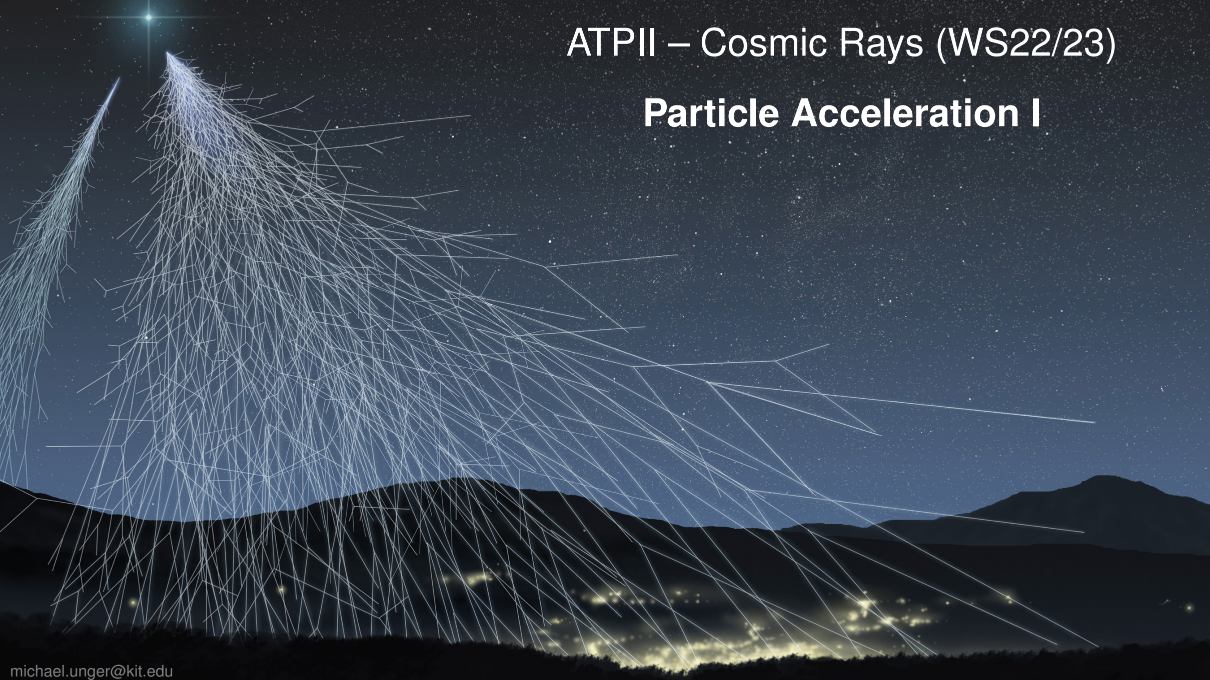


ATPII – Cosmic Rays (WS22/23)

Particle Acceleration I

michael.unger@kit.edu

PREVIOUSLY ON... Cosmic Rays

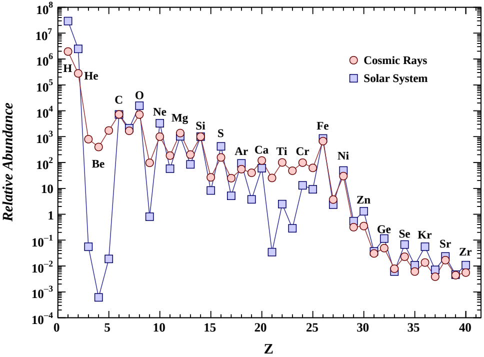


Figure 30.2: Cosmic ray elemental abundances compared to abundances in present-day solar system material. Abundances are normalised to Si=10^3. Cosmic ray abundances are from AMS-02 (H,He) [3,17], ACE/CRIS (Li-Ni) [18,19], and TIGER/SuperTIGER (Cu-Zr) [20,21]. Solar system abundances are from Table 6 of Ref. [22].

Particle Data Group 2022

- **Galaxy:**
  $\rightarrow$ thin disk $\rho_{\rm ISM} \approx 1\ {}^{m_p}/{\rm cm}^3$
  $\rightarrow$ turbulent magnetic fields
- **local CR energy density**
  $\rightarrow u_{\rm CR} \approx 1\ {}^{\rm eV}/{\rm cm}^3$
  $\rightarrow {\rm d}u_{\rm CR}/{\rm d}R \propto R^{-\gamma}$
- **secondary/primary ratios:**
  $\rightarrow$ CR "grammage" $\lambda = \int \rho(l){\rm d}l$
  $\rightarrow \lambda(R) \propto R^{-\delta}$, $\delta \sim {}^1/_3 \ldots {}^1/_2$
- **cosmic clocks:**
  $\rightarrow$ CR escape time $\tau_{\rm esc}$

$\Rightarrow$ CR diffusion in Galactic halo
$\Rightarrow$ injection spectrum $Q \propto R^{-\alpha}$
$\Rightarrow \alpha = \gamma - \delta = 2.1 \ldots 2.4$
$\Rightarrow Q_{\rm CR} \approx 10^{41}{}^{\rm erg}/_{\rm s} \sim 0.1 Q_{\rm SN}$

Cosmic-Ray Energy Spectrum

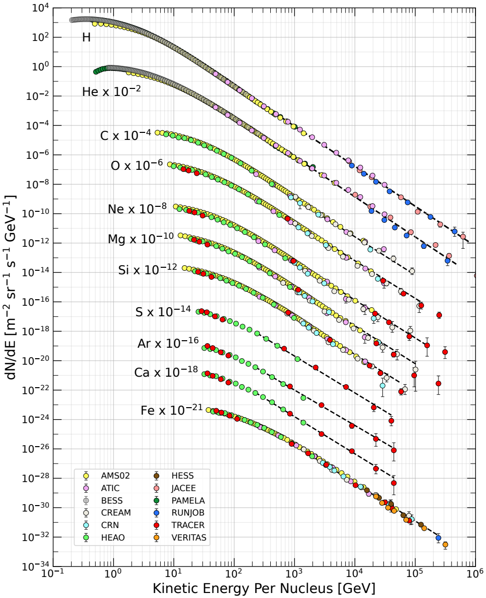


Figure 30.1: Fluxes of nuclei of the primary cosmic radiation in particles per energy-per-nucleus are plotted vs energy-per-nucleus using data from Refs. [1–15] The inset shows the H/He ratio as a function of rigidity [1,3].

- origin of power laws? ($\Phi \propto R^{-\alpha-\delta}$)
- value of spectral index? ($\alpha \sim 2.1 \ldots 2.4$)
- maximum rigidity?
- features? (knee, 2nd knee,...)

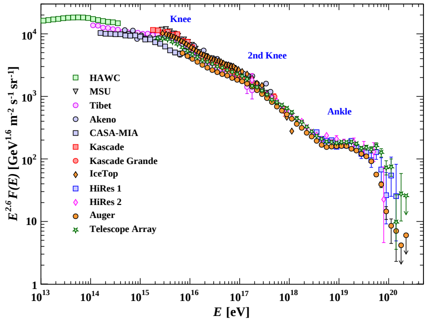


Figure 30.9: The all-particle spectrum as a function of E (energy-per-nucleus) from air shower measurements [106–119]

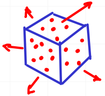
Origin of Power-Law Energy Spectra

differential spectrum

$$\boxed{\frac{dN}{dE} \sim E^{-\alpha}}$$

stochastic acceleration

accelerating region



escaping particles

integral spectrum

$$N(>E) \sim E^{-\alpha+1}$$

- energy gain during Δt while inside region:

$$E(t+\Delta t) = E(t)\,(1+\varepsilon)$$

- probability of escape during Δt : P

$\Rightarrow$ prob. after $n \cdot \Delta t$: $(1-P)^n$

- energy after $t = n\cdot\Delta t$:

$$E(t) = E_0(1+\varepsilon)^n$$

- to reach energy E :

$$\underline{n = \ln(E/E_0)/\ln(1+\varepsilon)} \quad (*)$$

$$N(>E) \sim \sum_{m=n}^{\infty} (1-P)^m$$

$$= (1-P)^n \sum_{m=n}^{\infty} (1-P)^{m-n}$$

$$= (1-P)^n \sum_{k=0}^{\infty} (1-P)^k = \underline{\frac{(1-P)^n}{P}} \quad (**)$$

$\left(k = m-n,\ \sum_{k=0}^{\infty} x^k = \frac{1}{1-x}\right)$

$(*)$ in $(**)$ $\Rightarrow$ $\boxed{N = \frac{1}{P}\left(\frac{E}{E_0}\right)^{-\alpha+1}}$

$\left(a^{\ln b} = b^{\ln a}\right)$

with spectral index $\boxed{\alpha - 1 = \ln\left(\frac{1}{1-P}\right) \Big/ \ln(1+\varepsilon)}$

$\boxed{\alpha \approx 1 + \frac{P}{\varepsilon}}$ $\quad (\ln(1+x) \approx x)$

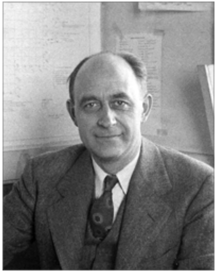
Fermi-Acceleration

PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

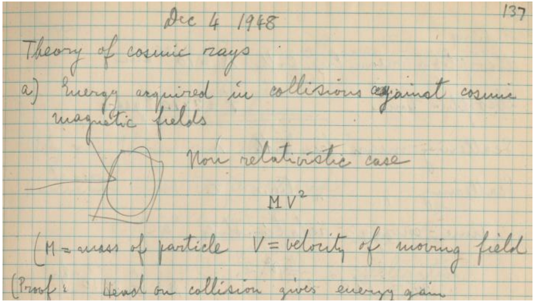
On the Origin of the Cosmic Radiation

Enrico Fermi
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays.



www.aip.org



www.symmetrymagazine.org

see also popularization in Frank Capra's "The Strange Case of Cosmic Rays" (1957) `www.youtube.com/watch?v=k_wt5AFjRQo` at around 00:45:00

Fermi Acceleration (2nd order)

- test particle: $m \ll M$, $v = c$, $E_1 \approx p_1$

- magnetized cloud: $\beta = \frac{V}{c}$

- particle energy in rest frame of cloud:

$$E_1' = \gamma E_1 (1 - \beta \cos\theta_1)$$

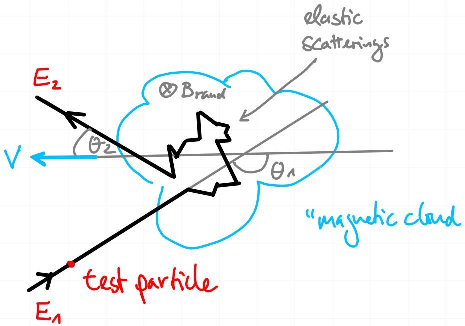
- elastic scattering: $E_1' = E_2'$

- back-transformation to lab-system:

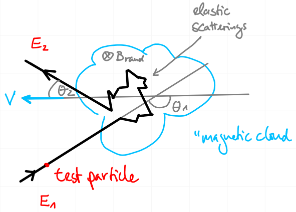
$$E_2 = \gamma E_2' (1 + \beta \cos\theta_2') = \gamma^2 E_1 (1 - \beta \cos\theta_1)(1 + \beta \cos\theta_2')$$

- energy change:

$$\varepsilon = \frac{E_2 - E_1}{E_1} = \frac{1 - \beta\cos\theta_1 + \beta\cos\theta_2' - \beta^2 \cos\theta_1 \cos\theta_2'}{1 - \beta^2} - 1$$



Fermi Acceleration (2nd order)



- energy change: $\varepsilon = \dfrac{E_2 - E_1}{E_1} = \dfrac{1 - \beta\cos\theta_1 + \beta\cos\theta_2' - \beta^2\cos\theta_1\cos\theta_2'}{1-\beta^2} - 1$

- average energy change:

1) θ_2' isotropic $P(\cos\theta_2') = \text{const}, \ \langle\cos\theta_2'\rangle = 0$

$$\rightarrow \langle\varepsilon\rangle_{\theta_2'} = \frac{1-\beta\cos\theta_1}{1-\beta^2} - 1$$

2) number of cloud encounters ~ relative velocity

$$P(\cos\theta_1) \sim 1 - \beta\cos\theta_1$$

$$\rightarrow \langle\cos\theta_1\rangle = \int_{-1}^{1} x\, P(x)\,dx \Big/ \int_{-1}^{1} P(x)\,dx$$

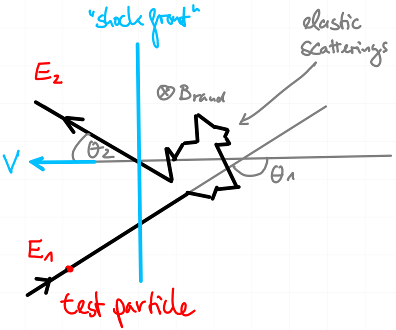
$$= \left[\tfrac{1}{2}x^2 - \tfrac{1}{3}\beta x^3\right]_{-1}^{1} \Big/ \left[x - \tfrac{1}{2}\beta x^2\right]_{-1}^{1}$$

$$= -\tfrac{2}{3}\beta / 2 = -\tfrac{1}{3}\beta$$

$$\Rightarrow \langle\varepsilon\rangle = \langle\langle\varepsilon\rangle_{\theta_2'}\rangle_{\theta_1} = \frac{1+\frac{1}{3}\beta^2}{1-\beta^2} - 1 = \frac{4}{3}\frac{\beta^2}{1-\beta^2}$$

$\beta \ll 1$ $\quad \boxed{\langle\varepsilon\rangle = \frac{4}{3}\beta^2}$ $\quad$ 2nd order in β → very slow!

Fermi Acceleration (1st order)



- $E_1 \to E_1' \to E_2' \to E_2$ exactly as before

$$\varepsilon = \frac{E_2 - E_1}{E_1} = \frac{1 - \beta\cos\theta_1 + \beta\cos\theta_2' - \beta^2\cos\theta_1\cos\theta_2'}{1-\beta^2} - 1$$

- average of $\cos\theta_2'$ and $\cos\theta_1$: projection of isotropic flux onto plane, $P(\cos\theta) \sim \cos\theta$

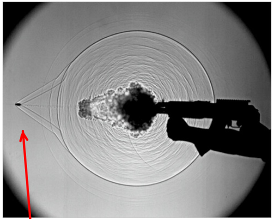
$$\Rightarrow \langle \cos\theta_2' \rangle = \int_0^1 x\,P(x)\,dx \Big/ \int_0^1 P(x)\,dx = \left[\tfrac{1}{3}x^3\right]_0^1 \Big/ \left[\tfrac{1}{2}x^2\right]_0^1 = \frac{2}{3}$$

$$\Rightarrow \langle \cos\theta_1 \rangle = \int_{-1}^0 x\,P(x)\,dx \Big/ \int_{-1}^0 P(x)\,dx = -\frac{2}{3}$$

- average energy change: $\langle \varepsilon \rangle = \dfrac{1 + \frac{4}{3}\beta + \frac{4}{9}\beta^2}{1-\beta^2} - 1$, $\beta \ll 1$ $\boxed{\langle \varepsilon \rangle = \frac{4}{3}\beta}$ 1st order in β!

Shock Waves

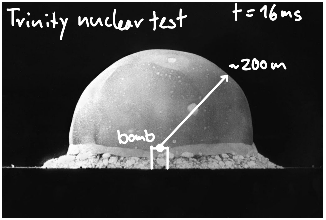
spherical blast wave



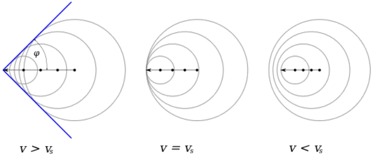
American Scientist **94** (2006) 22

Mach cone

bbc.co.uk

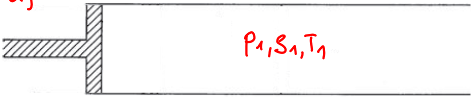


wikipedia



wikipedia

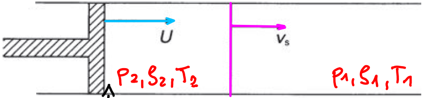
a)



piston moving at velocity $v = U$

shock wave, $v = v_s$

b)

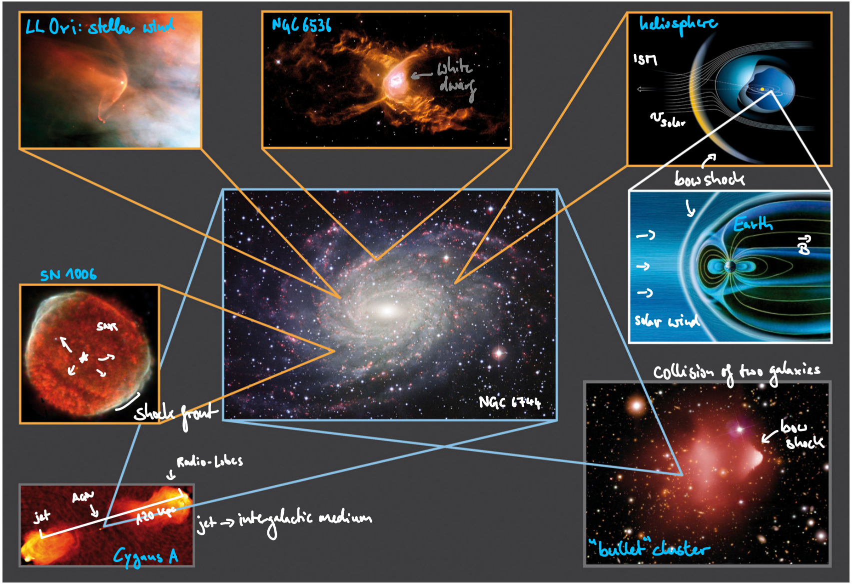


$P_2 > P_1, \rho_2 > \rho_1, T_2 > T_1$

'swept up material'

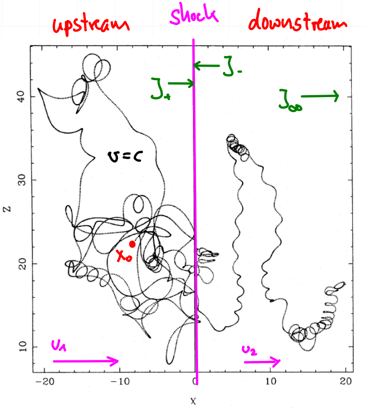
M.S. Longair, "High Energy Astrophysics"

Shock Waves in Astrophysics



Diffuse Shock Acceleration (DSA)

Shock rest frame:



K.R. Ballard and A.F. Heavens, MNRAS **259** (1992) 89

- magnetic scattering centers
- diffusion: $\langle (x-x_0)^2 \rangle = 2Dt$, $\langle x \rangle = x_0$

  (D: diffusion coefficient)
- advection: $\langle x \rangle = x_0 + u \cdot t$
- upstream: advection always 'wins',

  → particle always returns to the shock
- downstream: advection away from the shock

  $J_\infty = n_0 u_2$

  (downstream particle density n_0)
- downstream isotropic flux $\frac{c \cdot n_0}{4\pi}$
- downstream → upstream: $J_- = \int_0^{2\pi} d\varphi \int_0^1 \frac{c \cdot n_0}{4\pi} \cos\theta \, d\cos\theta = \frac{c n_0}{4}$
- $J_+ = J_- + J_\infty$, $P_{esc} = \frac{J_\infty}{J_+} = \frac{J_\infty}{J_- + J_\infty} = \frac{u_2}{c/4 + u_2} \approx \underline{4u_2/c}$

Diffuse Shock Acceleration (DSA)

- plug into results for stochastic acceleration

- $\alpha - 1 = \ln\left(\frac{1}{1-P_{esc}}\right) / \ln(1+\varepsilon)$ integral spectral index

  $\approx P_{esc}/\varepsilon$

DSA:

- $\langle \varepsilon \rangle = \frac{4}{3}\beta = \frac{4}{3}\frac{u_1 - u_2}{c}$ average energy gain

- $P_{esc} = 4u_2/c$ escape probability

$\Rightarrow$ Spectral index: $\frac{4}{3}\frac{u_1-u_2}{c\,4u_2/c} = \frac{1}{3}\left(\frac{u_1}{u_2}-1\right)$

- $\alpha - 1 = \frac{3}{u_1/u_2 - 1}$

$M = \frac{u}{c}$ ← flow velocity, ← speed of sound

- Strong shocks (Mach number $M \gg 1$)

$$\frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{\gamma+1}{\gamma-1}$$

$\frac{\gamma+1}{\gamma-1} = \frac{8/3}{2/3} = 4$

- ideal gas, adiabatic index $\gamma = \frac{5}{3} \rightarrow \frac{u_1}{u_2} = 4$

($\gamma = c_p/c_v$; heat capacities c at constant pressure/volume)

$\alpha = \frac{3}{4-1} + 1$

$\Rightarrow \boxed{\alpha = 2}$

$$\boxed{\left(\frac{dn}{dE}\right)_{DSA} \sim E^{-2}}$$

(does not depend on details of shock or diffusion!)