Please submit via ILIAS by **Monday February 27th**. Feel free to contact thomas.fitoussi@kit.edu and michael.unger@kit.edu in case of questions. A meeting to discuss the results will be scheduled afterwards.

A Heitler-Matthews-Greisen Model of Air Showers

Experiment with a simplified model for the electromagnetic component of air showers and summarize your results in a short summary note. Please also submit your code as part of the solution.

As in the Heitler-Matthews model, assume that the electromagnetic component of air showers is dominated by photons from the decay of neutral pions that were created in the first CR+air interaction. To generate the longitudinal profile (i.e. the number of electromagnetic particles as a function of slant depth) of one proton-induced shower proceed as follows: a) generate the depth of first interaction by drawing an exponentially distributed random number [1] with interaction length λ_p corresponding to a proton-air cross section of 550 mb at 10^{19} eV. Then, starting at this depth, add up the particles initiated by the photons from the decay of N neutral pions. For each photon, generate a random energy following the uniform distribution [2] in $\mathrm{d}N/\mathrm{d}E_\gamma$ expected for the two-body decay of the pion (similar to exercise 2 in problem set 4, see also Sec. 11.1 in CRPP [4]). Assume that each photon initiates an electromagnetic cascade following the Greisen profile as in lecture 9 (Eq. (15.30) in CRPP). The total p+air induced electromagnetic profile is given by the sum of the individual photon-induced profiles.

(a) Run a few hundred showers with $E_0 = 10^{18}$ eV, find the shower maximum (X_max) for each shower numerically (i.e. the position of the maximum of the sum of all Greisen profiles) and determine the mean of the X_max distribution, $\langle X_\text{max}\rangle$. Adjust the neutral pion multiplicity N, until the value of $\langle X_\text{max}\rangle \approx 800\ \text{g/cm}^2$ as obtained in full air shower simulations can be reproduced. Use this multiplicity in the following.

(b) Does your Heitler-Matthews-Greisen simulation reproduce the standard deviation of the X_max distribution of proton-induced showers which is $\sigma(X_\text{max}) \sim 60\ \text{g/cm}^2$?

(c) Instead of a fixed pion multiplicity, introduce additional shower-to-shower fluctuations by using a Poissonian random variable [3] with mean N. How does this change affect $\langle X_\text{max}\rangle$ and $\sigma(X_\text{max})$?

(d) Assume a nucleus-air interaction length of $\lambda_A \sim \lambda_p / A^{2/3}$ and a Poisson-distributed [3] number of participating nucleons with a mean value of $\langle n\rangle = A\,\lambda_A/\lambda_p$. Choose one (or both) of the extreme cases for the fragmentation of the spectator nucleon: full fragmentation into nucleons or no fragmentation, i.e. the spectator nucleus remains fully intact. Numerically verify the superposition theorem (cf. lecture 10) and calculate $\langle X_\text{max}\rangle$ and $\sigma(X_\text{max})$ for iron-induced showers. How do the values compare to the ones of fully simulated showers for which $\langle X_\text{max}\rangle \approx 700\ \text{g/cm}^2$ and $\sigma(X_\text{max}) \approx 20\ \text{g/cm}^2$?

(e) Draw a few examples of longitudinal profiles (N vs X) obtained with your simulation at $E_0 = 10^{19}$ eV for proton and iron primaries. Do your simulated profiles qualitatively resemble the ones shown in the picture on the next page?

[1] e.g. `numpy.random.exponential` or `TRandom::Exp()`

[2] e.g. `numpy.random.uniform` or `TRandom::Uniform()`

[3] e.g. `numpy.random.poisson` or `TRandom::Poisson()`

[4] T.K. Gaisser, R. Engel, E. Resconi Cosmic Rays and Particle Physics Cambridge University Press (2016);

[5] air shower simulation program CORSIKA

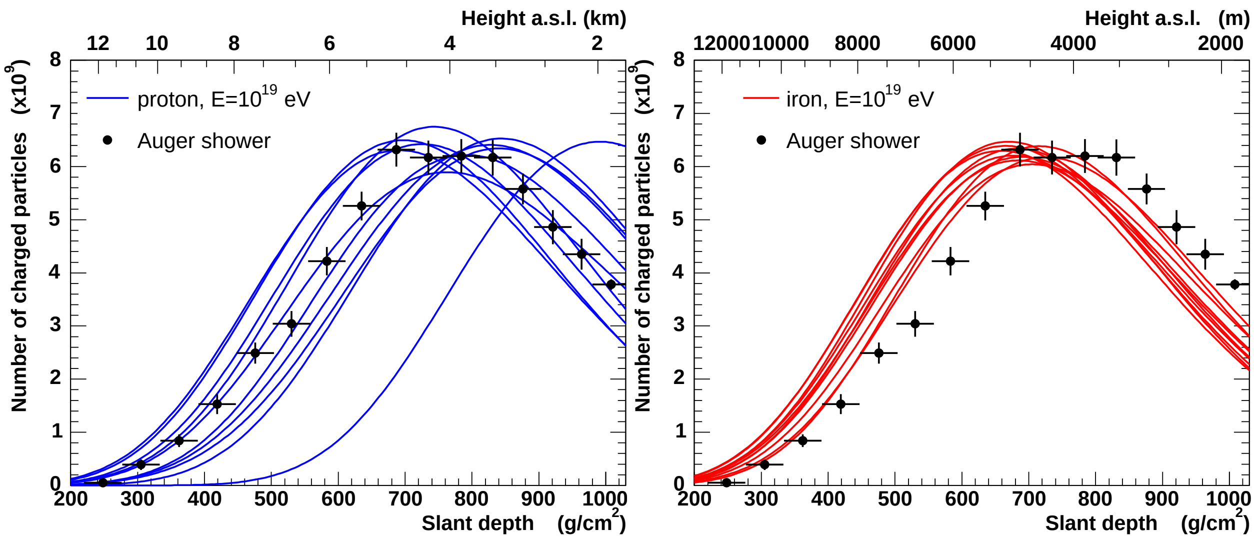

Figure 1: Longitudinal particle profiles for proton (left) and iron (right) induced showers simulated with CORSIKA [5] and compared to one event measured by the Pierre Auger Observatory.