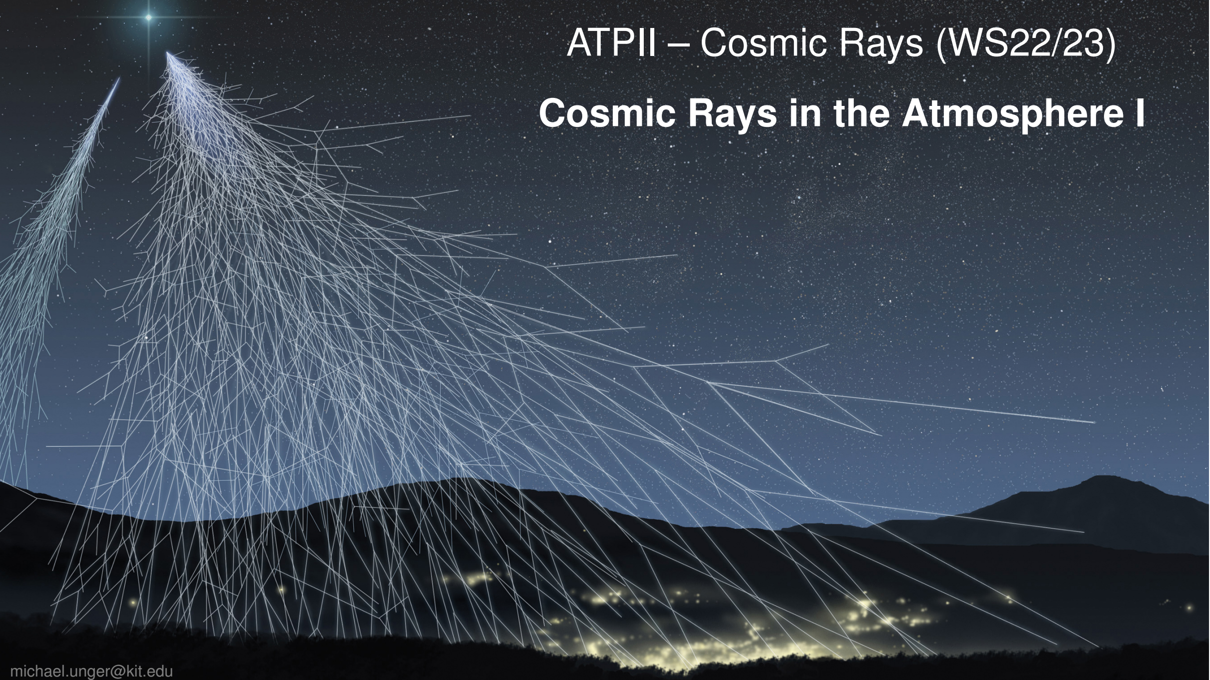
ATPII – Cosmic Rays (WS22/23)

Cosmic Rays in the Atmosphere I



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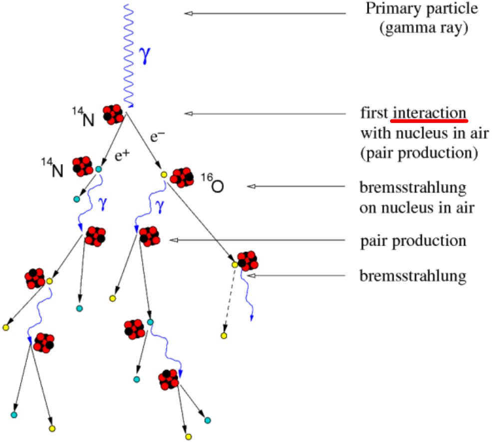
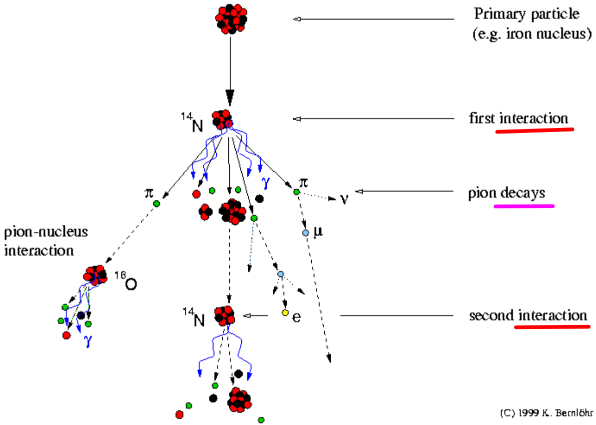
Particle Cascade in the Atmosphere / Air Shower





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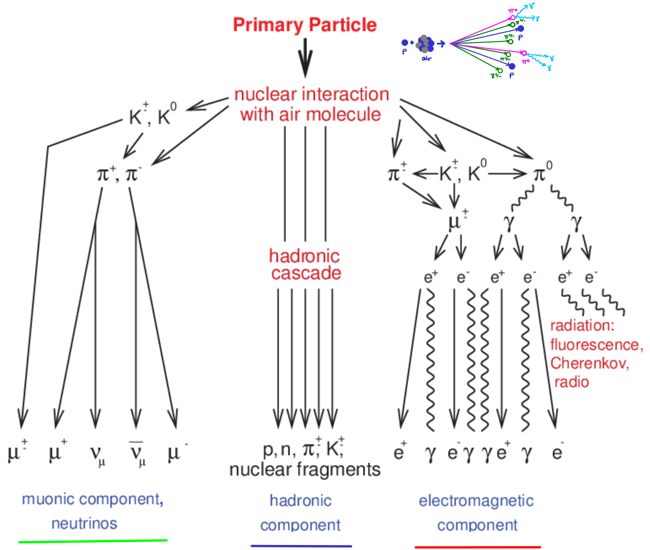
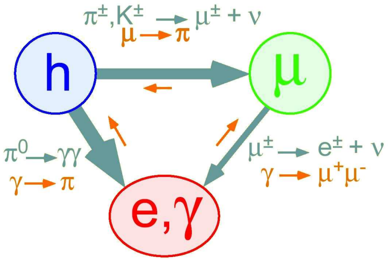
Particle Cascade in the Atmosphere / Air Shower





⇒ complicated coupled particle transport through atmosphere

⇒ numerical solutions or Monte Carlo

e.g. CORSIKA (dev. at IAP!)

$\boxed{E = 10^{11}\ \mathrm{eV}}$

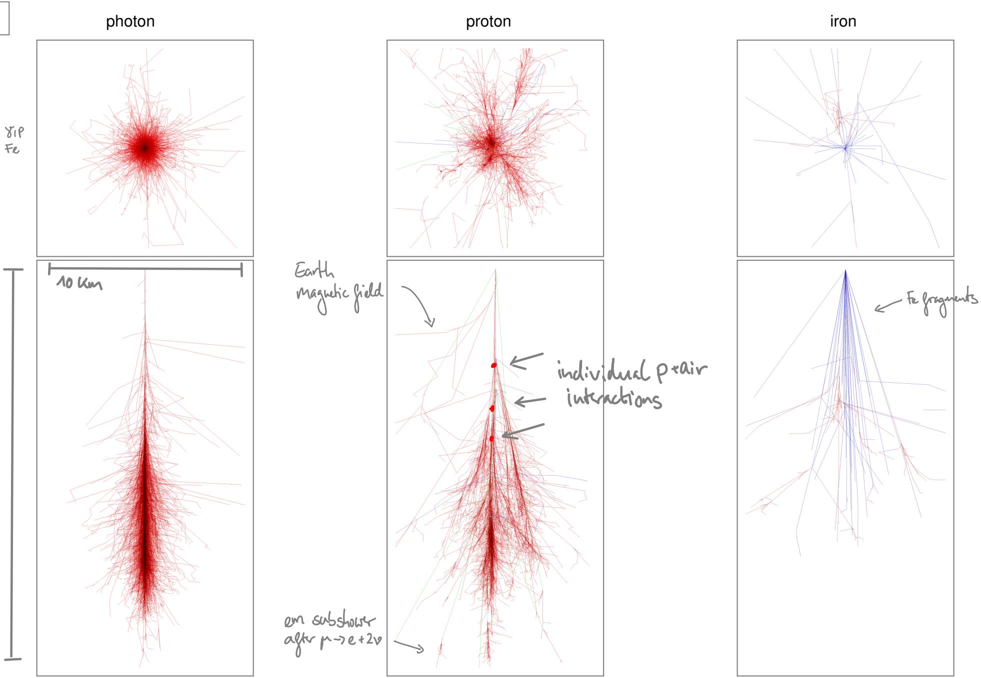
$\downarrow$

E_{tot}!

$\Rightarrow E_{kin} = E_{tot} - Mc^2$

$\approx \begin{cases} 100\ \mathrm{GeV} & \gamma, p \\ 44\ \mathrm{GeV} & \mathrm{Fe} \end{cases}$

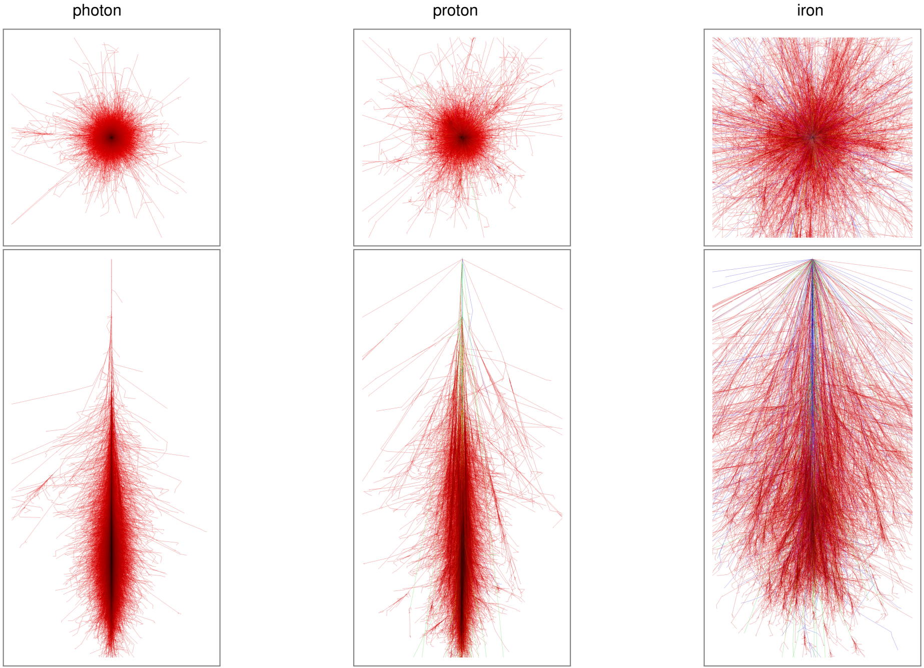
$\Rightarrow E_{kin}/n < 1\ \mathrm{GeV}$ for Fe



electrons, positrons, gammas muons hadrons; height: 30 km, width: ± 5 km

https://www-zeuthen.desy.de/~jknapp/fs/iron-showers.html

$E = 10^{12}$ eV



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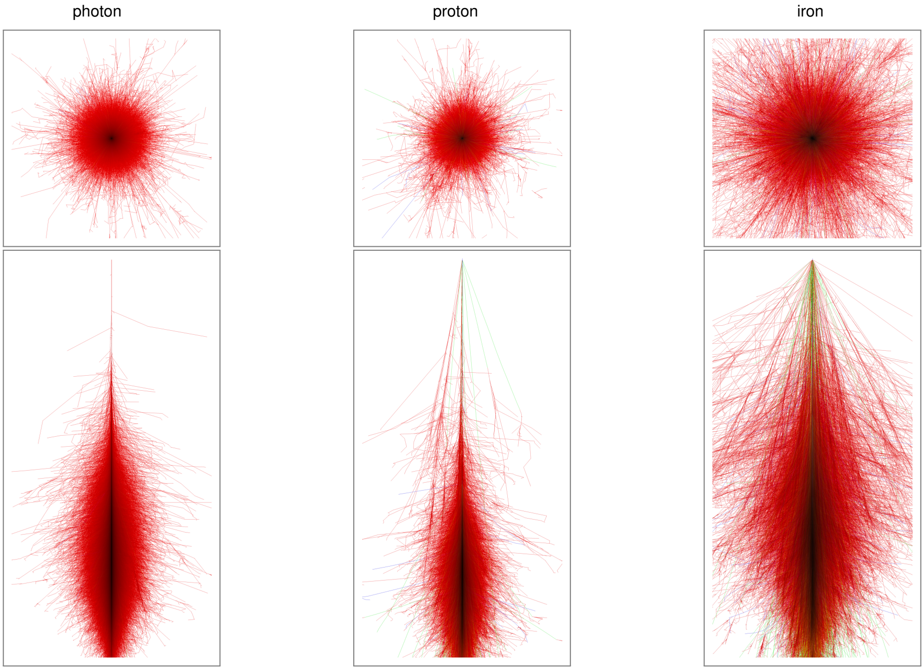
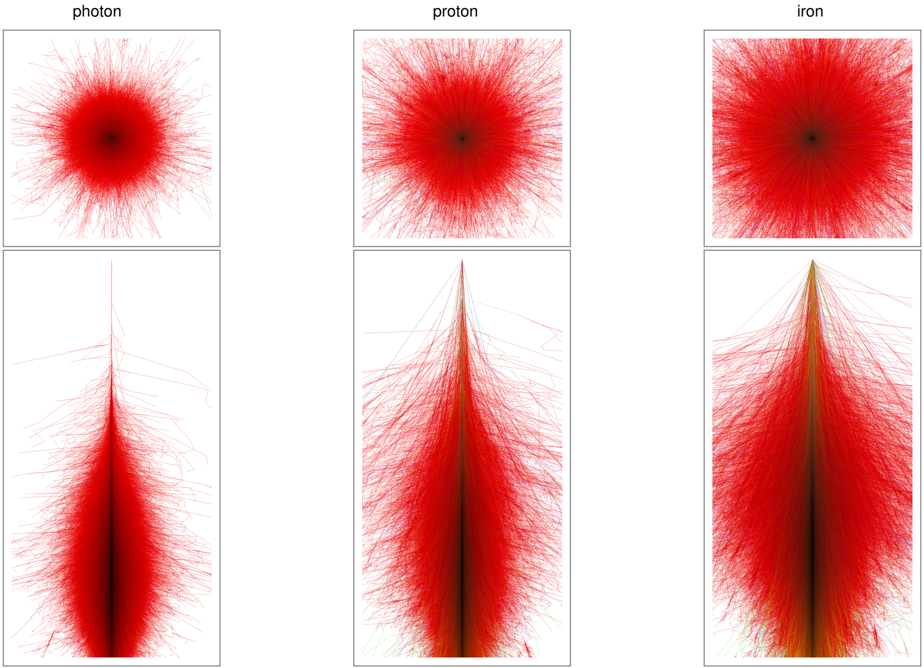


electrons, positrons, gammas muons hadrons; height: 30 km, width: ± 5 km

https://www-zeuthen.desy.de/~jknapp/fs/iron-showers.html

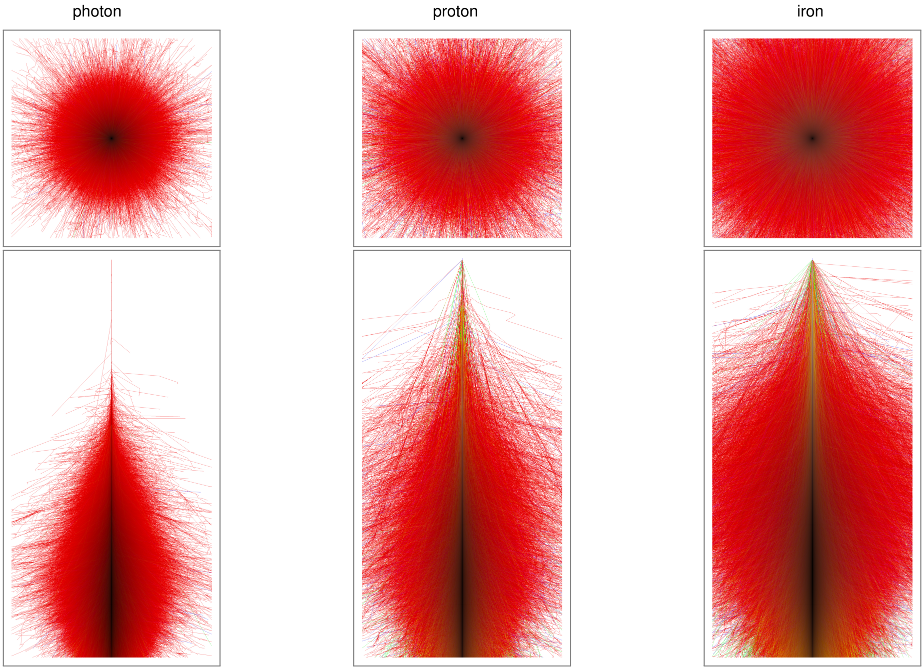
$E = 10^{14}$ eV



electrons, positrons, gammas muons hadrons; height: 30 km, width: ± 5 km

https://www-zeuthen.desy.de/~jknapp/fs/iron-showers.html

$E = 10^{15}$ eV



electrons, positrons, gammas muons hadrons; height: 30 km, width: ± 5 km

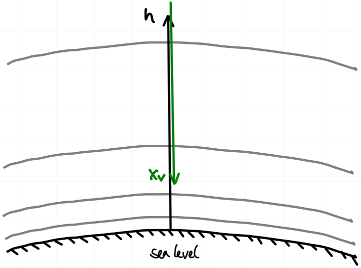
`https://www-zeuthen.desy.de/~jknapp/fs/iron-showers.html`

Atmosphere

- height above sea level h
- air density $\rho(h)$
- vertical depth X_v

$$X_v = \int_h^\infty \rho(h')\,dh'$$

$[x_v] = g/cm^2 \Rightarrow$ "grammage"



- isothermal atmosphere:

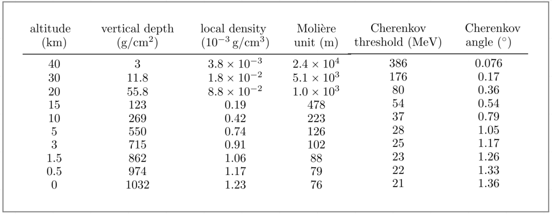
$$\rho(h) = \rho_0\, e^{-h/h_0}$$

$$X_v = X_0\, e^{-h/h_0}$$

- $X_0 \approx 1030\ g/cm^2$ at sea level

- scale height $h_0 \approx 8.4$ km at sea level, ≈ 6.4 km high altitudes above $h \approx 10$ km

h X_v $\rho(h)$ $\underbrace{}$ lateral spread due to Coulomb scattering $\underbrace{}$ see lecture 2



altitude (km)	vertical depth (g/cm^2)	local density (10^{-3} g/cm^3)	Molière unit (m)	Cherenkov threshold (MeV)	Cherenkov angle (°)
40	3	3.8×10^{-3}	2.4×10^4	386	0.076
30	11.8	1.8×10^{-2}	5.1×10^3	176	0.17
20	55.8	8.8×10^{-2}	1.0×10^3	80	0.36
15	123	0.19	478	54	0.54
10	269	0.42	223	37	0.79
5	550	0.74	126	28	1.05
3	715	0.91	102	25	1.17
1.5	862	1.06	88	23	1.26
0.5	974	1.17	79	22	1.33
0	1032	1.23	76	21	1.36

Atmosphere

- slant depth:

$$X = \int_{\ell}^{\infty} \rho(h(\ell'))\, d\ell'$$

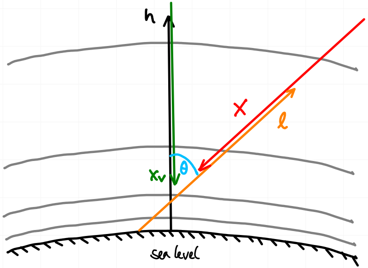
- zenith angle θ $h/\ell = \cos\theta$

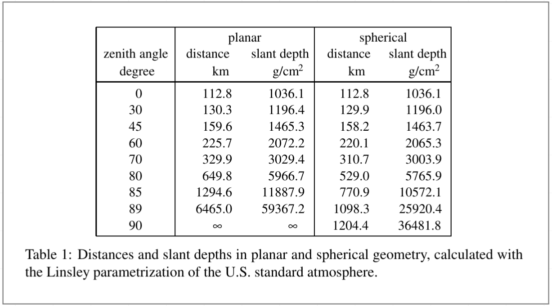
- flat atmosphere approximation for $\theta \lesssim 65°$

$$X = X_v / \cos\theta$$

- horizontal thickness of curved atmosphere:

$$X(\theta = 90°) \approx 3.5 \cdot 10^4 \ g/cm^2$$





zenith angle degree	planar distance km	planar slant depth g/cm^2	spherical distance km	spherical slant depth g/cm^2
0	112.8	1036.1	112.8	1036.1
30	130.3	1196.4	129.9	1196.0
45	159.6	1465.3	158.2	1463.7
60	225.7	2072.2	220.1	2065.3
70	329.9	3029.4	310.7	3003.9
80	649.8	5966.7	529.0	5765.9
85	1294.6	11887.9	770.9	10572.1
89	6465.0	59367.2	1098.3	25920.4
90	∞	∞	1204.4	36481.8

Table 1: Distances and slant depths in planar and spherical geometry, calculated with the Linsley parametrization of the U.S. standard atmosphere.

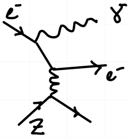
D. Heck FZKA-6954 2004

Electromagnetic Interactions (recap lecture 2)

interactions with nuclei of material (Z)

energy loss

$$\left\langle -\frac{dE}{dx}\right\rangle_{\text{brems, pair}} = \frac{E}{X_0} \quad \leftrightarrow \quad E(x) = E_0 e^{-x/X_0}$$



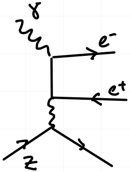
bremsstrahlung

radiation length :

$$X_0 \sim \Big(\underset{\text{projectile}}{\frac{1}{m^2}} \overbrace{\frac{Z^2}{A}\rho}^{\text{material}}\Big)^{-1}$$

⇒ electron radiation length in air :

$$\boxed{X_0^{air} = 36.6\ g/cm^2}$$



pair production

critical energy :

$$\left\langle -\frac{dE}{dx}\right\rangle_{\text{brems}} \sim E$$

$$\left\langle -\frac{dE}{dx}\right\rangle_{\text{ion}} \sim \text{const}$$

$$E_{crit} \text{ when } \left\langle -\frac{dE}{dx}\right\rangle_{\text{brems}} = \left\langle -\frac{dE}{dx}\right\rangle_{\text{ion}}$$

⇒ critical energy in air :

$$\boxed{E_{crit}^{air} = 84\ \text{MeV}}$$

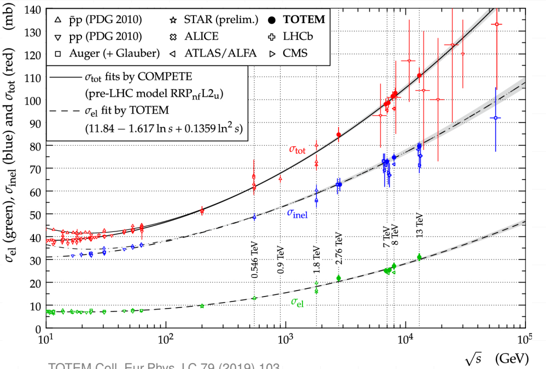
Hadronic Interactions

- charge radius (e+p scattering):

$$r_p = 0.88 \cdot 10^{-15}\ m$$

$\rightarrow \sigma_{pp} \approx (2r_p)^2 \pi \approx 100\ mb$

(b: "barn", $1\ b = 10^{-28}\ m^2$)



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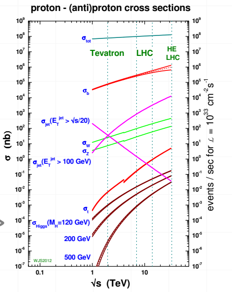
- inelastic cross section: $\sigma_{inel} = \underbrace{\sigma_{tot}}_{\text{total}} - \underbrace{\sigma_{ela}}_{\text{elastic}}$
  ≙ particle production

$\sigma_{inel} \approx 35\ mb$

$(10\ GeV < E_{lab} < 1\ TeV)$



- particle production: $p+p \rightarrow p+p + m\cdot\pi^{+/-} + n\cdot\pi^0$ (and $K^{+/-}$, Λ, K^0, n, Higgs ...)

but...

- pion multiplicity: $m \approx 2\cdot n$, $K_\pi = \dfrac{\Gamma(p+p \rightarrow \pi^{+/-} + X)}{\Gamma(p+p \rightarrow \pi^0 + X)} = \dfrac{m}{n} \approx 2$

Hadronic Interactions

reminder lecture 4:

- interaction length: j+air → X

mass density

$$\lambda_j = \ell_j \varrho = \frac{\varrho}{n_A \sigma_j^{air}} = \frac{\langle A \rangle m_p}{\sigma_j^{air}}$$

$[\lambda] = g/cm^2$ $\quad$ $[\ell] = cm$ $\quad$ number density $\quad$ cross section

- typical values: @ 10 TeV

$\lambda_N \approx 80\ g/cm^2$ p+air/n+air

$\lambda_\pi \approx 100\ g/cm^2$ π+air

- average air mass: $\langle A \rangle = 14.6$ (78.09% N, 20.95% O, 0.93% Ar)
- nucleon + nucleus interactions:

$$\sigma(p+A) \sim A^{2/3}$$

← geometrical size of nucleus with A spherically packed nucleons

- nucleus + nucleus interactions:

$$\sigma(A_1 + A_2) \approx \pi R_0^2 \left(A_1^{1/3} + A_2^{1/3} - \delta\right)^2$$

$(\delta = 1.12,\ R_0 = 1.47\ fm)$

- Glauber model of n+A scattering (see CRPP A6 and Glauber + Matthiae Nucl. Phys. B 21 (1970) 135)

Spectrum-weighted moments

- inclusive cross section: $j + \text{air} \rightarrow a + X$

'Feynman scaling'

- inclusive energy distribution of particles of type a:

$$E_a \frac{1}{\sigma_{inel}} \frac{d\sigma_{ja}(E_p,E_a)}{dE_a} = E_a \frac{dn(E_j,E_a)}{dE_a} \equiv F_{ja}(E_j,E_a) \approx F_{ja}\left(\frac{E_a}{E_j}\right)$$

- 'Z-factor': $\boxed{Z_{pa} = \int_0^1 x^{\gamma-2} F_{ja}(x)\,dx}$ $\left(x = \frac{E_a}{E_j}\right)$

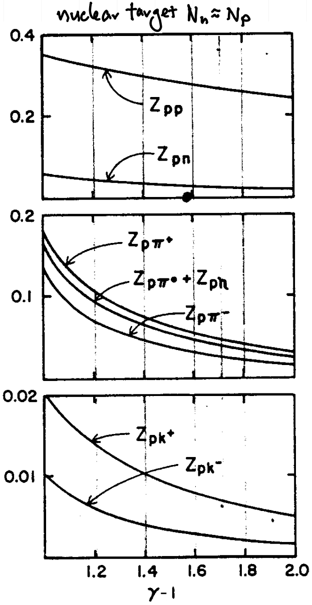
'spectrum weighted moments'

- $\gamma = 2$: Z_{ja} = average fraction of initial energy going to particle type a

- if energy spectrum j is a power law: $\phi_j \sim \frac{dN_j}{dE_j} = N_j^0 \cdot E^{-\gamma}$

$$\phi_a(E_a) = \int_{E_a}^{\infty} \phi_j^0 E_j^{-\gamma} \frac{1}{E_a} F_{ja}\left(\frac{E_a}{E_j}\right) dE_j = \underline{\phi_j^0 Z_{ja} \cdot E_a^{-\gamma}}$$

⇒ see problem set 4!



T.K. Gaisser, "Cosmic Rays & Particle Physics"

Particle Decay

- decay length

air density

decay length in cm

$$d_j = \varrho \, \gamma_j \, \underbrace{c \, \tau_j}$$

← lifetime

$[d] = g/cm^2$

Lorentz factor E/m_j

using $\varrho = -\frac{dX_v}{dh} = \frac{X_v}{h_0} \approx \frac{X \cos\theta}{h_0}$

$$\Rightarrow \quad d_j = \frac{X \cos\theta}{h_0} \; \frac{E c \tau}{m_j c^2}$$

$$\boxed{d_j = \frac{E X \cos\theta}{\varepsilon_j}}$$

$$\varepsilon_j = m_j c^2 \, \frac{h_0}{c \tau_j}$$

e.g.:

$\varepsilon_\mu = 1$ GeV

$\varepsilon_{\pi^\pm} = 115$ GeV

$\varepsilon_{K^\pm} = 850$ GeV

$\varepsilon_{\pi^0} = 3.5 \cdot 10^{10}$ GeV

$d_j \gg \lambda_j \Rightarrow$ interactions dominate

(and vice versa)

meson decay:

⇒ see problem set 4!

branching ratios:

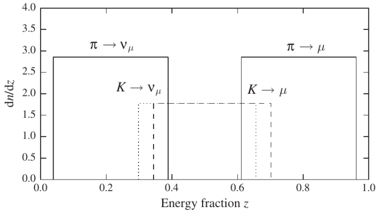
$\pi^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu)$ ~ 100%

$K^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu)$ ~ 63.5%

decay kinematics:

- two-body decay in meson rest frame: $m_\pi \approx m_\mu \Rightarrow$ most energy to μ

  $m_K \gg m_\nu \Rightarrow$ similar energy to $\mu + \nu$
- boost to lab frame (isotropic in rest frame):



Ⓟ $z_\mu : z_\nu \approx 0.8 : 0.2$

Ⓚ $z_\mu : z_\nu \approx 0.5 : 0.5$

Figure 6.1 Decay distributions for π-decay and K-decay into $\mu \, \nu_\mu$ for 200 MeV/c parent mesons. z is the ratio of the total lab energy of the decay product to that of the parent.

Gaisser, Engel & Resconi, “Cosmic Rays & Particle Physics”