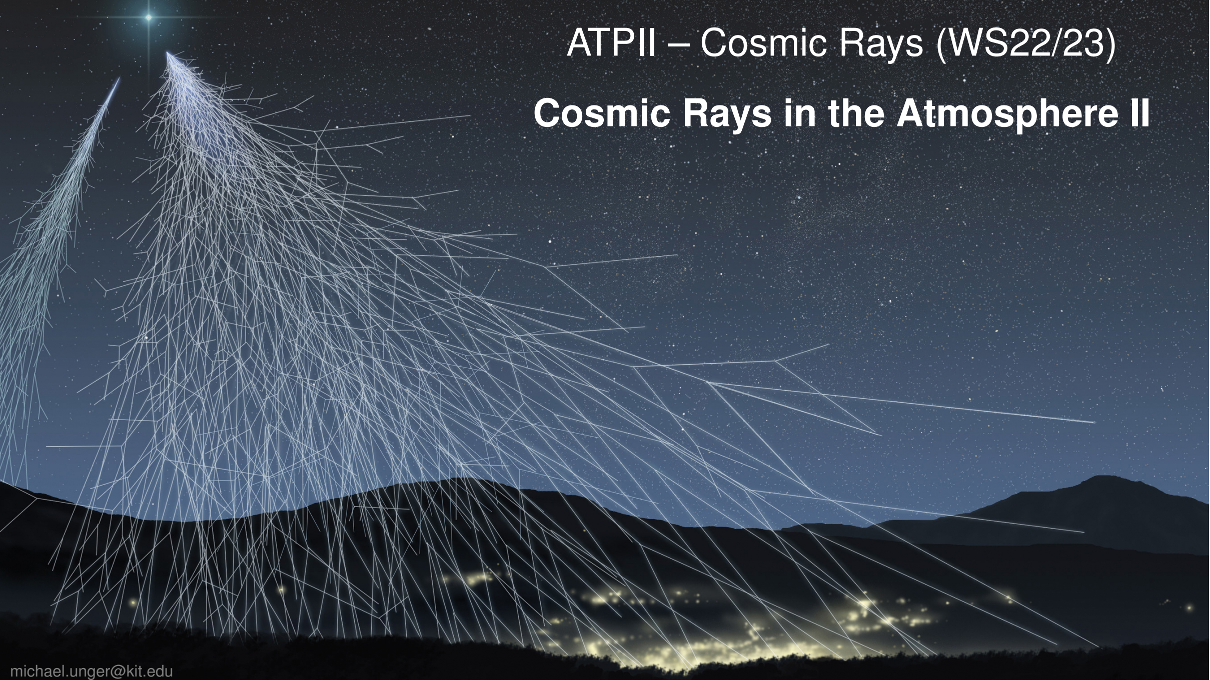
ATPII – Cosmic Rays (WS22/23)

Cosmic Rays in the Atmosphere II



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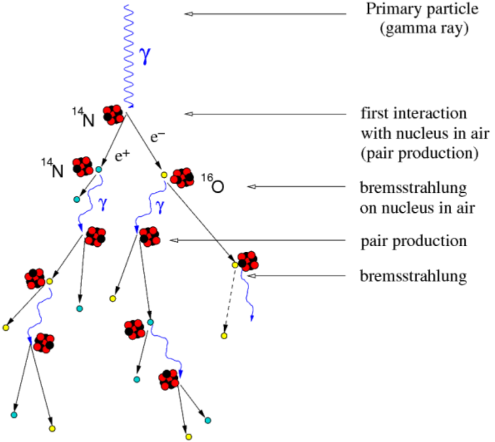
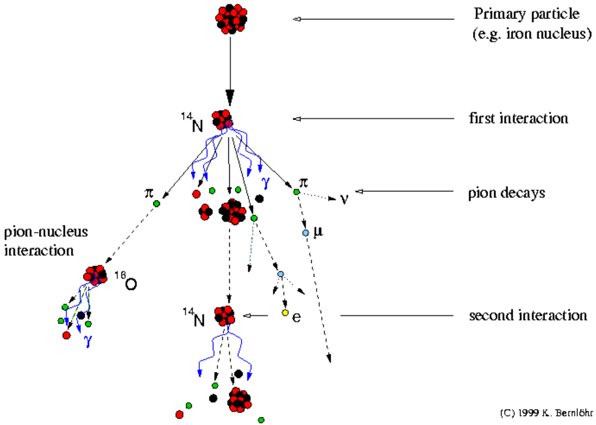
Particle Cascade in the Atmosphere / Air Shower





Recap: Atmosphere

- **vertical depth X_v**

$$X_v = \int_h^\infty \rho(h')\,dh'$$

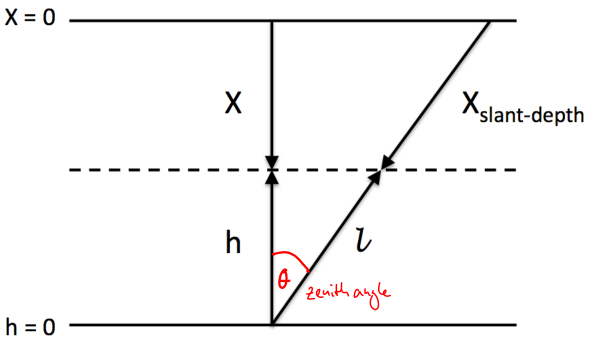
$[X_v] = g/cm^2 \Rightarrow$ "grammage"

- isothermal atmosphere:

$$\rho(h) = \rho_0\, e^{-h/h_0}$$
$$X_v = X_0\, e^{-h/h_0}$$

- $X_0 \approx 1030\ g/cm^2$ at sea level

- **scale height** $h_0 \approx 8.4$ km at sea level, ≈ 6.4 km high altitudes
  above $h \approx 10$ km



- **slant depth:**

$$X = \int_l^\infty \rho(h(l'))\,dl'$$

- flat atmosphere approximation for $\theta \lesssim 65°$

$$X = X_v / \cos\theta$$

Recap: Interactions and Decay

- decay length $d_j = \frac{E \times \cos\theta}{\varepsilon_j}$; $\varepsilon_j = m_j c^2 \frac{h_0}{c\tau_j}$ ← lifetime $[d] = g/cm^2$

Table 5.3 *Decay constants for various particles*

Particle	$c\tau$ (cm)	ϵ (GeV)
$\mu^{\pm}$	6.59×10^4	1.0
$\pi^{\pm}$	780	115
π^0	2.5×10^{-6}	3.5×10^{10}
$K^{\pm}$	371	850
K_S	2.68	1.2×10^5
K_L	1534	208
$D^{\pm}$	0.031	3.7×10^7
D^0	0.012	9.9×10^7

Table 5.4 *Atmospheric interaction and attenuation lengths for γ = 2.7 in air, in units of g/cm^2.*

Λ: attenuation length → see today

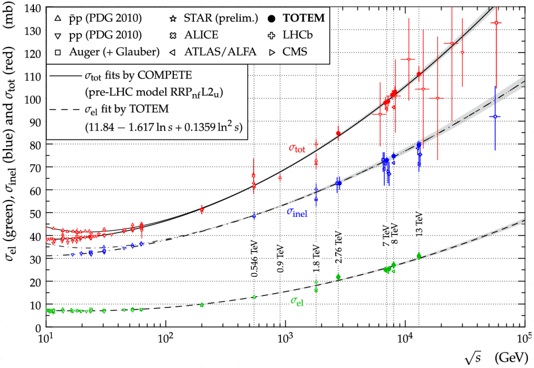
E_{lab} (GeV)	λ_N	Λ_N	λ_π	Λ_π	λ_K	Λ_K
100	88	120	116	155	134	160
1000	85	115	111	148	122	147
10000	79	106	101	135	110	133
100000	72	97	87	114	95	114

- interaction length: j + air → X

mass density

$$\lambda_j = \ell_j \varrho = \frac{\varrho}{n_A \sigma_j^{air}} = \frac{\langle A \rangle m_p}{\sigma_j^{air}}$$

$[\lambda] = g/cm^2$ $[\ell] = cm$ number density Cross section



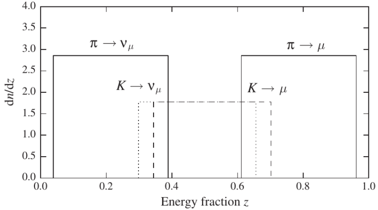


Figure 6.1 Decay distributions for π-decay and K-decay into $\mu\, \nu_\mu$ for 200 MeV/c parent mesons. z is the ratio of the total lab energy of the decay product to that of the parent.

Recap: Spectrum-weighted moments "z-factors"

- inclusive cross section: $j + \text{air} \to a + X$
- inclusive energy distribution of particles of type a:

$$E_a \frac{dn(E_j, E_a)}{dE_a} = F_{ja}(E_j, E_a) \approx F_{ja}\left(\frac{E_a}{E_j}\right)$$

'Feynman scaling'

- 'z-factor': $\boxed{Z_{pa} = \int_0^1 x^{\gamma-2} F_{ja}(x)\, dx}$ $\left(x = \frac{E_a}{E_j}\right)$

- if energy spectrum j is a power law: $\phi_j \sim \frac{dN_j}{dE_j} = N_j^0 \cdot E^{-\gamma}$

$$\phi_a(E_a) = Z_{ja} \cdot \phi_j^0\, E_a^{-\gamma}$$

Table 5.2 *Spectrum-weighted moments at 1 TeV*

Index p-air	$\gamma = 2$	$\gamma = 2.7$
$\pi^0(+\eta)$	0.206	0.0459
π^+	0.206	0.0489
π^-	0.156	0.0324
K^+	0.030	0.0071
K^-	0.018	0.0036
$K_L + K_S$	0.043	0.0092
$p + \bar{p}$	0.217	0.126
$n + \bar{n}$	0.114	0.052

Gaisser, Engel & Resconi, "Cosmic Rays & Particle Physics"

$Z_{p\pi^+} > Z_{p\pi^-}$ (charge)

$Z_{pp} \gg Z_{p\pi^\pm}$ (baryon number)

$Z_{K^+} \gg Z_{K^-}$ (associate hyperon production $p + N \to K^+ + K^- + X$, $p + N \to K^+ + \Lambda + X$)

Cascade Equations

evolution of particle fluxes in the atmosphere:

$$\frac{dN_i(E_i,X)}{dX} = \underbrace{-\frac{N_i(E_i,X)}{\lambda_i}}_{\text{interaction}} \underbrace{-\frac{N_i(E_i,X)}{d_i}}_{\text{decay}} + \underbrace{\sum_{j=i}^{N} \int_{E}^{\infty} \frac{F_{ji}(E_i,E_j)}{E_i} \frac{N_j(E_j,X)}{\lambda_j} dE_j}_{\text{production}}$$

boundary conditions:

"inclusive particle flux": $N(E,0) = N_0(E) \approx 1.7\ E^{-2.7} \frac{\text{nucleons}}{\text{cm}^2\ \text{sr}\ \text{s}\ \text{GeV/A}}$ nucleon flux at top of atmosphere

single air shower: $N(E,0) = A \cdot \delta\left(E - \frac{E_0}{A}\right) \delta(t-t_0)$ (superposition approximation: $(E_0, A) \hat{=} A \times (E/A, 1)$)

Nucleon Flux $N_N(E,X) \equiv N(E,X)$

$Z_{NN} = Z_{pp} + Z_{pn}$

$(Z_{pp} = Z_{nn},\ Z_{pn} = Z_{np})$

no decay, no production $j \neq i$ (e.g. no $\pi^+ + \text{air} \to p + X$; $Z_{\pi^+ N} \approx 0.02 \ll Z_{NN} \approx 0.18$)

$$\frac{dN(E,X)}{dX} = -\frac{N(E,X)}{\lambda_N} + \int_E^\infty \frac{N(E',X)}{\lambda_N(E')} F_{NN}(E,E') \frac{dE'}{E}$$

Ansatz: $N(E,X) = G(E)\, g(X)$ factorization and change of variables $x = E/E'$

$\left(E' = E/x,\ \frac{dE'}{dx} = -\frac{E}{x^2}\right.$

$\to dE' = -\frac{E}{x^2}\,dx$

$x(E'=\infty) = 0$

$\left. x(E'=E) = 1 \right)$

$$\Rightarrow \quad G g' = -\frac{G g}{\lambda_N} + g \int_0^1 \frac{G(E/x)\, F_{NN}(x,E)}{\lambda_N(E/x)} \frac{dx}{x^2}$$

$$\Rightarrow \quad \frac{g'}{g} = -\frac{1}{\lambda_N(E)} + \frac{1}{G(E)} \int_0^1 \frac{G(E/x)\, F_{NN}(x,E)}{\lambda_N(E/x)} \frac{dx}{x^2}$$

$$\approx -\frac{1}{\lambda_N}\left(1 - \int_0^1 x^{\gamma-2} F_{NN}(x)\,dx\right) = -\frac{1 - Z_{NN}}{\lambda_N}$$

$\lambda_N \approx \text{const},\ F(x,E) \approx F(x)$ (Feynman scaling)

+ boundary condition: $N(E,0) = E^{-\gamma} g(0)$

Solution:

$$\boxed{N(E,X) = N_0\, e^{-X/\Lambda} E^{-\gamma}}$$

attenuation length

$$\boxed{\Lambda = \frac{\lambda_N}{\underbrace{1 - Z_{NN}}_{\text{"regeneration"}}}}$$

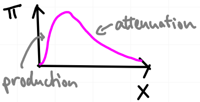
Meson Flux

$N_\pi(E,X) \equiv \Pi(E,X)$ similar for $K^\pm, D^\pm$

$$\frac{d\Pi}{dX} = -\left(\frac{1}{\lambda_\pi} + \frac{1}{d_\pi}\right)\Pi + \int_0^1 \frac{\Pi(E/x)\, F_{\pi\pi}(x,E)}{\lambda_\pi(E/x)} \frac{dx}{x^2} + \int_0^1 \frac{N(E/x)\, F_{N\pi}(x,E)}{\lambda_N(E/x)} \frac{dx}{x^2}$$

(note $x = \frac{E}{E'}$, X: slant depth)

- boundary condition: $\Pi(0,E) = 0$ $\leftrightarrow$ no primary mesons



- extreme cases:

  a) $E \gg \varepsilon_\pi \Rightarrow$ no decay $\quad \Pi(E,X) = N(E,0)\, \frac{Z_{N\pi}}{1-Z_{NN}}\, \frac{\Lambda_\pi}{\Lambda_\pi - \Lambda_N} \left(e^{-X/\Lambda_\pi} - e^{-X/\Lambda_N}\right) \sim E^{-\gamma}$ $\quad \Lambda_\pi > \Lambda_N$

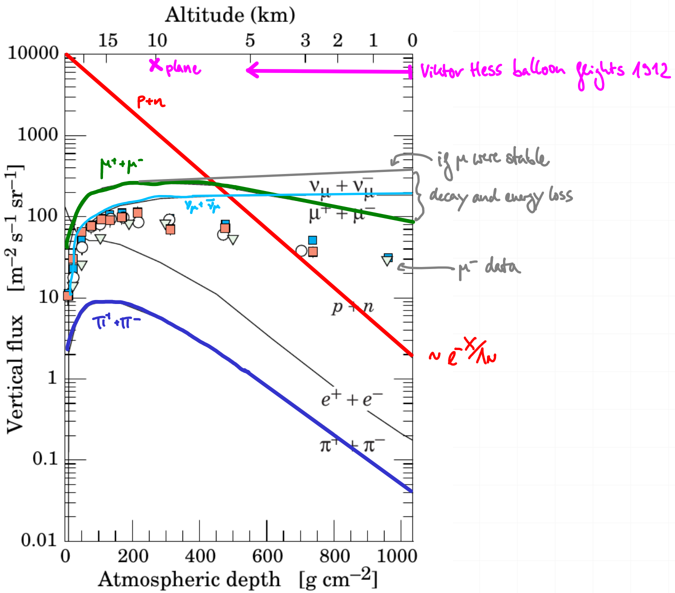
  $\Rightarrow$ same shape as primary spectrum

  b) $E \ll \varepsilon_\pi \Rightarrow$ decay dominates $\quad \Pi(E,X) = N(E,0)\, \frac{Z_{N\pi}}{\lambda_N}\, e^{-X/\Lambda_N}\, \frac{X E \cos\theta}{\varepsilon_\pi}$ $\quad \sim E^{-\gamma+1}$

  $\Rightarrow$ harder at low energies $E < \varepsilon_\pi$

- decaying mesons $\Rightarrow$ source term for $i = \mu$ and ν_μ $\quad P_{i\pi} = \int_{E_{min}}^{E_{max}} \frac{dn_{i\pi}(E,E')}{dE}\, \frac{\Pi(E',X)}{d_\pi(E')}\, dE'$ $\quad$ ($E_{min/max}$, $\frac{dn}{dE}$ from kinematics)

Inclusive Particle Fluxes in the Atmosphere



PDG 2022

Muon Spectrum at Sea Level

⇒ important prediction of atmospheric ν → atmospheric ν-oscillation
→ background for astrophysical νs
⇒ see summer

muons from $\pi^\pm$ and $K^\pm$

nucleon spectrum (top of atmosphere)

branching ratio $K \to \mu + \nu$

$$\boxed{\frac{dN_\mu}{dE_\mu} \simeq S_\mu(E_\mu)\,\frac{N_0(E_\mu)}{1-Z_{NN}}\left\{\mathcal{A}_{\pi\mu}\,\frac{1}{1+\mathcal{B}_{\pi\mu}\cos\theta\,E_\mu/\epsilon_\pi} + 0.635\,\mathcal{A}_{K\mu}\,\frac{1}{1+\mathcal{B}_{K\mu}\cos\theta\,E_\mu/\epsilon_K}\right\} \quad (6.36)}$$

(Similar for ν, see Eq. 6.29 in CRPP)

zenith

115 GeV

850 GeV

$A_{i\mu} \sim Z_{Ni}$ (meson production)

$B_{i\mu}$: kinematics etc.

higher energy fraction to ν in $K \to \mu\nu$

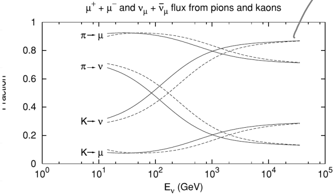


Fig. 1. Fraction of muons and muon neutrinos from pion decay and from kaon decay vs. neutrino energy. Solid lines for vertical, dashed lines for zenith angle 60°.

T.K. Gaisser Earth, Planets and Space 62 (2010) 195

follows ≈ primary spectrum

$\theta \approx 0°$

softening due to $\lambda_i < d_i$

$S_\mu(E_\mu)$
(decay + dE_μ/dX)

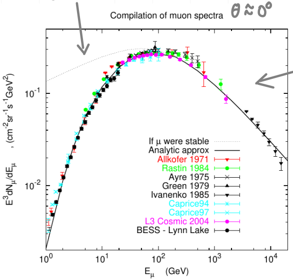


Figure 8: Summary of measurements of the vertical muon intensity at the ground. The solid line shows an analytic calculation [52]. The dotted line shows the spectrum in the absence of decay and energy loss, or equivalently the muon production spectrum integrated over the atmosphere.

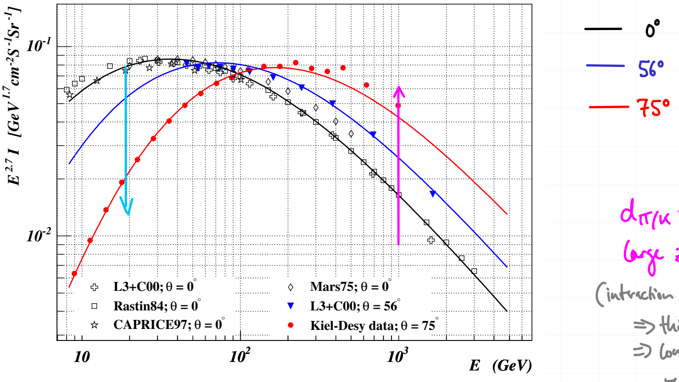
T.K. Gaisser Phys.Scripta T 121 (2005) 51

Muon Spectrum at Sea Level

muons from $\pi^{\pm}$ and $K^{\pm}$

$$\frac{\mathrm{d}N_\mu}{\mathrm{d}E_\mu} \simeq S_\mu(E_\mu)\,\frac{N_0(E_\mu)}{1-Z_{NN}}\left\{\mathcal{A}_{\pi\mu}\,\frac{1}{1+\mathcal{B}_{\pi\mu}\,\underline{\cos\theta\,E_\mu/\epsilon_\pi}} + 0.635\,\mathcal{A}_{K\mu}\,\frac{1}{1+\mathcal{B}_{K\mu}\,\underline{\cos\theta\,E_\mu/\epsilon_K}}\right\} \quad (6.36)$$

larger path length from production to ground
$\Rightarrow$ more muon decay



arXiv:1509.06176

$d_{\pi/K} < \lambda_{\pi/K}$ at large zenith angle

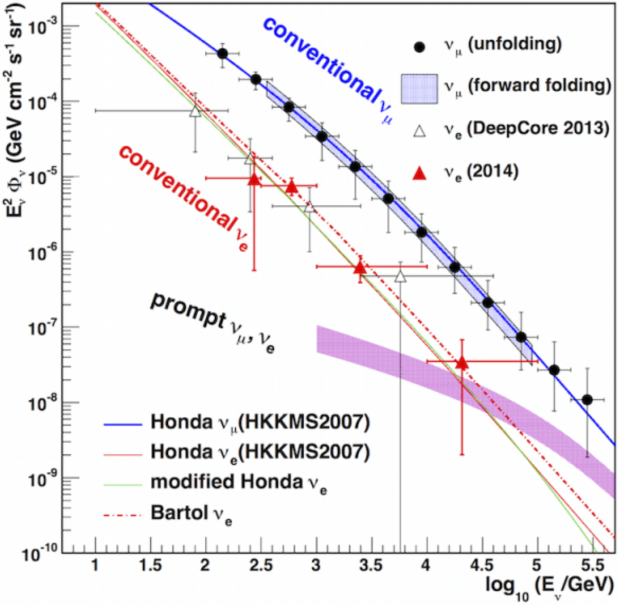
(interaction at higher altitude
$\Rightarrow$ thinner atmosphere
$\Rightarrow$ longer path (in cm) to reach grammage λ)

Atmospheric Neutrinos

"Conventional ν_μ" : $\pi^\pm, K^\pm$

"Conventional ν_e" mostly
$K_L^0 \to \pi^\pm e^\mp \nu_e$ ($\approx 41\%$)

"prompt" : charmed mesons $D^\pm, D^0$
$\hookrightarrow \alpha \approx 0$



IceCube data

IceCube Coll. PRD 2015

Electromagnetic Cascades

Bruno Rossi + Kenneth Greisen Rev. Mod. Phys. 1941

photon flux $\gamma(W,t)$ and e^+e^- flux $\pi(E,t)$, $t = X/X_0$

$$\frac{d\gamma}{dt} = -\frac{\gamma(W,t)}{\lambda_{pair}} + \int_W^\infty \pi(E',t)\,\frac{dn_{e\to\gamma}}{dW\,dt}\,dE'$$

$$\frac{d\pi}{dt} = -\frac{\pi(E,t)}{\lambda_{brems}} + \int_E^\infty \pi(E',t)\,\frac{dn_{e\to e}}{dE\,dt}\,dE' + 2\int_E^\infty \gamma(W',t)\,\frac{dn_{\gamma\to e}}{dE\,dt}\,dW'$$

"approximation A":

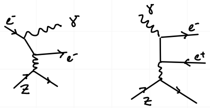
- $E \gg E_c$ → neglect ionization losses
- scaling of bremsstrahlung + pair production

$$E'\,\frac{dn_{e\to\gamma}}{dW\,dt} = f\left(\frac{W}{E'}\right) \qquad \text{Brems.}$$

$$W'\,\frac{dn_{\gamma\to e}}{dE\,dt} = g\left(\frac{E}{W'}\right) \qquad \text{pair}$$

$$E'\,\frac{dn_{e\to e}}{dE\,dt} = f\left(1-\frac{E}{E'}\right) \qquad \text{Brems.}$$

interactions with nuclei of material (Z)



bremsstrahlung pair production

boundary condition: photon induced shower

$$\pi(E,0) = \delta(E,E_0)$$

Solution: "Greisen Profile"

$$\boxed{N(t) = \frac{0.31}{\sqrt{\ln(E_0/E_c)}}\, e^{t\left(1-\frac{3}{2}\ln s\right)}}$$

$E_{\gamma,e^{\pm}} > 20\ \text{MeV}$

shower age s: $s = \dfrac{3t}{t+2t_{max}}$

shower maximum at $t_{max} = \ln\dfrac{E_0}{E_c}$

$$N(t_{max}) = \frac{0.31}{\sqrt{\ln(E_0/E_c)}}\,\frac{E_0}{E_c}$$

γ-induced air shower profile

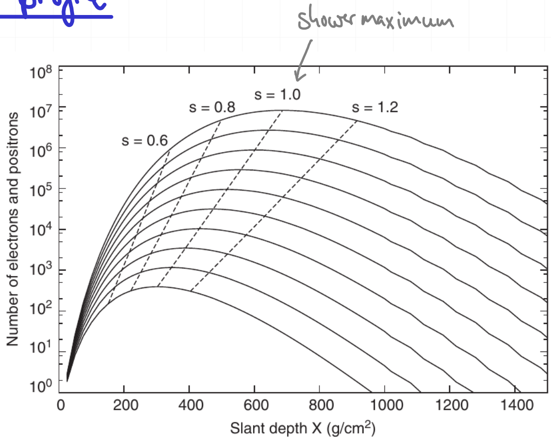


Figure 15.2 Shower size as a function of slant depth for photon-initiated showers in half-decade intervals of primary energy from 316 GeV (lowest curve) to 10^7 GeV (highest curve). The dashed lines trace the locus of size at specific shower ages across the same range of energies.

Gaisser, Engel & Resconi, "Cosmic Rays & Particle Physics"