light cone emitted at the height of 15, 10, 5 and 1 km, when projected to the ground, for an ultra-relativistic electron that moves vertically downwards? (Hint: use $n(h) = 1 + 0.000283\rho(h)/\rho(0)$ for the height-dependence of the refractive index in air) **(1 point)**

(c) The water-Cherenkov detectors of the Pierre Auger Observatory are cylindrical tanks with a $10\,\mathrm{m}^2$ top area and a height of 120 cm. They are filled with 12 tons of ultra-pure water which acts as a detection medium where Cherenkov photons are observed with photomultiplier tubes. In this detector, how many Cherenkov photons are emitted between wavelengths of 300 to 550 nm by a vertically crossing muon ($E_\mu \gg E_\mathrm{thresh}$)? (Hint: use Eq.(34.43) or (34.44) from [1]). **(1 point)**

(d) The Cherenkov light is observed by three photomultipliers, each with an active diameter of 23 cm. Estimate the number of photoelectrons released per photocathode if the quantum efficiency in the wavelength range mentioned above is $q_\mathrm{eff} = 0.12$. Take into account that diffusely-reflecting walls with reflectivity $R = 0.96$ are randomly scattering the Cherenkov photons around the detector. You can neglect the absorption of photons in the water. **(1 point)**

2. Maximum detection distance of fluorescence telescopes

The telescopes of the Pierre Auger Observatory have an optical aperture of $1.5\,\mathrm{m}^2$ and detect the fluorescence light of air showers of energy E with a camera made of 440 pixel photomultipliers. A charged secondary particle isotropically emits on average around 4 photons per meter of its travel distance. The number of charged particles N_ch in a shower maximum is in a good approximation $N_\mathrm{ch} \approx E/1.6\,\mathrm{GeV}$. Consider a vertical air shower at a distance r from the telescope with the shower maximum at the level of the telescope height. You can neglect the lateral extent of showers.

(a) How many fluorescence photons enter the aperture from the region around the shower maximum in the view of one pixel (half of the opening angle is $\alpha = 0.75°$). For the propagation of the fluorescence light take into account the mean atmospheric attenuation length $\Lambda = 20\,\mathrm{km}$. The total efficiency, including the quantum efficiency of the photomultipliers, is $\varepsilon = 0.1$. On average, how many photoelectrons N_pe are detected by a photomultiplier pixel? **(1 point)**

(b) How many background photons N_bg are detected in a time interval Δt if the background flux of the ultraviolet night sky is $\Phi_\mathrm{bg} = 6\times10^5/\mathrm{m}^2\,\mathrm{sr}\,\mu\mathrm{s}$? Set Δt to the time it takes the shower to traverse the view of the pixel (parallel movement). **(1 point)**

(c) To minimize the probability of accidental signals a pixel is considered as triggered only if its signal-to-noise ratio $k = N_\mathrm{pe}/\sqrt{N_\mathrm{bg}}$ in the time interval Δt is larger than $k_\mathrm{min} = 4.5$. What is to this threshold corresponding rate of accidental pixel triggers, induced by the background, if you assume a Poisson-distributed N_bg? **(1 point)**

(d) Use the partial results derived above and the minimal signal-to-noise ratio k_min to determine the relationship between the shower energy E and the maximal distance r_max from which the shower can still trigger a pixel. Numerically obtain the maximal distance r_max at which we can detect air showers with energy 10^{18} eV and 10^{20} eV. **(1 point)**

3. Flux and measurement uncertainty

A satellite experiment is measuring the flux of protons above the atmosphere of the Earth. The energy of each measured proton can be determined with only finite precision. The measured energy is usually log-normally distributed or, in other words, the logarithm of the ratio between the true energy E_{true} and the reconstructed energy E_{rec} is normally distributed with a variance σ^2, i.e.

$$\frac{\mathrm{d}P}{\mathrm{d}\epsilon} = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{\epsilon^2}{2\sigma^2}\right) \qquad \text{where} \qquad \epsilon = \ln\frac{E_{\mathrm{rec}}}{E_{\mathrm{true}}}.$$

How large is the actual flux of protons when the measured flux follows the form of a power law

$$\Phi(E_{\mathrm{rec}}) = \frac{\mathrm{d}^4 N}{\mathrm{d}E_{\mathrm{rec}}\,\mathrm{d}A\,\mathrm{d}\Omega\,\mathrm{d}t} = \Phi_0 \left(\frac{E_{\mathrm{rec}}}{E_0}\right)^{-\gamma} ?$$

Hint: as an ansatz use the fact that the functional form of the true spectrum is (up to the factor Φ_0) the same as the observed spectrum. **(2 points)**

[1] Particle Data Group "Passage of particles through matter" https://pdg.lbl.gov/2020/reviews/rpp2020-rev-passage-particles-matter.pdf