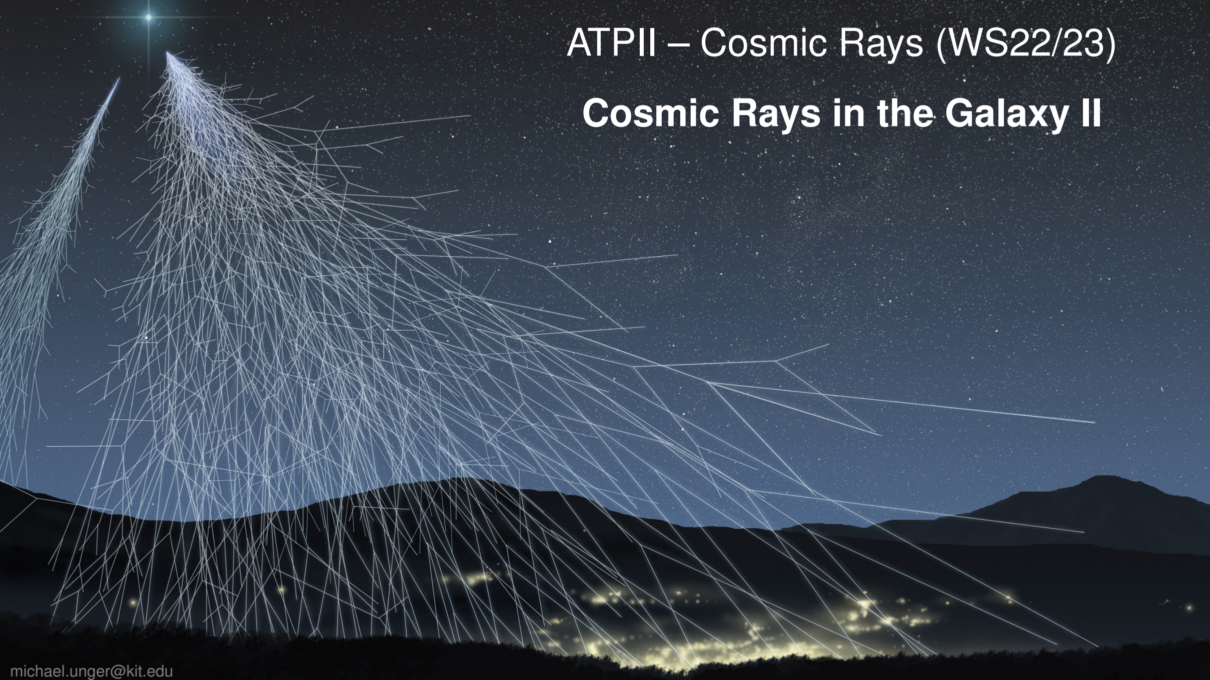
ATPII – Cosmic Rays (WS22/23)

Cosmic Rays in the Galaxy II



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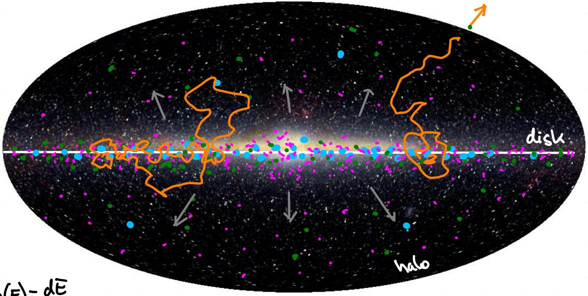
Propagation of Cosmic Rays in the Galaxy

- CR density n_i
- sources Q_i
- interstellar medium ϱ
- diffusion $D_i \tau_{esc}$
- energy loss/gain $b(E) = \frac{dE}{dt}$



http://map.gsfc.nasa.gov

- interactions:

  $A_i + p \to A_k + X$

  $\Rightarrow$ loss + gain

- decay:

  $A_i \to A_k + X$

  $\Rightarrow$ (mostly) loss

- advection $\vec{u}$ of cosmic rays

  e.g. galactic wind

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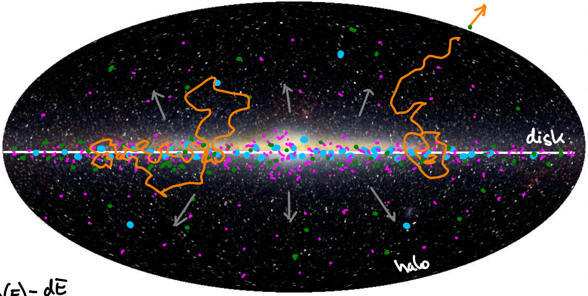

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transport equation $n, Q = f(\mathbf{x}, E, t), \rho = f(\mathbf{x})$

$$\frac{\partial n_i}{\partial t} = Q_i + \nabla(D_i \nabla n_i) - \frac{\partial}{\partial E}(b_i(E) n_i) - \nabla \mathbf{u}\, n_i - p_i n_i + \frac{v\,\rho}{m} \sum_{k \geq i} \int \frac{\mathrm{d}\sigma_{ik}(E, E')}{dE} n_k(E') \mathrm{d}E'$$

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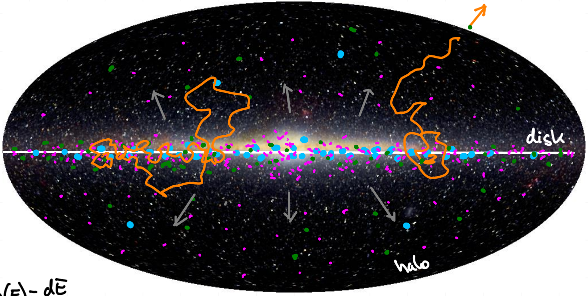

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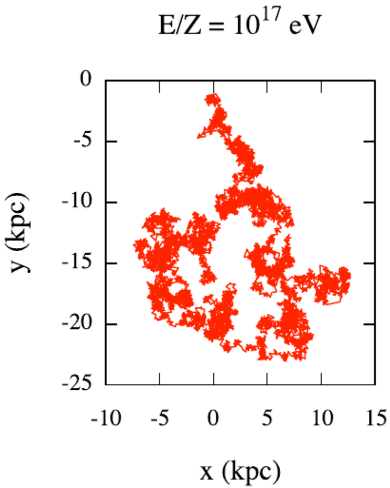
Scattering of Particles in Magnetic Fields

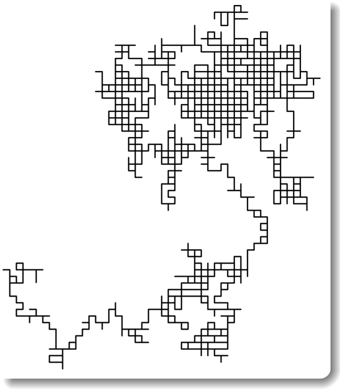
see lecture 3



$\vec{B} = (0, 0, 3)\ \mu\text{G}$, $b_{\text{rms}} = 1\ \mu\text{G}$, arXiv:1305.4364

2D Random Walk

$n = 2500$



wikipedia

Scattering of Particles in Magnetic Fields

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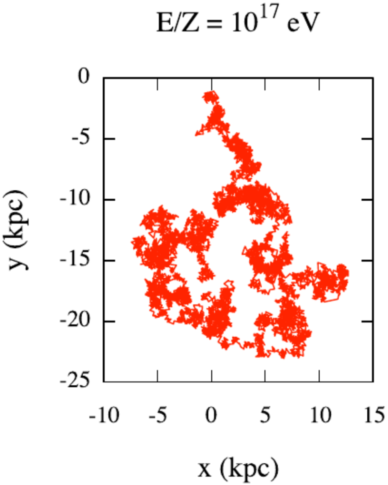

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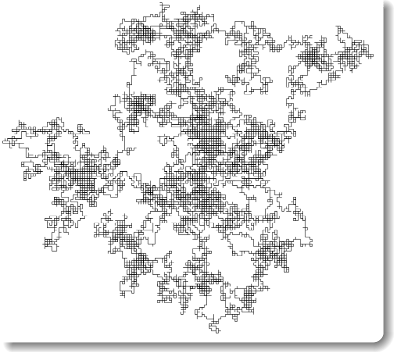
2D Random Walk

$$n = 25000$$



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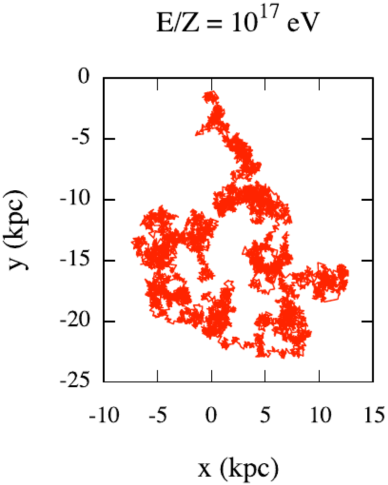

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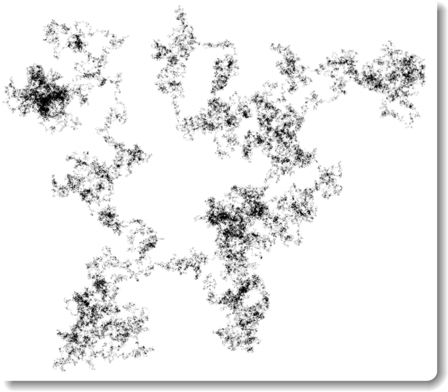
2D Random Walk

$$n = 2 \times 10^6$$



wikipedia

Random Walk

"scattering length" λ

- e.g. 1D coin flip $\quad z = \begin{cases} \lambda & p=0.5 \\ -\lambda & p=0.5 \end{cases}$

  or Gaussian step

- after n steps: $\quad X_n = \sum_{i=1}^{n} z_i$

- expectation value:

$$E(X_n) = E\left(\sum z_i\right) = \sum E(z_i) = 0$$

$$E(X_n^2) = E\left(\left(\sum z_i\right)^2\right) = E\left(\sum z_i^2 + 2\sum_{i<j} z_i z_j\right)$$

$$= E\left(\sum z_i^2\right) + 2\underbrace{E\left(\sum z_i z_j\right)}_{0} \overset{\text{coin}}{=} n\cdot\lambda^2$$

- three dimensions: $\quad E(r^2) = E(x^2) + E(y^2) + E(z^2) = n\cdot\lambda^2\cdot 3$
- d dimensions: $\quad E(r^2) = n\cdot d\cdot\lambda^2 \quad$ or $\quad n\lambda_r^2 \quad \left(\lambda = \frac{\lambda_r}{\sqrt{d}}\right)$

expectation value:

X, Y random variables (rVs)

$$E(x) = \sum x\, p(x)$$

$$E(x+y) = E(x) + E(y)$$

$$E(a\cdot x) = a\cdot E(x)$$

$$E(x\cdot y) = E(x)\,E(y) \quad \text{if } x \text{ and } y \text{ \underline{uncorrelated} rvs}$$

diffusion coefficient D:

$$\langle r^2\rangle \equiv 2\cdot d\cdot D t$$

$$\Rightarrow D = \frac{n\cdot\lambda \;\; \lambda\cdot d}{2\, t \;\; d}$$

$\rightarrow v!$

$$\boxed{D = \frac{\lambda v}{2}}$$

units: length²/time, e.g. $\frac{cm^2}{s}$

Diffusion

equivalent: $\frac{dN}{dt} + \oint \vec{j}\, d\vec{S} = 0$

- Continuity equation: $\frac{\partial n}{\partial t} + \nabla \vec{j} = 0$ (1)

change of density with time

in/outgoing flux of particles

- Fick's first law:

(phenomenological definition of D)

$\vec{j}_{diff} = -D \nabla n$ (2)

in general $D = f(\vec{x})$!

diffusive flux

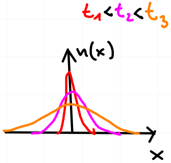
- (2) in (1) $\boxed{\frac{\partial n}{\partial t} = \nabla (D \nabla n)}$

D = const $\boxed{\frac{\partial n}{\partial t} = D \nabla^2 n}$

diffusion equation

- example: 1D diffusion equation

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2} \quad (3)$$



1D coin flip:

central limit theorem:

sum of independent rvs → gaussian pdf

mean $\mu = 0$

variance $\sigma^2 = n \cdot \lambda^2 = \lambda \cdot v \cdot t = 2 \cdot D \cdot t$

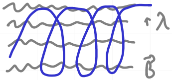
$$n(x,t) \sim \sqrt{\frac{1}{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$= \sqrt{\frac{1}{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right) \quad (4)$$

(4) in (3) → solution to 1D diff. eq.!

Diffusion of charged particles in turbulent magnetic fields

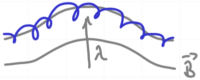
$r_L \gg \lambda$



recap: (lecture 2+3)

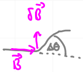
- charged particle in $\vec{B}$ → spiral path, pitch angle $\alpha = \measuredangle(\vec{v}, \vec{B})$

$r_L \ll \lambda$



- Larmor radius / gyro radius: $r_L = \frac{R}{Bc}$ with rigidity $R = \frac{pc}{z \cdot e}$ for $\alpha = \frac{\pi}{2}$
- perturbation $\delta\vec{B}$ length scale λ ⇒ scattering if $r_L \sim \lambda$

- diffusion coefficient (order of magnitude)



→ inclination of disturbance wrt ambient field B $\Delta\theta \sim \delta B / B$

→ pitch angle scattering $\Delta\alpha \sim \Delta\theta$

→ random scattering by 1 radian → $\sqrt{n} \cdot \Delta\alpha = 1$, $n = \frac{1}{\Delta\alpha^2}$

→ scattering length $\lambda_{sc} = n \cdot \lambda \overset{\text{resonant}}{\approx} n \cdot r_L = \frac{r_L}{\Delta\alpha^2} = \frac{r_L}{(\delta B / B)^2}$

→ turbulent cascade (lecture 3)

$E(k) \sim k^{-\beta}$

→ magnetic field at resonance

$\delta B^2 \sim B^2(k) \cdot k \sim k^{-\beta} \cdot k$ at $k \approx \frac{1}{r_L}$

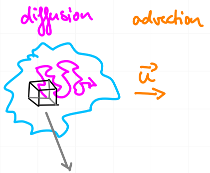
⇒ $\underline{D \sim \lambda_{scatt} \sim r_L^{-\beta+2}}$

⇒ Kolmogorov, $\beta = \frac{5}{3}$ $D \sim R^{1/3}$

⇒ Kraichnan, $\beta = \frac{3}{2}$ $D \sim R^{1/2}$

Advection

bulk motion of fluid / plasma, velocity $\vec{u}$



particle density n in stationary volume element

e.g.: • oil spill in ocean diffuses and is advected by water flow
• galactic wind transports plasma away from plane

→ add advective flux into continuity equation $\vec{j} = \vec{j}_{diff} + \vec{j}_{adv}$

where $\vec{j}_{adv} = \vec{u} \cdot n$

→ using again Fick's 1st law we obtain the

diffusion-advection equation $$\boxed{\frac{\partial n}{\partial t} = D \nabla^2 n - \vec{v} \nabla n}$$ ($D = \text{const}$, $\vec{v} = \text{const}$)

Particle Decay

loss of particles of type a $$\boxed{\frac{\partial n_a}{\partial t} = -\frac{1}{\gamma \tau_a} n_a}$$

• Lorentz-factor $\gamma = \frac{E_a}{m_a}$
• lifetime τ_a

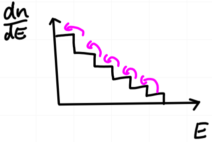
(can be source term of particle type b, energy E_b)

Energy Loss

change of energy: $-\frac{dE}{dt} = b(E)$ e.g. energy loss due to ionization (see lecture II)

(but also: energy gain if $b<0$, e.g. re-acceleration)

at time t: $n(E)\,\Delta E$ particles in $[E, E+\Delta E]$



at time $t+\Delta t$: replaced by particles in $[E', E'+\Delta E']$

$$E' = E + b(E)\,\Delta t\,, \quad E'+\Delta E' = (E+\Delta E) + b(E+\Delta E)\,\Delta t$$

e.g. Longair 7.5.1

$\Rightarrow$ diffusion-loss equation $\boxed{\frac{\partial n}{\partial t} = \frac{d}{dE}\big(b(E)\,n\big)}$

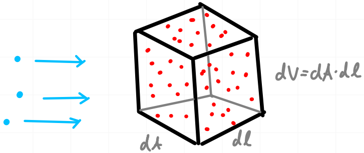
Particle Absorption

$\boxed{a+b\to X}$

particles type b, mass density ρ_b

particles type a, flux ϕ_a

"projectiles"



"target"

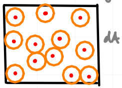
number of target particles:

$$dN_b = \frac{\rho_b}{m_b}\, dA\, dl \qquad (\text{number density } \rho/m)$$

particle absorption:

$$\frac{dN_a}{dt} = -\underbrace{\overset{\text{cross section}}{\sigma_{abs}}\,\frac{\rho_b}{m_b}\, dA\, dl}_{\text{"area" of target } dN_b\cdot\sigma}\;\phi_a$$

face-on view of target



all directions: $\phi_a = \frac{1}{4\pi}\frac{dN}{dt\,dA}$ ⇒ see lecture 1

particle density: $n = 4\pi/\beta\cdot c\;\phi_a$

$$\Rightarrow \quad \frac{\partial n_a}{\partial t} = -\sigma_{abs}\,\frac{\rho_b}{m_b}\,\beta\cdot c\cdot n$$

$$\boxed{\frac{\partial n_a}{\partial t} = -\frac{\beta c \rho}{\lambda_{abs}}\cdot n}$$

with absorption length $\lambda_{abs} = \frac{m_b}{\sigma_{abs}}$

(units: $[\lambda_{abs}]$ = mass/area
e.g. g/cm² → "column depth")

Particle Production

$\boxed{a+b\to j}$ $(+X)$

e.g. nuclear fragmentation

$$\underset{a}{{}^{12}C_{CR}} + \underset{b}{p_{ISM}} \to \underset{j}{{}^{11}B_{CR}} + \underbrace{p_{CR} + p_{ISM}}_{X}$$

$\frac{E_a}{m_a} \approx \frac{E_j}{m_j}$ energy per nucleon conserved

e.g. inelastic collision

$$\underset{a}{p_{CR}} + \underset{b}{p_{ISM}} \to \underbrace{p+p+p}_{X} + \underset{j}{\bar{p}} + \underbrace{\pi^+ + \pi^- + \pi^0 + \dots}_{X}$$

$E_a > E_j$

e.g. anti-proton production

number of secondary particles produced by a beam of particles (all energies) in target volume: $\frac{dN_j}{dt\,dV} = \underbrace{\sigma_{a,b\to j}}_{\text{inclusive production cross section}} \frac{\rho_b}{m_b} \phi_a$

isotropic projectile flux, energy spectrum: $\phi_a(E_a) = \frac{1}{4\pi}\frac{dN_a}{dE\,dt\,dA}$

$$\frac{dN_j}{dE_j\,dt\,dV} = 4\pi \frac{\rho_b}{m_b} \int \frac{d\sigma_j(E_a)}{dE_j} \phi(E_a)\,dE_a$$

or

using $n = \frac{4\pi}{\beta c}\phi$

$$\boxed{\frac{dn_j}{dE\,dt} = \frac{\rho_b}{m_b} \int \frac{d\sigma_j}{dE_j} \beta_a c\, n_a(E_a)\,dE_a}$$

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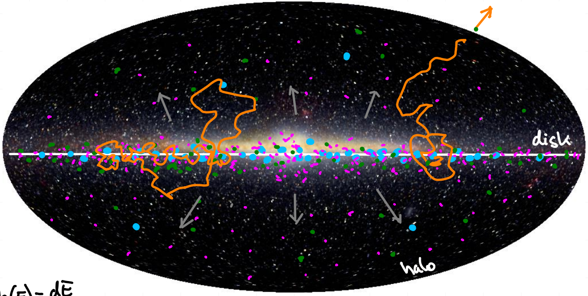

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source, diffusion, energy loss/gain, advection, absorption + decay, production

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$p_i = \frac{1}{\gamma \tau_i} + \frac{\beta c \rho}{\lambda}$

sum over parent particles