# Direct $CP$ Violation

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**Flavor Physics Lectures**
**III / XII**

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# Reading material and references

**Lecture material based on several textbooks and online lectures/notes.**
**Credits for material and figures include:**

Literature

- Perkins, Donald H. (2000), *Introduction to High Energy Physics*.
- Griffiths, David J. (2nd edition), *Introduction to Elementary Particles*.
- Stone, Sheldon (2nd edition), $B$ *decays*.

Online Resources

- Belle/BaBar Collaborations, *The Phyiscs of the* $B$*-Factories*.
  http://arxiv.org/abs/1406.6311
- Bona, Marcella (University of London), $CP$ *Violation Lecture Notes*,
  http://pprc.qmul.ac.uk/ bona/ulpg/cpv/
- Richman, Jeremy D. (UCSB), *Heavy Quark Phyiscs and* $CP$ *Violation*.
  http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver_houches12.pdf
- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*,
  http://www.hep.phy.cam.ac.uk/ thomson/partIIIparticles/welcome.html
- Grossman, Yuval (Cornell University), *Just a Taste. Lectures on Flavor Physics*,
  http://www.lepp.cornell.edu/ pt267/files/notes/FlavorNotes.pdf
- Kooijman, P. & Tuning, N., $CP$ *Violation*,
  https://www.nikhef.nl/ h71/Lectures/2015/ppII-cpviolation-29012015.pdf

# Recap & outline

Last time:

- From first principles, we saw explicitely that the eigenstates of the $\mathcal{H}_w$ (mass eigenstates $K_S$ and $K_L$) are **not equal** to the strong eigenstates of definite flavor $K^0(d\bar{s})$ and $\overline{K}^0(\bar{d}s)$.
- We then saw that the $K_S$ and $K_L$ mass eigenstates are **not eigenstates of $\boldsymbol{CP}$**, i.e., $CP$ is violated to a small degree $\varepsilon = 2 \times 10^{-3}$.
- $CP$-violation in mixing leads to $\Gamma(K^0_{t=0} \to \overline{K}^0) \neq \Gamma(\overline{K}^0_{t=0} \to K^0)$, which is quantified by $|\frac{q}{p}| \neq 1$.

Today, we'll:

- Introduce another type of $CP$ violation:
  **$\boldsymbol{CP}$ violation in decay (aka "direct" $\boldsymbol{CPV}$)**.
- To understand it fully, we'll need to revisit our $\mathcal{C}$ and $\mathcal{P}$ operators in more detail and introduce the weak and strong phases.

# $CP$ violation in decay

Good news: *Conceptually, much less complicated than mixing!*

- This "direct" $CPV$ is observed by comparing the decay rate of particles $\Gamma(P \to f)$ and anti-particles $\Gamma(\overline{P} \to \overline{f})$, where:

  $P$ is a pseudoscalar meson

  $f$ and $\overline{f}$ are $CP$-conjugate final states (i.e., eigenstates of $CP$).

- Stated simply, if

  $\Gamma(P \to f) \neq \Gamma(\overline{P} \to \overline{f}) \quad \Rightarrow \quad CP$ Violation in decay

  We can express this as an asymmetry:

  $$\mathcal{A}_{CP} = \frac{\Gamma(P \to f) - \Gamma(\overline{P} \to \overline{f})}{\Gamma(P \to f) + \Gamma(\overline{P} \to \overline{f})} \neq 0$$

$\Rightarrow$ ***Lets prove this in the next slides***

# Condition for direct $CP$ violation

**Denote our initial and final states as $|P\rangle$ and $|f\rangle$, respectively.**

Define the action of the $CP$ operator on these states by:

$$CP|P\rangle = e^{2i\theta(P)}|\overline{P}\rangle$$

$$CP|f\rangle = e^{2i\theta(f)}|\overline{f}\rangle$$

where $e^{2i\theta(X)}$ is the *intrinsic* $CP$ phase factor associated with $X$.

**If $CP$ is conserved, it commutes with the hamiltonian: $[H, CP] = 0$.**

Write the amplitude for the $P \to f$ decay as:

$$A \equiv A(P \to f) = \langle f|H|P\rangle$$

The amplitude for the $CP$-conjugate process is:

$$\overline{A} \equiv \overline{A}(\overline{P} \to \overline{f}) = \langle \overline{f}|H|\overline{P}\rangle$$

# Condition for direct $CP$ violation

**Now lets see how the amplitudes are related** (assuming $CP$ is conserved):

$$A = \langle f|H|P\rangle = \langle f|(CP)^\dagger(CP)H(CP)^\dagger(CP)|P\rangle$$
$$= \langle \overline{f}|(CP)H(CP)^\dagger|\overline{P}\rangle \cdot e^{2i(\theta(P)-\theta(f))}$$
$$= \langle \overline{f}|H|\overline{P}\rangle \cdot e^{2i(\theta(P)-\theta(f))} \qquad \text{(using } [H, CP] = 0\text{)}$$
$$= \overline{A} \cdot e^{2i(\theta(P)-\theta(f))}$$

$$\Rightarrow \frac{\overline{A}}{A} = e^{-2i(\theta(P)-\theta(f))}$$

**Remove the dependence on the intrinsic phases by taking the magnitude of the amplitudes:**

$$\left|\frac{\overline{A}}{A}\right| = \left|e^{-2i(\theta(P)-\theta(f))}\right| = 1$$ $CP$ conservation *independent of phase convention*

**Arrive at our condition for $DCPV$:**

$$\left|\frac{\overline{A}}{A}\right| \neq 1 \quad \Rightarrow$$ $CP$ Violation in decay

# $CP$-violating phase

In the standard model, $CP$-conjugate amplitudes differ from the original amplitude by at most a phase factor.

$\Rightarrow$ *If only a single amplitude contributes to a given decay process, there cannot be an observable $CP$ asymmetry.*
Do you see why? (⇒ remember $\Gamma = |A|^2$)

Now suppose that there is more than one amplitude $A_j$ for a given decay, and that each amplitude has an associated $CP$-violating phase $\phi_j$.

- Write the overall amplitude for the $P \to f$ decay as:

  $$A \equiv A(P \to f) = \langle f|H|P\rangle = \sum_j A_j = \sum_j a_j e^{i\phi_j}$$

  These phases $\phi_j$ which change sign under $CP$ are the so-called **weak phases**. Can be **phases from the CKM matrix**, *but can also be due to new physics*.

- Similarly for the $\overline{P} \to \overline{f}$ decay:

  $$\overline{A} \equiv \overline{A}(\overline{P} \to \overline{f}) = \langle \overline{f}|H|\overline{P}\rangle = \sum_j \overline{A}_j = \sum_j a_j e^{-i\phi_j}$$

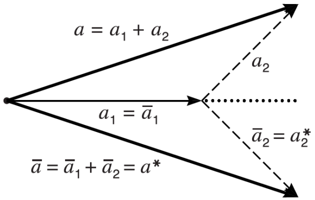
  where the $CP$-violating phases have changed sign.

# *Only* $CP$-violating weak phases

$$\left|\frac{\overline{A}}{A}\right| = \left|\frac{\sum_j |a_j| e^{-i\phi_j}}{\sum_j |a_j| e^{i\phi_j}}\right| = \left|\frac{\sum_j |a_j| \cos\phi_j - i\sum_j |a_j| \sin\phi_j}{\sum_j |a_j| \cos\phi_j + i\sum_j |a_j| \sin\phi_j}\right| = 1$$

Why?

$\Rightarrow$ *The numerator is just the complex conjugate of the denominator, so the magnitude of the amplitudes are the same.*




Case for 2 interfering amplitudes $a_1$ and $a_2$ with only weak phases.

The $CP$-conjugate amplitude $\bar{a} = \bar{a}_1 + \bar{a}_2$ has the same magnitude as the original amplitude $a$, *and there is no $CP$ asymmetry*

Something is missing...

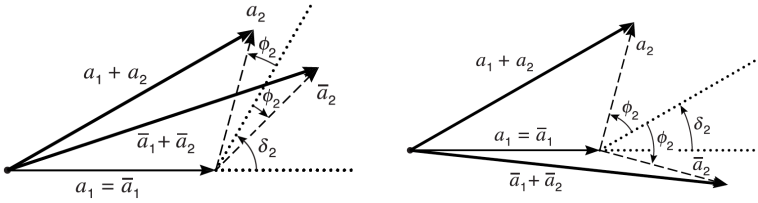
**What about the coefficients $a_j$?**

# Introduce the strong phase

**The coefficients $a_j$ are complex and are of the form $a_j = |a_j|\, e^{i\delta_j}$**, where $\delta_j$ are non-$CP$-violating phases that can arise, e.g., from strong interactions in the final state (exchanging gluons - no $CPV$ *that we know of*).

*These $\delta_j$ do not change sign under $CP$*

$$\left|\frac{\overline{A}}{A}\right| = \left|\frac{\sum_j |a_j| e^{i(\delta_j - \phi_j)}}{\sum_j |a_j| e^{i(\delta_j + \phi_j)}}\right| = \left|\frac{\sum_j |a_j| \cos(\delta_j - \phi_j) - i \sum_j |a_j| \sin(\delta_j - \phi_j)}{\sum_j |a_j| \cos(\delta_j + \phi_j) + i \sum_j |a_j| \sin(\delta_j + \phi_j}\right| \neq 1$$




Case for 2 interfering amplitudes $a_1$ and $a_2$ with weak ($\phi$) **and strong ($\delta$)** phases.

# Direct $CPV$ asymmetry

We can express this result as:

$$|\overline{A}|^2 - |A|^2 = 2\sum_{i,j} |a_i|\,|a_j|\ \sin(\phi_i - \phi_j)\sin(\delta_i - \delta_j)$$

We are usually concerned with only 2 amplitudes, so we can write the asymmetry as:

$$\mathcal{A}_{\mathcal{CP}} = \frac{|\overline{A}|^2 - |A|^2}{|\overline{A}|^2 + |A|^2} = \frac{2|a_1||a_2|\ \sin(\phi_1-\phi_2)\ \sin(\delta_1-\delta_2)}{|a_1|^2 + |a_2|^2 + |a_1||a_2|\ \cos(\phi_1-\phi_2)\ \cos(\delta_1-\delta_2)}$$

**These results have important implications:**

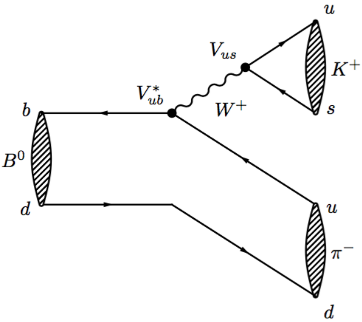
To observe $CP$-violating effects by comparing $\Gamma(P \to f)$ and $\Gamma(\overline{P} \to \overline{f})$ we need:

1. A minimum of 2 amplitudes contributing to a given decay process.
2. *Both* $CP$-violating and non-$CP$-violating phases.

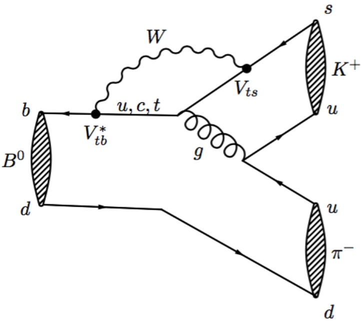
***⇒ Lets look at an example in $B^0 \to K^+\pi^-$ decays***

# $B^0 \to K^+\pi^-$ decays

Amplitude 1 $(\sim V_{ub}^* V_{us})$




Amplitude 2 $(\sim V_{tb}^* V_{ts})$




Recall the Wolfenstein phase convention of the CKM matrix elements

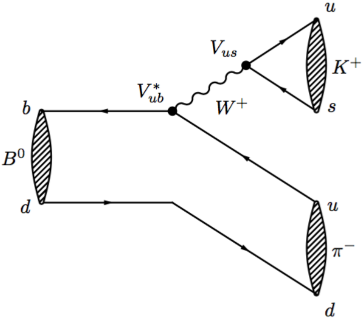
*(Lecture 1, Slide 31)*

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}| & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^4)$$

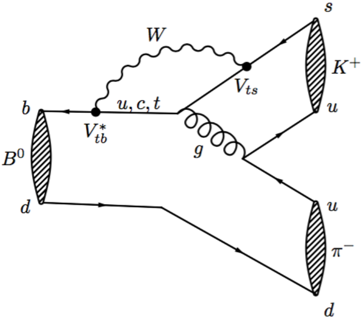
and note which matrix elements contain the $CP$ violating phase

# $B^0 \to K^+\pi^-$ decays

Tree amplitude $(\sim V_{ub}^* V_{us})$




Penguin amplitude $(\sim V_{tb}^* V_{ts})$




$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}| & |V_{tb}| \end{pmatrix}$$

- Tree amplitude contains the $CP$-violating phase $\gamma$. The relative strong phase shift between the tree and penguin diagrams is $\delta$.

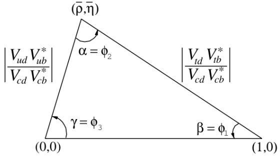
$$\Gamma(B^0 \to K^+\pi^-) \propto \left|A_P - A_T e^{-i\gamma} e^{i\delta}\right|^2$$

$$\Gamma(\overline{B}^0 \to K^-\pi^+) \propto \left|A_P - A_T e^{i\gamma} e^{i\delta}\right|^2$$

# What can we deduce from these amplitudes?

- If we compute
  $\frac{\Gamma(B^0\to K^+\pi^-)+\Gamma(\overline{B}^0\to K^-\pi^+)}{2} \propto |A_P|^2 \left(1 - 2\frac{A_T}{A_P}\cos\gamma\,\cos\delta + \left(\frac{A_T}{A_P}\right)^2\right)$
- and use the fact that $B^+ \to K^0\pi^+$ is almost entirely a penguin process:
  $\Gamma(B^+ \to K^0\pi^+) = \Gamma(B^- \to \overline{K}^0\pi^-) \propto |A_P|^2$
- we can compute the ratio:
  $R \equiv \frac{\Gamma(B_d\to K^\pm\pi^\mp)}{\Gamma(B^\pm\to K^0\pi^\pm)} = \left(1 - 2\frac{A_T}{A_P}\cos\gamma\,\cos\delta + \left(\frac{A_T}{A_P}\right)^2\right)$ (note the $|A_P|^2$ term is gone)
- The minimum value of $R$ (as a function of $\frac{A_T}{A_P}$) is attained when $\frac{A_T}{A_P} = \cos\gamma\,\cos\delta$, and is given by
  $R \geq 1 - \cos^2\gamma\,\cos^2\delta \geq \sin^2\gamma$

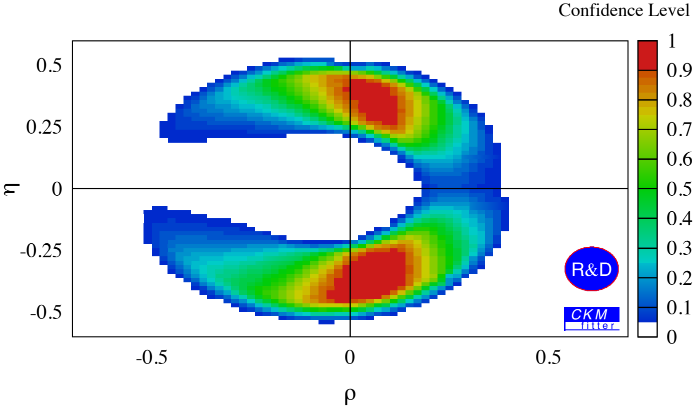
- We now have a constraint in the $\rho - \eta$ plane
  $R \geq \frac{\eta^2}{\rho^2+\eta^2} = \sin^2\gamma$
  (If you don't see it, make a mini right triangle of origin (0,0), base $\rho$ and height $\eta$)




$\Rightarrow$ *Constraint from only 2 $K\pi$ measurements!*

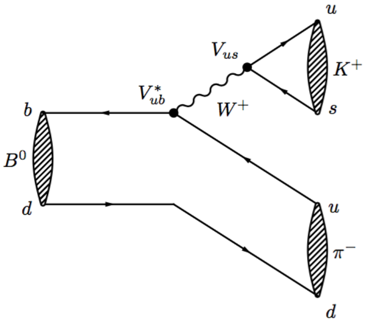
# Constraint in the $(\rho, \eta)$ plane

We can make a constraint on the apex of the Unitary Triangle using the $B \to K\pi$ and $B \to \pi\pi$ *(future lecture)* results.




(Note: these are very old [2002], but the point is to illustrate the idea)
http://ckmfitter.in2p3.fr/www/archives/ckm_charmless2002.html

# What about $B^+ \to K^+\pi^0$ decays?

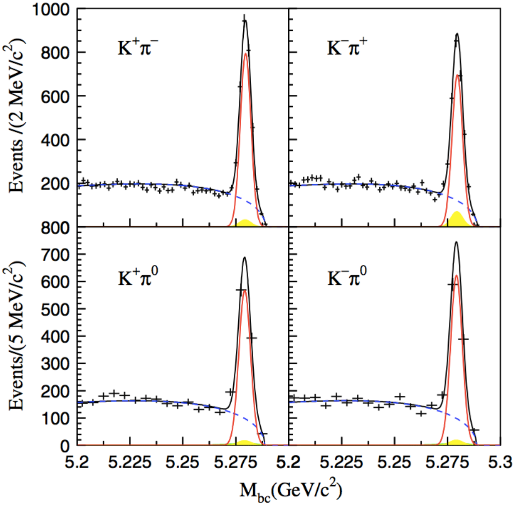



If we replace the spectator $d$-quark by a $u$-quark in both $B^0 \to K^+\pi^-$ diagrams, the $\pi^-$ becomes a $\pi^0$, and we have the tree and penguin diagrams for $B^+ \to K^+\pi^0$ decays.

We should expect to see around the same $A_{CP}$ since we're only changing the spectator quark, no?

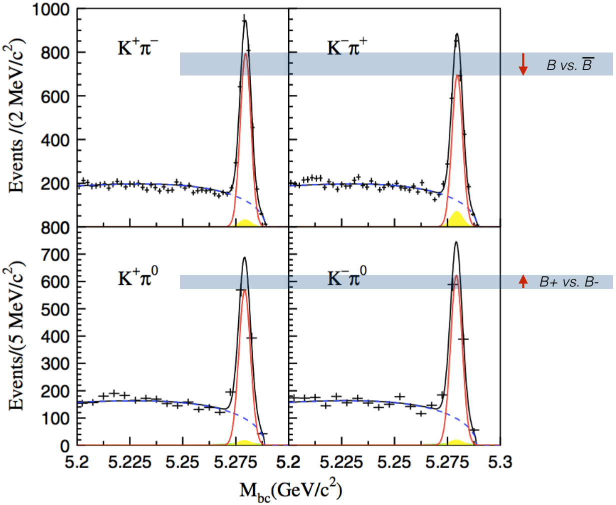
# Not quite!

Measurements of $DCPV$ in $B^+ \to K^+\pi^0$ found to be different than $B^0 \to K^+\pi^-$



# Not quite!

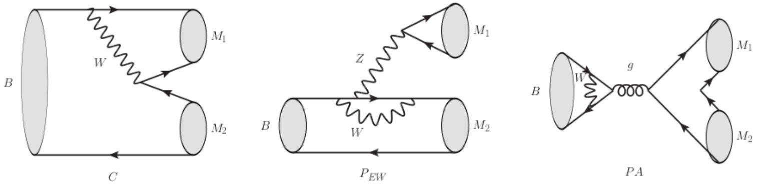
Measurements of $DCPV$ in $B^+ \to K^+\pi^0$ found to be different than $B^0 \to K^+\pi^-$




$$\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-} = 0.112 \pm 0.027 \pm 0.007 \ \ (4\sigma)$$

# Additional SM Diagrams or New Physics?

- The difference could be due to:
  - Neglected diagrams contributing to $B$ decays (theoretical uncertainty is still large).
    $K^+\pi^- : T + P + P_{EW}^C$
    $K^+\pi^0 : T + P + C + P_{EW} + P_{EW}^C + PA$




  - Some unknown NP effect that violates Isospin.

⇒ **In combination with other $K\pi$ measurements** *and with the larger Belle II dataset, strong interaction effects can be controlled and the validitiy of the SM can be tested in a model-independent way.*

# $B \to K\pi$: Current experimental status

Complete set of measurements from Belle and BaBar.

| $\mathcal{B}(10^{-6})$ | | | |
|---|---|---|---|
| Mode | BABAR | Belle | LHCb |
| $K^+\pi^-$ | $19.1 \pm 0.6 \pm 0.6$ | $20.0 \pm 0.34 \pm 0.60$ | |
| $K^+\pi^0$ | $13.6 \pm 0.6 \pm 0.7$ | $12.62 \pm 0.31 \pm 0.56$ | |
| $K^0\pi^+$ | $23.9 \pm 1.1 \pm 1.0$ | $23.97 \pm 0.53 \pm 0.71$ | |
| $K^0\pi^0$ | $10.1 \pm 0.6 \pm 0.4$ | $9.68 \pm 0.46 \pm 0.50$ | |

| $A_{CP}$ | | | |
|---|---|---|---|
| Mode | BABAR | Belle | LHCb |
| $K^+\pi^-$ | $-0.107 \pm 0.016^{+0.006}_{-0.004}$ | $-0.069 \pm 0.014 \pm 0.007$ | $-0.080 \pm 0.007 \pm 0.003$ |
| $K^+\pi^0$ | $0.030 \pm 0.039 \pm 0.010$ | $0.043 \pm 0.024 \pm 0.002$ | |
| $K^0\pi^+$ | $-0.029 \pm 0.039 \pm 0.010$ | $-0.011 \pm 0.021 \pm 0.006$ | $-0.022 \pm 0.025 \pm 0.010$ |
| $K^0\pi^0$ | $-0.13 \pm 0.13 \pm 0.03$ | $0.14 \pm 0.13 \pm 0.06$ | |

# $B \to K\pi$: Test-of-sum Rule

Asymmetry (test-of-sum) rule for NP nearly free of theoretical uncertainties, where the SM can be tested by measuring all observables: [PLB 627, 82(2005), PRD 58, 036005(1998)]

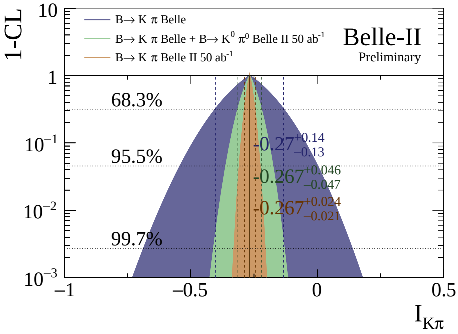
$$I_{K\pi} = \mathcal{A}_{K^+\pi^-} + \mathcal{A}_{K^0\pi^+}\frac{\mathcal{B}(K^0\pi^+)}{\mathcal{B}(K^+\pi^-)}\frac{\tau_{B^0}}{\tau_{B^+}} - 2\mathcal{A}_{K^+\pi^0}\frac{\mathcal{B}(K^+\pi^0)}{\mathcal{B}(K^+\pi^-)}\frac{\tau_{B^0}}{\tau_{B^+}} - 2\mathcal{A}_{K^0\pi^0}\frac{\mathcal{B}(K^0\pi^0)}{\mathcal{B}(K^+\pi^-)}$$

$\left(I_{K\pi} = -0.0088^{+0.0016+0.0131}_{-0.0017-0.0091}\right)$ [@NNLO] PLB 750(2015)348-355

$$\boldsymbol{I_{K\pi} = -0.270 \pm 0.132 \pm 0.060} \textbf{ [Belle]}$$

- Most demanding measurement is $K^0\pi^0$ final state: $\mathcal{A}_{K^0\pi^0} = 0.14 \pm 0.13 \pm 0.06$.
  Belle, PRD 81, 011101(R) (2010)
- With Belle II, the uncertainty on $\mathcal{A}_{K^0\pi^0}$ from time-dep. analysis is expected to reach $\sim 4\%$.

  $\Rightarrow$ *Sufficient for NP studies*



# Modified $P_{EW}$ Sector

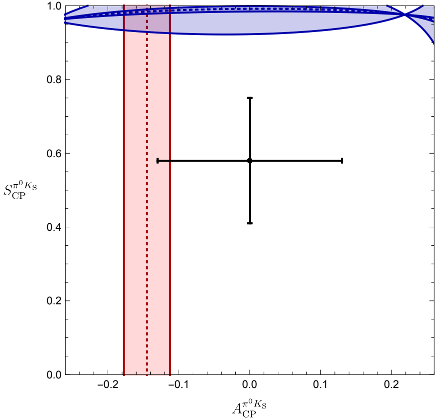
- Data point is the WA for $\mathcal{A}_{K^0\pi^0}$ and $\mathcal{S}_{K^0\pi^0}$.
- The $\mathcal{A}_{K^0\pi^0}$ value obtained from the sum rule with WA inputs for all other $\mathcal{A}_{K\pi}$ and $\mathcal{B}(K\pi)$ values.
- Isospin relation involving tighter constraints from CKM angle $\gamma$:

$$\sqrt{2}\mathcal{A}_{K^0\pi^0} + \mathcal{A}_{K^+\pi^0} = -\left(\hat{T} + \hat{C}\right)\left(e^{i\gamma} - qe^{i\phi}e^{i\omega}\right).$$

EW penguin effects described by

$$qe^{i\phi}e^{i\omega} \equiv -\left(\hat{P}_{EW} + \hat{P}^{C}_{EW}\right) / \left(\hat{T} + \hat{C}\right).$$

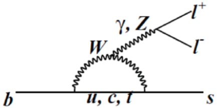
R. Fleischer *et al.*, arXiv:1712.02323, Moriond QCD




- Discrepancy can be resolved if:
  $CP$ asymmetries move by $\approx 1\sigma$; $\mathcal{B}(K^0\pi^0)$ moves by $\approx 2.5\sigma$.
- **Or NP from EW $Z$ penguins that couple to quarks:**
  ***Includes models with extra $Z'$ bosons, which can be used to resolve anomalies in $B \to K^{(*)}\ell\ell$ measurements.***

# Aside: $\mathcal{R}_{K^{(*)}}$ Anomaly *(To be discussed in future lectures)*

- Within the SM, decays proceed via one loop diagram: JHEP0712:040,2007

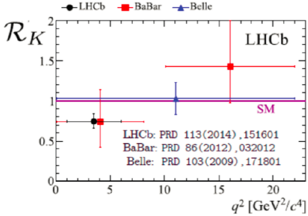



$$\mathcal{R}_K = \frac{\mathcal{B}(B^+ \to K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \to K^+ e^+ e^-)} = 1.00030^{+0.00010}_{-0.00007}$$

- LHCb reported a 2.6$\sigma$ deviation for the dilepton invariant mass squared region $1 < q^2 < 6\ \text{GeV}^4/c^2$:

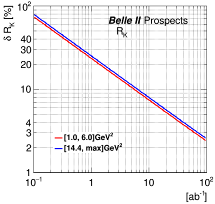
$$\mathcal{R}_K = 0.745^{+0.090}_{-0.074} \pm 0.036$$

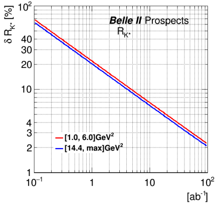
Phys. Rev. Lett. **113** 151601 (2016)

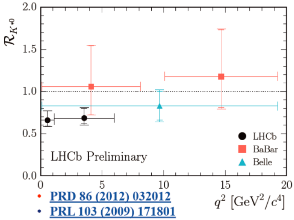



- Electrons and muons have the same $\varepsilon$ at Belle II:

$\Rightarrow$ *Both low and high* $q^2$ *regions possible.*




- PRD 86 (2012) 032012
- PRL 103 (2009) 171801

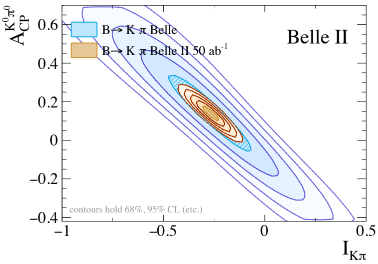
## More data:

### Extrapolate Belle measurements to 5 and 50 $\mathrm{ab}^{-1}$

- Systematic uncertainties scale primarily with integrated luminosity, with the exception of $A_{CP}$ measurements of channels with $K_S^0$:
  $\Rightarrow$ *asymmetry of $K^0$/ $\overline{K}^0$ interactions in material* ($\sigma_{ired} \approx 0.2\%$)
  Phys. Rev. D **84**, 111501 (2011)
- Ideally separate the reducible and irreducible systematic errors (unchanged throughout data accumulation) when extrapolating.
  - Few modes are systematically limited, so treat all syst. errors as reducible (with few exceptions, e.g., $K_S^0\pi^0$, next slide).
  - Apply scaling to all stat. and syst. errors to Belle results via:

$$\sigma_{Belle\ II} = \sqrt{(\sigma_{stat}^2 + \sigma_{syst}^2)\frac{\mathcal{L}_{Belle}}{\mathcal{L}_{BelleII}} + \sigma_{ired}^2}$$

- Perform a 2D scan of $\mathcal{A}_{K^0\pi^0}$ vs. $I_{K\pi}$ for different Belle II scenarios.
- The only possible correlated errors for the $A_{CP}$ measurements are caused by the detector bias, which is estimated with different methods for each channel. $\Rightarrow$ *Assume that the bias errors are not correlated.*
- Additionaly the systematic uncertainties are conservatively provided and they are still smaller than the statistical errors.




Projections for the $B \to K\pi$ isospin sum rule parameter, $I_{K\pi}$, at the Belle measured central value.

| Scenario | $\mathcal{A}_{K^0\pi^0}$ Value | Stat. | (Red., Irred.) | $I_{K\pi}$ |
|---|---|---|---|---|
| Belle | 0.14 | 0.13 | $(0.06, 0.02)$ | $-0.27 \pm 0.14$ |
| Belle + $B \to K^0\pi^0$ at Belle II 5 ab$^{-1}$ | | 0.05 | $(0.02, 0.02)$ | $-0.27 \pm 0.07$ |
| Belle II 50 ab$^{-1}$ | | 0.01 | $(0.01, 0.02)$ | $-0.27 \pm 0.03$ |

# $K^*\pi$ and $K^{(*)}\rho$ systems

Expect analogous sum rules by replacing:

**$K \to K^*$**

$$I_{K^*\pi} = \mathcal{A}_{K^{*+}\pi^-} + \mathcal{A}_{K^{*0}\pi^+} \frac{\mathcal{B}(K^{*0}\pi^+)}{\mathcal{B}(K^{*+}\pi^-)} \frac{\tau_{B^0}}{\tau_{B^+}} - 2\mathcal{A}_{K^{*+}\pi^0} \frac{\mathcal{B}(K^{*+}\pi^0)}{\mathcal{B}(K^{*+}\pi^-)} \frac{\tau_{B^0}}{\tau_{B^+}} - 2\mathcal{A}_{K^{*0}\pi^0} \frac{\mathcal{B}(K^{*0}\pi^0)}{\mathcal{B}(K^{*+}\pi^-)}$$

**$\pi \to \rho$**

$$I_{K\rho} = \mathcal{A}_{K^+\rho^-} + \mathcal{A}_{K^0\rho^+} \frac{\mathcal{B}(K^0\rho^+)}{\mathcal{B}(K^+\rho^-)} \frac{\tau_{B^0}}{\tau_{B^+}} - 2\mathcal{A}_{K^+\rho^0} \frac{\mathcal{B}(K^+\rho^0)}{\mathcal{B}(K^+\rho^-)} \frac{\tau_{B^0}}{\tau_{B^+}} - 2\mathcal{A}_{K^0\rho^0} \frac{\mathcal{B}(K^0\rho^0)}{\mathcal{B}(K^+\rho^-)}$$

**$K \to K^*$ & $\pi \to \rho$**

$$I_{K^*\rho} = \mathcal{A}_{K^{*+}\rho^-} + \mathcal{A}_{K^{*0}\rho^+} \frac{\mathcal{B}(K^{*0}\rho^+)}{\mathcal{B}(K^{*+}\rho^-)} \frac{\tau_{B^0}}{\tau_{B^+}} - 2\mathcal{A}_{K^{*+}\rho^0} \frac{\mathcal{B}(K^{*+}\rho^0)}{\mathcal{B}(K^{*+}\rho^-)} \frac{\tau_{B^0}}{\tau_{B^+}} - 2\mathcal{A}_{K^{*0}\rho^0} \frac{\mathcal{B}(K^{*0}\rho^0)}{\mathcal{B}(K^{*+}\rho^-)}$$

For each set of decays[1], perform a 2D scan of $A_{CP}$ (for most limiting final state) vs. the isospin sum rule parameter.

$\Rightarrow$ Compare with (N)NLO calculations[2].

[1] For the PV & VV systems, BaBar $\mathcal{B}$ and $A_{CP}$ used for projections (Belle results n/a) - see BKUP slides.

[2] No NNLO calc. for VV system, as longitudinal $A_{CP}$ fraction n/a for all final states.

# Theory predictions PLB 750(2015)348-355

## Two-loop current–current operator contribution to the non-leptonic QCD penguin amplitude

CrossMark

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A B S T R A C T

The computation of direct CP asymmetries in charmless $B$ decays at next-to-next-to-leading order (NNLO) in QCD is of interest to ascertain the short-distance contribution. Here we compute the two-loop penguin contractions of the current–current operators $Q_{1,2}$ and provide a first estimate of NNLO CP asymmetries in penguin-dominated $b \to s$ transitions.



### 1. Introduction

Non-leptonic exclusive decays of $B$ mesons play a crucial role in studying the CKM mechanism of quark flavour mixing and in quantifying the phenomenon of CP violation. Direct CP violation is related to the rate difference of $\bar{B} \to f$ decay and its CP-conjugate and arises if the decay amplitude is composed of at least two partial amplitudes with different re-scattering ("strong") phases, which are multiplied by different CKM matrix elements. Very often useful information on the CKM parameters including the CP-violating phase can be obtained from combining different decay modes, whose partial amplitudes are related by the approximate flavour symmetries of the strong interaction [1], which are then determined from data.

The direct computation of the partial amplitudes is a complicated strong interaction problem, which can, however, be addressed in the heavy-quark limit. The QCD factorization approach [2–4] employs soft-collinear factorization in this limit to express the hadronic matrix elements in terms of form factors and convolutions of perturbative objects (hard-scattering kernels) with non-perturbative light-cone distribution amplitudes (LCDAs). At leading order in $\Lambda/m_b$,

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = i m_B^2 \Big\{ f_+^{BM_1}(0) \int_0^1 du \, T_i^I(u) \, f_{M_2} \phi_{M_2}(u) + (M_1 \leftrightarrow M_2) + \int_0^\infty d\omega \int_0^1 du dv \, T_i^{II}(\omega, v, u) \, \hat{f}_B \phi_B(\omega) \times f_{M_1} \phi_{M_1}(v) \, f_{M_2} \phi_{M_2}(u) \Big\}, \quad (1)$$

where $Q_i$ is a generic operator from the effective weak Hamiltonian. At this order the re-scattering phases are generated at the scale $m_b$ only, and reside in the loop corrections to the hard-scattering kernels. Beyond the leading order factorization does not hold, and re-scattering occurs at all scales. The leading contributions to the strong phases are therefore of order $\alpha_s(m_b)$ or/and $\Lambda/m_b$. It is of paramount importance for the predictivity of the approach for the direct CP asymmetries to know whether the short-distance or long-distance contribution dominates in practice, since apart from being parametrically small, both could be numerically of similar size.

The short-distance contribution to the direct CP asymmetries is fully known only to the first non-vanishing order (that is, $\mathcal{O}(\alpha_s)$) through the one-loop computations of the vertex kernels $T_i^I$ performed long ago [2,4,5]. A reliable result presumably requires the next-to-next-to-leading order $\mathcal{O}(\alpha_s^2)$ hard-scattering kernels, at least their imaginary parts. For the spectator-scattering kernels $T_i^{II}$

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For table on next slide:

- $A_{CP}$, $\Delta A_{CP}$, and $I_{xy}$ in %.
- The results listed in the Exp. (WA) column are taken from HFLAV 2014 results (arXiv:1412.7515).
- However, the Belle II fit projections were computed with results from ***a single experiment***: $K\pi$ Belle; $K^*\pi$ & $K\rho$ BaBar.
- The results of the GammaCombo fits are added in the last column. Also shown are the $A_{CP}$ input used in the 2D fit ($A_{CP}$ vs $I_{xy}$).
- The results of projecting to 5 and 50 $\mathrm{ab}^{-1}$ of Belle II data are shown in ()(), respectively.

# Comparison w/theory (Modified Table I)

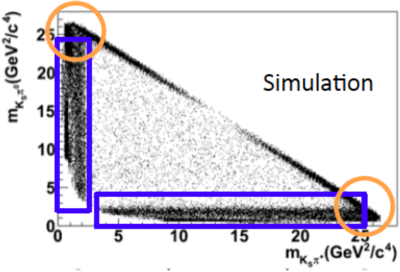
| $f$ | NLO | NNLO | NNLO + LD | Exp (WA) | Exp (GC fit and B2 proj.) |
|---|---|---|---|---|---|
| $\pi^- \bar{K}^0$ | $0.71^{+0.13\,+0.21}_{-0.14\,-0.19}$ | $0.77^{+0.14\,+0.23}_{-0.15\,-0.22}$ | $0.10^{+0.02\,+1.24}_{-0.02\,-0.27}$ | $-1.7 \pm 1.6$ | **Belle input** |
| $\pi^0 K^-$ | $9.42^{+1.77\,+1.87}_{-1.76\,-1.88}$ | $10.18^{+1.91\,+2.03}_{-1.90\,-2.62}$ | $-1.17^{+0.22\,+20.00}_{-0.22\,-6.62}$ | $4.0 \pm 2.1$ | |
| $\pi^+ K^-$ | $7.25^{+1.36\,+2.13}_{-1.36\,-2.58}$ | $8.08^{+1.52\,+2.52}_{-1.51\,-2.65}$ | $-3.23^{+0.61\,+19.17}_{-0.61\,-3.36}$ | $-8.2 \pm 0.6$ | |
| $\pi^0 \bar{K}^0$ | $-4.27^{+0.83\,+1.48}_{-0.77\,-2.23}$ | $-4.33^{+0.84\,+3.29}_{-0.78\,-2.32}$ | $-1.41^{+0.27\,+5.54}_{-0.25\,-6.10}$ | $1 \pm 10$ | $-14 \pm 13$ |
| $\Delta A_{CP}$ | $2.17^{+0.40\,+1.39}_{-0.40\,-0.74}$ | $2.10^{+0.39\,+1.40}_{-0.39\,-2.86}$ | $2.07^{+0.39\,+2.76}_{-0.39\,-4.55}$ | $12.2 \pm 2.2$ | |
| $I_{K\pi}$ | $-1.15^{+0.21\,+0.55}_{-0.22\,-0.84}$ | $-0.88^{+0.16\,+1.31}_{-0.17\,-0.91}$ | $-0.48^{+0.09\,+1.09}_{-0.09\,-1.15}$ | $-14 \pm 11$ | $-27 \pm 14(7)(3)$ |
| $\pi^- \bar{K}^{*0}$ | $1.36^{+0.25\,+0.60}_{-0.26\,-0.47}$ | $1.49^{+0.27\,+0.69}_{-0.29\,-0.56}$ | $0.27^{+0.05\,+3.18}_{-0.05\,-0.67}$ | $-3.8 \pm 4.2$ | **BaBar input** |
| $\pi^0 K^{*-}$ | $13.85^{+2.40\,+5.84}_{-2.70\,-5.86}$ | $18.16^{+3.11\,+7.79}_{-3.52\,-10.57}$ | $-15.81^{+3.01\,+69.35}_{-2.83\,-15.39}$ | $-6 \pm 24$ | $-6 \pm 24$ |
| $\pi^+ K^{*-}$ | $11.18^{+2.00\,+9.75}_{-2.15\,-10.62}$ | $19.70^{+3.37\,+10.54}_{-3.80\,-11.42}$ | $-23.07^{+4.35\,+86.20}_{-4.05\,-20.64}$ | $-23 \pm 6$ | |
| $\pi^0 \bar{K}^{*0}$ | $-17.23^{+3.33\,+7.59}_{-3.00\,-12.57}$ | $-15.11^{+2.93\,+12.34}_{-2.65\,-10.64}$ | $2.16^{+0.39\,+17.53}_{-0.42\,-36.80}$ | $-15 \pm 13$ | |
| $\Delta A_{CP}$ | $2.68^{+0.72\,+5.44}_{-0.67\,-4.30}$ | $-1.54^{+0.45\,+4.60}_{-0.58\,-9.19}$ | $7.26^{+1.21\,+12.78}_{-1.34\,-20.65}$ | $17 \pm 25$ | |
| $I_{K^*\pi}$ | $-7.18^{+1.38\,+3.38}_{-1.28\,-5.35}$ | $-3.45^{+0.67\,+9.48}_{-0.59\,-4.95}$ | $-1.02^{+0.19\,+4.32}_{-0.18\,-7.86}$ | $-5 \pm 45$ | $69 \pm 32(15)(6)$ |
| $\rho^- \bar{K}^0$ | $0.38^{+0.07\,+0.16}_{-0.07\,-0.27}$ | $0.22^{+0.04\,+0.19}_{-0.04\,-0.17}$ | $0.30^{+0.06\,+2.28}_{-0.06\,-2.39}$ | $-12 \pm 17$ | **BaBar input** |
| $\rho^0 K^-$ | $-19.31^{+3.42\,+13.95}_{-3.61\,-8.96}$ | $-4.17^{+0.75\,+19.26}_{-0.80\,-19.52}$ | $43.73^{+7.07\,+44.00}_{-7.62\,-137.77}$ | $37 \pm 11$ | |
| $\rho^+ K^-$ | $-5.13^{+0.95\,+6.38}_{-0.97\,-4.02}$ | $1.50^{+0.29\,+8.69}_{-0.27\,-10.36}$ | $25.93^{+4.43\,+25.40}_{-4.90\,-75.63}$ | $20 \pm 11$ | |
| $\rho^0 \bar{K}^0$ | $8.63^{+1.59\,+2.31}_{-1.65\,-1.69}$ | $8.99^{+1.66\,+3.60}_{-1.71\,-7.44}$ | $-0.42^{+0.08\,+19.49}_{-0.08\,-8.78}$ | $6 \pm 20$ | $5 \pm 26$ |
| $\Delta A_{CP}$ | $-14.17^{+2.80\,+7.98}_{-2.96\,-5.39}$ | $-5.67^{+0.96\,+10.86}_{-1.01\,-9.79}$ | $17.80^{+3.15\,+19.51}_{-3.01\,-62.44}$ | $17 \pm 16$ | |
| $I_{K\rho}$ | $-8.75^{+1.62\,+4.78}_{-1.66\,-6.48}$ | $-10.84^{+1.98\,+11.67}_{-2.09\,-9.09}$ | $-2.43^{+0.46\,+4.60}_{-0.42\,-19.43}$ | $-37 \pm 37$ | $-44 \pm 49(25)(11)$ |

# Improvements to theoretical predictions

- Complete the NNLO calculation of the leading-power penguin amplitude $a_p^4$.
- Compute the scalar penguin amplitude $a_p^6$ to the same precision.
- Attempt to improve the modelling of the weak-annihilation amplitudes.

# Experimental challenges: e.g., $K^{*+}(K_S\pi^+)\pi^0$

| Decay channel | $\mathcal{B}\,(10^{-6})$ |
|---|---|
| $K^0\pi^+\pi^0$ | $45.9 \pm 2.6 \pm 3.0 \pm 8.6$ |
| $K^{*0}(892)\pi^+$ | $14.6 \pm 2.4 \pm 1.3 \pm 0.5$ |
| $K^{*+}(892)\pi^0$ | $9.2 \pm 1.3 \pm 0.6 \pm 0.5$ |
| $K_0^{*0}(1430)\pi^+$ | $50.0 \pm 4.8 \pm 6.0 \pm 4.0$ |
| $K_0^{*+}(1430)\pi^0$ | $17.2 \pm 2.4 \pm 1.5 \pm 1.8$ |
| $\rho^+(770)K^0$ | $9.4 \pm 1.6 \pm 1.0 \pm 2.6$ |

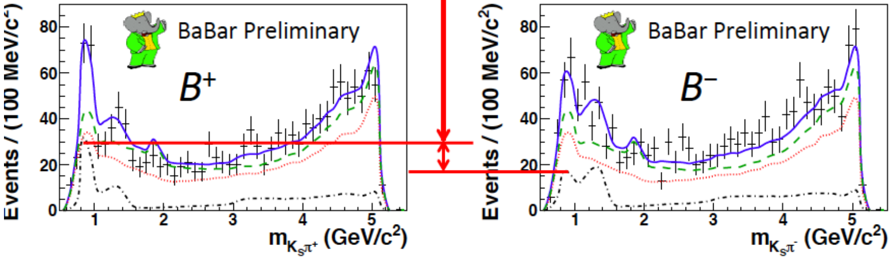



| Reference amplitude \ Resonances | Relative phases (°) $K^{*0}(892)\pi^+$ | $K^{*+}(892)\pi^0$ | $(K\pi)_0^{*0}\pi^+$ | $(K\pi)_0^{*+}\pi^0$ | $\rho^+(770)K_S^0$ |
|---|---|---|---|---|---|
| $B^+ \to K^{*0}(892)\pi^+$ | 0 | $-96 \pm 44$ | $174 \pm 11$ | $-91 \pm 43$ | $-122 \pm 38$ |
| $B^+ \to K^{*+}(892)\pi^0$ | – | 0 | $-90 \pm 42$ | $6 \pm 10$ | $-27 \pm 26$ |
| $B^+ \to (K\pi)_0^{*0}\pi^+$ | – | – | 0 | $95 \pm 42$ | $64 \pm 37$ |
| $B^+ \to (K\pi)_0^{*+}\pi^0$ | – | – | – | 0 | $-32 \pm 25$ |
| $B^+ \to \rho^+(770)K_S^0$ | – | – | – | – | 0 |

Slide credit: T. Latham, BaBar, BEACH 2014, Birmingham

- First evidence of direct CP violation in $B^+ \to K^{*+}\pi^0$
- 3.4σ significance estimated including statistical, systematic and model uncertainties
- $A_{CP}$ for $B^+ \to K^{*0}\pi^+$ consistent with zero (as expected)

| Decay channel | $A_{CP}$ |
|---|---|
| $K^0\pi^+\pi^0$ | $0.07 \pm 0.05 \pm 0.03 \pm 0.04$ |
| $K^{*0}(892)\pi^+$ | $-0.12 \pm 0.21 \pm 0.08 \pm 0.11$ |
| $K^{*+}(892)\pi^0$ | $-0.52 \pm 0.14 \pm 0.04 \pm 0.04$ |
| $K_0^{*0}(1430)\pi^+$ | $0.14 \pm 0.10 \pm 0.04 \pm 0.14$ |
| $K_0^{*+}(1430)\pi^0$ | $0.26 \pm 0.12 \pm 0.08 \pm 0.12$ |
| $\rho^+(770)K^0$ | $0.21 \pm 0.19 \pm 0.07 \pm 0.30$ |




Slide credit: T. Latham, BaBar, BEACH 2014, Birmingham

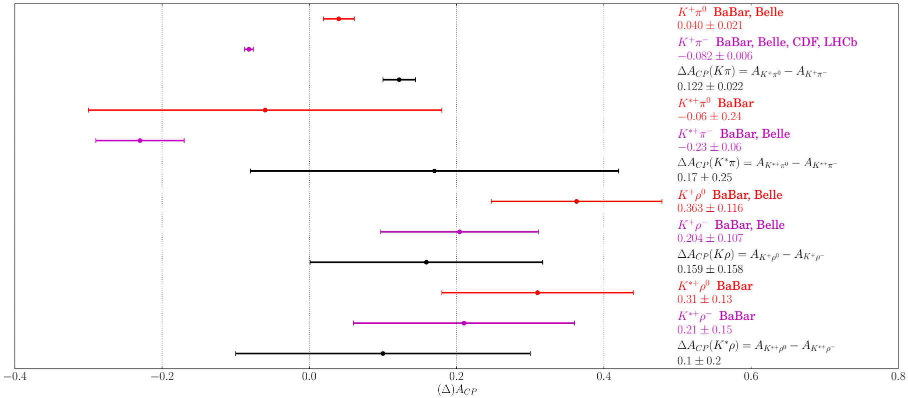
For $B \to VV$ decays, must separate out the longitudinal and transverse components:

- NNLO computation not possible for transverse amplitudes: power-suppressed and there is no QCD factorization theorem for them.
- For longitudinal component, comparison of NNLO computation to experiment not possible since $A_{CP}$ not available for individual helicity amplitudes in $K^{*+}\rho^-$.
- NLO computation available for comparison.
- Many modes still uncovered by Belle & LHCb.

| $\mathcal{B}(10^{-6})$ | | |
|---|---|---|
| Mode | BABAR | Belle |
| $K^{*+}\rho^-$ | $10.3 \pm 2.3 \pm 1.3$ | |
| $K^{*+}\rho^0$ | $4.6 \pm 1.0 \pm 0.4$ | |
| $K^{*0}\rho^+$ | $9.6 \pm 1.7 \pm 1.5$ | |
| $K^{*0}\rho^0$ | $5.1 \pm 0.6^{+0.6}_{-0.8}$ | $2.1^{+0.8+0.9}_{-0.7-0.5}$ |

| $A_{CP}$ | |
|---|---|
| Mode | BABAR |
| $K^{*+}\rho^-$ | $0.21 \pm 0.15 \pm 0.02$ |
| $K^{*+}\rho^0$ | $0.31 \pm 0.13 \pm 0.03$ |
| $K^{*0}\rho^+$ | $-0.01 \pm 0.16 \pm 0.02$ |
| $K^{*0}\rho^0$ | $-0.06 \pm 0.09 \pm 0.02$ |

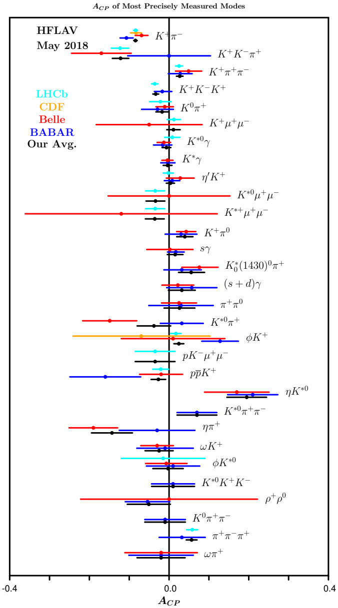
# Summary of WA $(\Delta)A_{CP}$ results for $K^{(*)}\pi$ and $K^{(*)}\rho$




*Uncertainty much improved in $K\pi$ but still too large in $K^*\pi$ and $K^{(*)}\rho$ systems to be conclusive.*

*Many Belle $\mathcal{B}$ and $A_{CP}$ measurements missing for the $PV$ and $VV$ channels. Challenging Dalitz plot and $VV$ analyses.*

# Summary of most precisely measured modes *Lots to study!*



# Aside - Why "Penguin"?

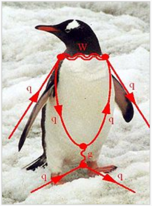
In quantum field theory, **penguin diagrams** are a class of Feynman diagrams which are important for understanding CP violating processes in the standard model. They refer to one-loop processes in which a quark temporarily changes flavor (via a W or Z loop), and the flavor-changed quark engages in some tree interaction, typically a strong one. For tree interactions where some quark flavors (e.g. very heavy ones) have much higher interaction amplitudes than others, such as CP-violating or Higgs interactions, these penguin processes may have amplitudes comparable to or even greater than those of the direct tree processes. A similar diagram can be drawn for leptonic decays.[1]

They were first isolated and studied by Mikhail Shifman, Arkady Vainshtein, and Valentin Zakharov.[2] The processes which they describe were first directly observed in 1991 and 1994 by the CLEO collaboration.

## Origin of the name [ edit ]

John Ellis was the first to refer to a certain class of Feynman diagrams as **penguin diagrams**, due in part to their shape, and in part to a legendary bar-room bet with Melissa Franklin. According to John Ellis:[3]

> Mary K. [Gaillard], Dimitri [Nanopoulos] and I first got interested in what are now called penguin diagrams while we were studying CP violation in the Standard Model in 1976... The penguin name came in 1977, as follows.
>
> In the spring of 1977, Mike Chanowitz, Mary K and I wrote a paper on GUTs predicting the b quark mass before it was found. When it was found a few weeks later, Mary K, Dimitri, Serge Rudaz and I immediately started working on its phenomenology. That summer, there was a student at CERN, Melissa Franklin who is now an experimentalist at Harvard. One evening, she, I, and Serge went to a pub, and she and I started a game of darts. We made a bet that if I lost I had to put the word penguin into my next paper. She actually left the darts game before the end, and was replaced by Serge, who beat me. Nevertheless, I felt obligated to carry out the conditions of the bet.
>
> For some time, it was not clear to me how to get the word into this b quark paper that we were writing at the time. Then, one evening, after working at CERN, I stopped on my way back to my apartment to visit some friends living in Meyrin where I smoked some illegal substance. Later, when I got back to my apartment and continued working on our paper, I had a sudden flash that the famous diagrams look like penguins. So we put the name into our paper, and the rest, as they say, is history.



Example of a penguin diagram

# Extra reading

- Richman, Jeremy D. (UCSB), *Heavy Quark Phyiscs and $CP$ Violation*.
  http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver_houches12.pdf
  Pages 171-183.
- Measurements of branching fractions and direct $CP$ asymmetries for $B \to K\pi$, $B \to \pi\pi$ and $B \to KK$ decays (Belle Collaboration, 2014)