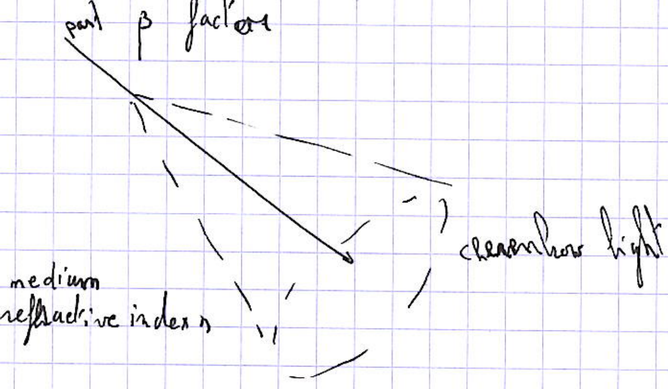
1. Particles detection with Cherenkov radiation



air : $n = 1.000\,283$

water : $n = 1.333$

(a) $E_{min} = m \left(1 - \frac{1}{n^2}\right)^{-1/2}$

	air	water
e^-	21,1 MeV	0.77 MeV
μ	4.385 GeV	159,79 MeV

from $\cos\theta = \frac{1}{\beta n}$ $\Rightarrow$ has to be ≥ 1

$\Rightarrow \beta_{min} = \frac{1}{n}$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1$

$\Rightarrow \gamma_{min} = \frac{1}{\sqrt{1-\beta_{min}^2}} = \frac{1}{\sqrt{1-\frac{1}{n^2}}}$

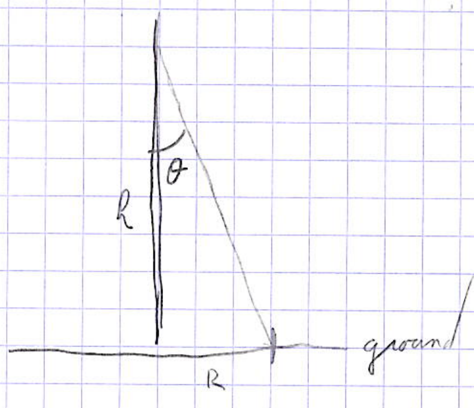
$E_{min} = \gamma_{min}\, m$

(b) $\cos\theta = \frac{1}{\beta n}$

approx $\beta = 1$ (ultra relativistic)

air: 1.38°

water. $41,4^\circ$



$\tan\theta = \frac{R}{h}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$

$= \sqrt{\frac{1}{\cos^2\theta} - 1}$

$\Rightarrow R = h\sqrt{\frac{1}{\cos^2\theta} - 1}$

$$\boxed{R = h\sqrt{(\beta n)^2 - 1}}$$

$n(h) = 1 + 0,000283 \frac{\rho(h)}{\rho(0)}$

@ 0 km. $\rho(0) = 1.23$

$h = 1$ km $\rho(h) = 1.115$ (mean)

5 km $\rho(5) = 0,74$

10 km $\rho(10) = 0.42$

15 km $\rho(15) = 0,19$

Gaisser et Al book

@ $h = 1$ km : $R = 22{,}6$ m

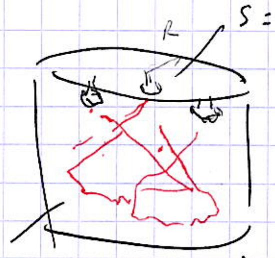
5 km $R = 92{,}3$ m

10 km $R = 139{,}$ m

15 km $R = 140$ m.

(c)

$S = 10\ m^2$



$L = 120$ cm

120 tons of pure water.

$S = \pi R^2 \Rightarrow R = \sqrt{\frac{S}{\pi}}$ $\Rightarrow$ perimeter $P = 2\pi R$

1,78 m.

total area surface $A_{tot} = 2S + LP$

$$\boxed{A_{tot} = 2S + L\, 2\pi \sqrt{\frac{S}{\pi}}}$$

$$\frac{d^2N}{d\lambda\, dl} = \frac{2\pi \alpha z^2}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2}\right)$$

α = fine structure constant

$\alpha \simeq \frac{1}{137}$

approx $\beta = 1$

$$N = \int_0^L \int_{\lambda_1}^{\lambda_2} \frac{dN}{d\lambda\, dl}\, d\lambda\, dl$$

z. number of charges: $(= 1)$

approx $n(\lambda) = cst.$

$$= L\ 2\pi\alpha z^2 \left(1 - \frac{1}{\beta^2 n^2}\right) \underbrace{\int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^2}}_{\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]}$$

$\left.\begin{array}{l}\lambda = 300\ nm \\ \lambda_2 = 55\ nm.\end{array}\right\}$ $N_0 \simeq 36\ 471$ photons.

(d) total number of photons . 0 reflection N_0

1 reflection $R N_0$ $\quad$ R: reflectivity

2 reflections $R^2 N_0$

$\vdots$

$$\Rightarrow \text{total: } N_0 \sum_{i=1}^{\infty} R^i = \frac{N_0}{1 - R} = N_{photons}$$

$R = 0.96$ $\quad N_{photons} = 911\ 796.$

Area PMT : $A_{PMT} = 3\pi R_{PMT}^2$ $\quad R_{PMT} = 23$ cm

Proba hit PMT : $P_{PMT} = \frac{A_{PMT}}{A_{tot}} = \frac{3\pi R_{PMT}^2}{2S + 2\pi L\sqrt{\frac{S}{\pi}}}$ $\quad P_{PMT} \sim 1.5\%$

Total number of photons detected:

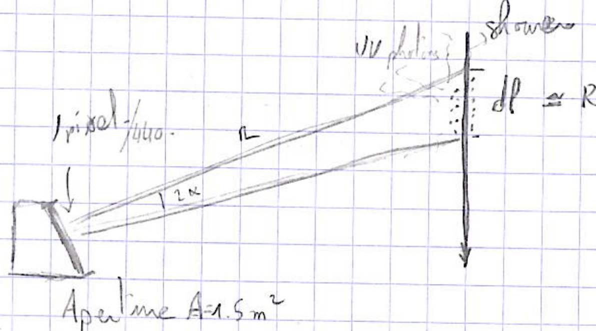
$$N_{pe} = P_{PMT}\; q_{eff}\; N_{photons}$$

q_{eff} ← quantum efficiency

$$N_{PE} = P_{PMT}\; q_{eff}\; \frac{N_0}{1-R}$$

$$q_{eff} = 0.12 \Rightarrow N_{PE} = 1630$$

2. Maximum detection distance of fluorescence telescope



$dl \simeq R\tan(2\alpha) \Rightarrow \Delta t = \frac{R}{c}\tan(2\alpha)$

(a) $dN_\gamma^{(iso)} = Y\, dN_e \rightarrow N_\gamma^{(iso)} = \int_{dl} dN_\gamma^{(iso)} = Y N_e\, dl = Y N_e\, R\tan(2\alpha)$ $\quad Y = 4/m$.

Number of γ reaching the telescope

$$N_\gamma^{(tel)} = N_\gamma^{(iso)} \underbrace{\frac{A}{4\pi R^2}}_{\text{solid angle}} \times \underbrace{e^{-R/\Lambda}}_{\text{attenuation}}$$

$N_{pe} = N_\gamma^{(tel)} \times \varepsilon$ efficiency PMT

$$= Y N_e\, R\tan(2\alpha)\, \frac{A}{4\pi R^2}\, e^{-R/\Lambda}\, \varepsilon$$

$$\boxed{N_{pe} = \frac{\varepsilon A}{4\pi}\tan(2\alpha)\, Y N_e\, \frac{e^{-R/\Lambda}}{R}}$$

$N_e \sim \left(\frac{E_{prim}}{E_0}\right)$

$E_0 = 1.6$ GeV

(c)

(b) $N_{bg} = \phi_{bg}\, \Delta t\, A\, \Omega$

$\Omega = 2\pi(1-\cos\alpha)$

$\Delta t = \frac{R}{c}\tan 2\alpha$

$= \phi_{bg} A \frac{R2\pi}{c} \underbrace{\tan 2\alpha}_{\sim 2\alpha} \underbrace{(1-\cos\alpha)}_{\sim \frac{\alpha^2}{2}}$

$\sim \phi_{bg} \frac{A 2\pi R}{c} \alpha^3$

Probability of k background photon:

$$P(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

Proba of $X \geq k$

$$P(X\geq k) = \sum_{i=k}^{\infty} P(i) = \sum_{i=k}^{\infty} \frac{\lambda^i}{i!} e^{-\lambda}$$

$$= e^{-\lambda} \sum_{i=k}^{\infty} \frac{\lambda^i}{i!}$$

Incomplete Gamma function (upper limit)

$$\Rightarrow P(X\geq k) = Q(\lambda, k) = \frac{\Gamma(\lambda, k)}{\Gamma(\lambda)} = \frac{\int_\lambda^\infty t^{\lambda-1} e^{-t}\,dt}{\int_0^\infty t^{\lambda-1} e^{-t}\,dt}$$

$\lambda = \langle X\rangle = \langle N_{bg}\rangle$, $k =$ [redacted] $+ k_{min}\sqrt{\langle N_{bg}\rangle}$

(d) above baseline

$$N_{pe} = k_{min}\sqrt{N_{bg}}$$

$$\frac{\mathcal{E}A}{4\pi}\tan(2\alpha)\,\gamma\, N_e(E)\, \frac{e^{-\frac{R_{max}}{\Lambda}}}{R_{max}} = k_{min}\sqrt{\phi_{bg}\frac{AR2\pi}{c}\tan(2\alpha)\,(1-\cos\alpha)}$$

$$N_e(E) = k_{min}\, R_{max}\, \frac{e^{R_{max}/\Lambda}}{\mathcal{E}\,\gamma}\sqrt{\phi_{bg}\frac{32\pi^3 R_{max}}{c\tan(2\alpha) A}(1-\cos\alpha)}$$

$$N_e^2 = \frac{k_{min}^2}{\mathcal{E}^2\gamma^2} R_{max}^2 e^{2R_{max}/\Lambda}\, \phi_{bg}\frac{32\pi^3 R_{max}(1-\cos\alpha)}{c\tan(2\alpha)\, A}$$

$$N_e^2 = \underbrace{\frac{k_{min}^2}{\mathcal{E}^2\gamma^2}\,\phi_{bg}\frac{32\pi^3(1-\cos\alpha)}{Ac\tan(2\alpha)}}_{K} R_{max}^3\, e^{2R_{max}/\Lambda}$$

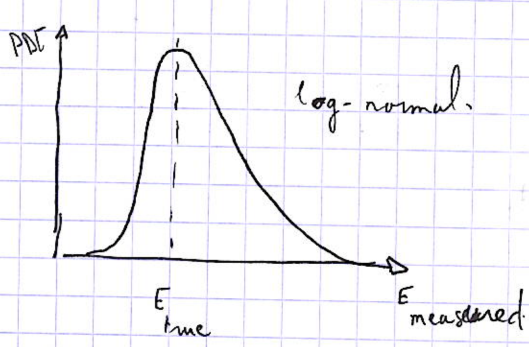
$$\frac{N_e^2}{K} = R_{max}^3\, e^{2R_{max}/\Lambda}$$

$\hookrightarrow$ solve graphically / numerically

1 EeV $\sim$ 8 km

100 EeV $\sim$ 42 km

3. Flux and measurement uncertainty



$$= \frac{dP}{d\varepsilon} = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\varepsilon^2/2\sigma^2}$$

$$\varepsilon = \ln \frac{E_{rec}}{E_{true}}$$

$$\phi(E_{rec}) = \phi_0 \left(\frac{E_{rec}}{E_0}\right)^{-\gamma}$$

$$g(E_{rec}, E_{true}) = \frac{dP}{dE_{rec}} = \frac{dP}{d\varepsilon}\left|\frac{d\varepsilon}{dE_{rec}}\right| = \frac{1}{E_{rec}}\frac{dP}{d\varepsilon}$$

$$\phi(E_{true}) = ?$$

$$\phi(E_{true}) = \int_{-\infty}^{\infty} \phi(E_{rec})\, g(E_{rec}, E_{true})\, dE_{rec} \quad \text{convolution}$$

$$= \phi_0 \int_{-\infty}^{\infty} \left(\frac{E_{rec}}{E_0}\right)^{-\gamma} \frac{1}{\sqrt{2\pi}\,\sigma} \frac{e^{-\frac{(\ln E_{rec} - \ln E_{true})^2}{2\sigma^2}}}{E_{rec}}\, dE_{rec}$$

$$= \frac{\phi_0 E_0^{\gamma}}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} E_{rec}^{-\gamma} \frac{e^{-\frac{(\ln E_{rec} - \ln E_{true})^2}{2\sigma^2}}}{E_{rec}}\, dE_{rec}$$

$$= \frac{\phi_0 E_0^{\gamma}}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{-\gamma \ln E_{rec} - \frac{(\ln E_{rec} - \ln E_{true})^2}{2\sigma^2}}}{E_{rec}}\, dE_{rec}$$

let $z = \ln E_{rec} \Rightarrow dz = \frac{dE_{rec}}{E_{rec}}$ $\quad z' = \ln E_{true}$

$$\phi(E_{true}) = \frac{\phi_0 E_0^{\gamma}}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\gamma z}\, e^{-\frac{(z - z')^2}{2\sigma^2}}\, dz$$

$$= \frac{\phi_0 E_0^{\gamma}}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\gamma z}\, e^{\frac{-(z - z')^2}{2\sigma^2}}\, dz$$

$$= \frac{\phi_0 E_0^{\gamma}}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\gamma z - \frac{z^2}{2\sigma^2} + \frac{z z'}{\sigma^2} - \frac{z'^2}{2\sigma}}\, dz$$

$$= \frac{\phi_0}{\sigma\sqrt{2\pi}} \left(\frac{E_{true}}{E_0}\right)^{-\gamma} \int_{-\infty}^{+\infty} e^{+\gamma z' - \gamma z}\, e^{-\frac{(z - z')^2}{2\sigma^2}}\, dz$$

$$z = \gamma - \gamma' = \ln \epsilon_{rec} - \ln \epsilon_{true}$$

$$\phi(\epsilon_{true}) = \underbrace{\frac{\phi_0}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\gamma z} \, e^{-\frac{z^2}{2\sigma^2}} \, dz}_{f(\epsilon_{true}) = cst} \left(\frac{\epsilon_{true}}{\epsilon_0}\right)^{-\gamma}$$

integral of a Gaussian:

$$\int_{-\infty}^{\infty} k e^{-f x^2 + g x + h} \, dx = \int_{-\infty}^{\infty} k \, e^{-f\left(x - \frac{g}{2f}\right)^2 + \frac{g^2}{4f} + h} \, dx$$

$$= k \sqrt{\frac{\pi}{f}} \, e^{\left(\frac{g^2}{4f} + h\right)}$$

in our case $k = 1$

$f = \frac{1}{2\sigma^2}$

$g = -\gamma$

$h = 0$

$$\Rightarrow \int_{-\infty}^{\infty} e^{-\gamma z} \, e^{-\frac{z}{2\sigma^2}} \, dz = \sqrt{2\pi} \, \sigma \, e^{-\frac{\gamma^2}{2}\sigma^2}$$

$$\Rightarrow \boxed{\phi(\epsilon_{true}) = \phi_0 \, e^{-\frac{\gamma^2 \sigma^2}{2}} \left(\frac{\epsilon_{true}}{\epsilon_0}\right)^{-\gamma}}$$